Hierarchical Forecasting and Reconciliation in The Context of Temporal Hierarchy

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Abstract - *The purpose of this study is to find a suitable forecast aggregation strategy for forecasting temporally aggregated hierarchical data series when the base level data exhibits a seasonal pattern. The study employs 10-year monthly data of foreign tourists visited in Kerala. Forecasting is essential for the four levels of hierarchy; the monthly, quarterly, half yearly and annual foreign tourist visit data. The forecasting strategies deliberated in the project are; bottom-up approach, top-down approach, and the optimal combination approach with Ordinary least square (OLS) for reconciliation. The performance of different strategies is compared using the Mean Absolute Percentage Error (MAPE). The exponential smoothing techniques; single exponential smoothing, double exponential smoothing and triple exponential smoothing are used for forecasting individual series. The study concludes that the suitable forecast aggregation strategy for forecasting temporally aggregated hierarchical data series when the base level data exhibits a seasonal pattern is bottom-up approach. Bottom-up approach outperform all top-down approaches and optimal combination approaches on average and across all levels.*

Key Words: Temporal Hierarchy, Forecast reconciliation, Single exponential smoothing, Double exponential smoothing, Triple exponential smoothing, Bottom-up, Top-down, Optimal combination.

1. INTRODUCTION

Demand forecasting plays a crucial role in the operations of modern organizations. It supports a variety of business decisions, from operational, to tactical, to strategic level, such as capacity planning, resource planning, advertising and promotional planning, demand planning, analyzing competition effects, tactical production planning, among others. Forecasting is valuable to businesses because it gives the ability to make informed business decisions and develop data-driven strategies. Financial and operational decisions are made based on current market conditions and predictions on how the future looks. Past data is aggregated and analyzed to find patterns, used to predict future trends and changes. Forecasting allows your company to be proactive instead of reactive. Accordingly, practitioners need to define the forecast objective in terms of forecast horizon and time bucket (e.g., daily, weekly, monthly, quarterly, half yearly, etc.), so as to support the appropriate decisions. Forecasting is the process of making predictions of the future based on past and present data and analysis of trends. A commonplace example might be estimation of some variable of interest at some specified future date. Prediction is a similar, but more general term. Forecasting is a problem that arises in many economic and managerial contexts, and hundreds of forecasting procedures have been developed over the years, for many different purposes, both in and outside of business enterprises. Accurate demand forecasts lead to efficient operations and high levels of customer service, while inaccurate forecasts will inevitably lead to inefficient, high-cost operations and/or poor levels of customer service. Risk and uncertainty are central to forecasting and prediction; it is generally considered good practice to indicate the degree of uncertainty attaching to forecasts. In any case, the data must be up to date in order for the forecast to be as accurate as possible. The recognition of the best forecasting techniques is very crucial in the forecasting field, since the forecasts are used to drive the budget, distribution, and production planning processes.

The main objectives of the current study are as follows:

- To develop a set of aggregation strategies to forecast the demand of product hierarchies based on temporal (time) aggregation corresponding to a specific class of data.

    1. Identify different forecasting techniques through literature survey.

    2. Find the appropriate hierarchical forecast reconciliation for the available data set.

    3. Perform Bottom up and Top-down approaches of forecasting.

    4. Compare the forecast approaches and develop a set of aggregation strategies.

The remainder of the paper is organized as follows. chapter 2 describes the most popular HF methods found in the literature, while Section 3 presents the proposed forecasting technique and reconciliation approach. Section 4 presents the results and findings. Section 5 concludes the work.

2. LITERATURE REVIEW

The journals and publications related to hierarchical forecasting are referred and the findings from each journal are tabulated in Table. 1.

Table. 1: List of Journals

SL. NO	TOPIC & AUTHOR	TYPE OF DATA & MODEL	FORECASTING STRATEGIES	FORECASTING TECHNIQUES	ERROR/PERFORMANCE MEASURES
1	Andrea Silvestrini & David Veredas (2008), Temporal aggregation of univariate and multi variate time series models: A survey, Journal of Economic Surveys	Emperical data	Temporal aggregation	AR, ARIMA, ARIMA with seasonality, ARIMAX and GARCH models.	MAE, RMSE, & MAPE
2	Rob J Hyndman, Roman A Ahmed, George Athanasopoulos,& Han Lin Shang (2010), Optimal combination forecasts for hierarchical time series	Simulated data (ARIMA) &Emperical data	Top-down approach, Bottom-up approach, & Optimal combination	ARIMA method, ANOVA model	MAE, RMSE, & MAPE
3	Alysha M. DE LIVERA, Rob J. HYNDMAN, and Ralph D. SNYDER (2012), Forecasting Time Series With Complex Seasonal Patterns Using Exponential Smoothing, Journal of the American Statistical Association	Emperical data	Top-down approach, Bottom-up approach	Exponential smoothing, ARMA, Box-Cox transformation	RMSE
4	Bahman Rostami-Tabar, M. Zied Babai, Aris Syntetos, & Yves Ducq (2013), Demand Forecasting by Temporal Aggregation	Emperical data	Top-down approach, Bottom-up approach	Single exponential smoothig, AR, MA, ARMA, ARIMA	MSE(Mean square d error)
5	Nikolaos Kourentzes, & Fotios Petropoulos (2016), Forecasting with multivariate temporal aggregation: The case of promotional modelling, Int. J. Production Economics	Emperical data	Top-down approach, Bottom-up approach, & Optimal combination	Exponential smoothing, MAPA(Multiple Aggregation Prediction Algorithm)	Scaled Mean Error (sME) and Scaled Mean Absolute Error (sMAE)
6	George Athanasopoulos, Rob J. Hyndman, Nikolaos Kourentzes, & Fotios Petropoulos(2017), Forecasting with Temporal Hierarchies	Simulated data(ARIMA) & Emperical data	Bottom-up approach & Top-down approach	ARIMA	RMAE, MASE
7	Bahman Rostami-Tabar, M. Zied Babai, Mohammad Ali, & John E. Boylan (2019), The impact of temporal aggregation on supply chains with ARMA(1,1) demand processes, European Journal of Operational Research	Emperical data	Bottom-up approach & Top-down approach	Single exponential smoothing, ARMA	Minimum mean squared error(MMSE), Bullwhip effect
8	Sushil Punia, Surya P. Singh, & Jitendra K. Madaan (2020), A cross-temporal hierarchical framework and deep learning for supply chain forecasting, Computers & Industrial Engineering	Emperical data	Top-down approach & Bottom-up approach	Deep learning technique (Long short term memory [LSTM] networks)	MAE, RMSE, & MAPE
9	Peter Nystrupa, Erik Lindströmb, Pierre Pinsonc, & Henrik Madsen (2020), Temporal hierarchies with autocorrelation for load forecasting, European Journal of Operational Research	Simulated data & Emperical data	Bottom-up approach	Autocovariance matrix within each aggregation level, Markov structure, GLASSO(graphical least absolute shrinkage and selection operator), & Shrinkage estimator of the cross-	RMSE, RMSSE, RMSPE, RRMSE

				correlation matrix.	
10	Filotas Theodosiou, &Nikolaos Kourentzes (2021), Deep Learning Temporal Hierarchies for Interval Forecasts, International Conference on AI in Finance	Emperical data	Bottom-up approach	Deep Temporal hierarchical forecasting (LSTM [Long short term memory networks])	MAE, RMSE, & MAPE

Test data are used for identifying and comparing the forecasting techniques. Generally, error measures such as RMSE, MAPE, and MAE are used in the papers related to temporal aggregation. The forecasting techniques generally used are Exponential smoothing, AR, MA, ARMA, and ARIMA models. Few studies are included the reconciliation techniques on temporal forecasting. The characteristics of demand can be significant in selecting appropriate forecasting strategies. Top-down and Bottom-up approaches are common strategies used for temporal hierarchical forecasting. An optimal combination method outperforms the bottom-up and top-down approach. In hierarchical forecasting the base forecasts are aggregated to top level by using Bottom-up approach, and allocated downwards using Top-down approach.

3. METHODOLOGY

The data collection is an important step for this study. Data selected should be analyzed to know the nature of the data. Trend, seasonality, cyclic, irregularity, etc. of the data should be check before forecasting. The nature of the data gives the idea of what kind of techniques should follow for forecasting. For the study the seasonality index is calculated and plotted the graph of the data.

From Kerala tourism statistics, the data of number of foreign tourists visited in Kerala are available. The monthly data up to 2019 is available. For this study, over 10-year data of number of foreign tourists visited in Kerala are selected. The data divided in to training data and test data for calculations. The first 6-year data (2010 – 2015) are taken as training set and the next 4-year data (2016 – 2019) are taken as test data. The test data are used for validation. Comparison of different forecasting approaches are performed using the error variation in the forecasted value and actual value of test data set.

The data analysis is carried out by plotting the graph of the data set and by ACF and PACF plots. For temporal aggregation the data is hierarchically represented as Monthly, Quarterly, Half yearly, and Yearly. The hierarchy used in this study is shown in fig. 1. The base level (level 3) consists of 12 months data (M1, M2, M3, M4, M5, M6, M7, M8, M9, M10, M11, M12). Level 2 consists of 4 quarters data (Q1, Q2, Q3, Q4). Level 1 consists of 2 half years data (H1, H2). The top level (level 0) consists of yearly data. These are represented in a hierarchy.

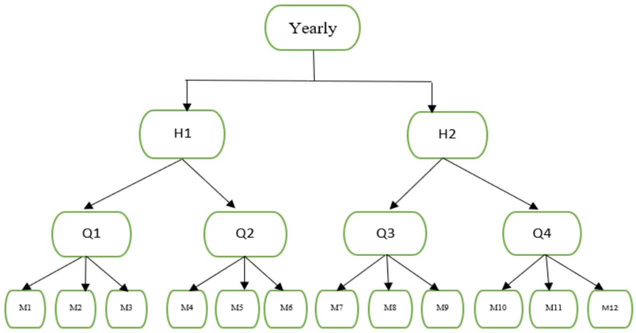

Fig -1: Temporal hierarchy for this study

M represents the months, Q for quarters and H for half years. From the hierarchy M1, M2, and M3 add up to get Q1. Similarly, all levels of data were generated using the base level.

The nature of the data set can be understood from the corresponding plots. For monthly, quarterly, and half yearly data, the graph shows a seasonality. The seasonal data can't use for single and double exponential smoothing techniques. So, the data are deseasonalized by dividing the data with corresponding seasonality index. For calculating the seasonality index the training data set are used. The plots for monthly, quarterly, and half yearly data are shown in fig. 2.

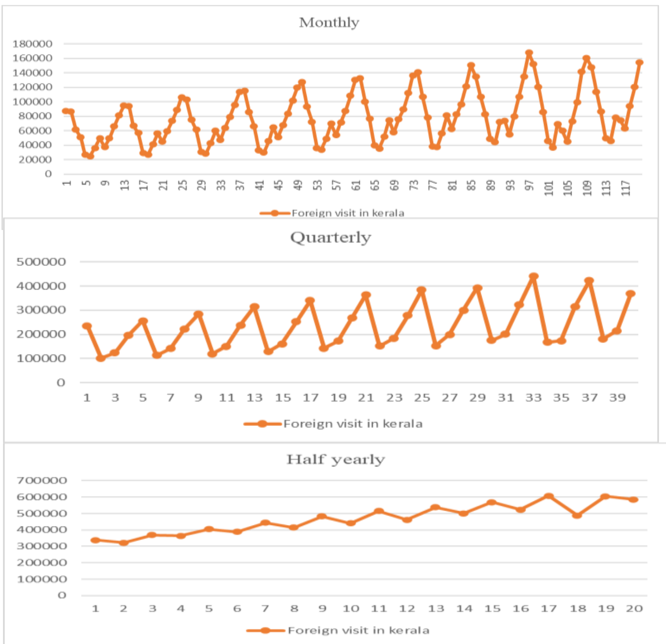

Fig -2: Plot of monthly, quarterly, and half yearly data (Number of foreign tourists visited vs time)

The deseasonalized data plot for monthly, quarterly, and half yearly are shown in fig. 3.

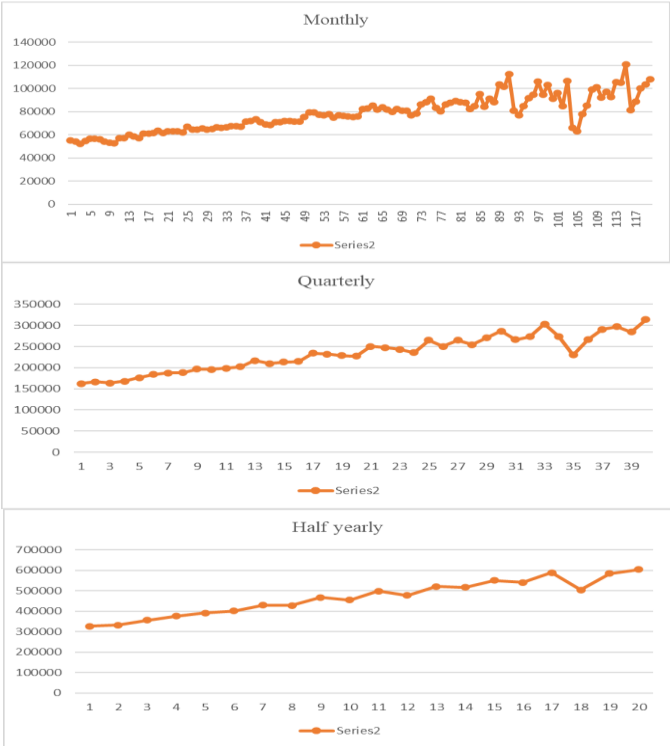

Fig -3: Plot of deseasonalised data of monthly, quarterly, and half yearly (Number of foreign tourists visited vs time)

The yearly data is seeming to be non-seasonal. The plot of yearly data shows a trend line. Triple exponential smoothing technique is only applicable for seasonal data. Here triple exponential smoothing can't be applicable for yearly data. The plot of yearly data is shown in fig. 4.

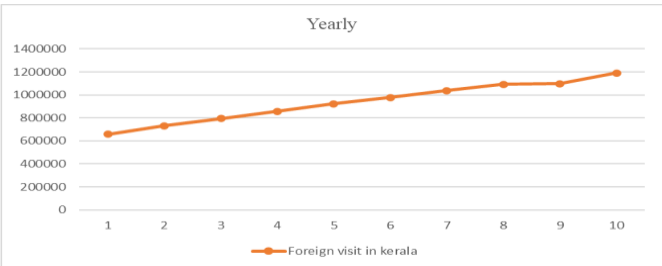

Fig -4: Plot of yearly data (Number of foreign tourists visited vs time)

3.1 Forecasting techniques

Single exponential smoothing, double exponential smoothing, and triple exponential smoothing techniques are used for forecasting. The forecasts of individual nodes in each level were forecasted using the three methods. The individual forecasts then reconciled to generate more accurate forecast. Single, double, and triple exponential smoothing are done using the forecast equations. α, ß, and Γ values are set for choosing best forecast.

SES assumes a fairly steady time-series data with no significant trend, seasonal or cyclical component. Here, the weights assigned to past data decline exponentially with the most recent observations assigned higher weights.

In single ES, the forecast at time (t + 1) is given by (Winters, 1960)

$$F_{t+1} = \alpha Y_t + (1-\alpha)F_t$$

Parameter α is called the smoothing constant and its value lies between 0 and 1. Since the model uses one smoothing constant, it is called single exponential smoothing.

Using the equation, the forecasts for test data are calculated. For each node in each level, the forecasts are calculated. The forecast error measured using MAPE and RMSE calculations.

Holt's method of double exponential smoothing is used in this study. One of the drawbacks of single exponential smoothing is that the model does not do well in the presence of trend. This can be improved by introducing an additional equation for capturing the trend in the time-series data. Double exponential smoothing uses two equations to forecast the future values of the time series, one for forecasting the level (short term average value) and another for capturing the trend.

Level (or Intercept) equation (Lt):

$$L_t = \alpha Y_t + (1-\alpha)F_t$$

he trend equation is given by (Tt):

$$T_t = ß(L_t - L_{t-1}) + (1-ß)T_{t-1}$$

α and ß are the smoothing constants for level and trend, respectively, and $0 < \alpha < 1$ and $0 < ß < 1$.

In this study, Holt-Winter model of triple exponential smoothing are used for calculations. Single and double exponential smoothing techniques discussed so far can handle data as long as the data do not have any seasonal component associated with it. However, when there is seasonality in the time-series data, techniques such as moving average, exponential smoothing, and double exponential smoothing are no longer appropriate. In most cases, the fitted error values (actual demand minus forecast) associated with simple exponential smoothing and Holt's method will indicate systematic error patterns that reflect the existence of seasonality. For example, presence of seasonality may result in all positive errors, except for negative values that occur at fixed intervals. Such pattern in error would imply existence of seasonality. Such time series data require the use of a seasonal method to eliminate the systematic patterns in error.

Triple exponential smoothing is used when the data has trend as well as seasonality. The following three equations which account for level, trend, and seasonality are used for forecasting (for a multiplicative model, Winters 1960):

Level (or Intercept) equation:

$$L_t = \alpha(Y_t/S_{t-c}) + (1-\alpha)[L_{t-1} + T_{t-1}]$$

Trend equation:

$$T_t = ß(L_t - L_{t-1}) + (1-ß)T_{t-1}$$

Seasonal equation:

$$S_t = \Gamma(Y_t/L_t) + (1-\Gamma)S_{t-c}$$

The forecast F_{t+1} using triple exponential smoothing is given by:

$$F_{t+1} = [L_t + T_t] * S_{t-c}$$

Where c is the number of seasons (if it is monthly seasonality, then c = 12; in case of quarterly seasonality c = 4; and in case of daily data c = 7).

3.2 Forecasting approaches for hierarchical and grouped time series

This study mainly focused on Bottom-up, Top-down and Optimal combination approaches. The Optimal combination approach used in this study are discussed in section 3.1 and section 3.3.

Bottom-up (BU) approach first generates the base forecasts in the bottom level of the forecasting structure, using a forecasting model. All other forecasts in the structure are generated through aggregating of the base forecast to the higher levels, in a manner which is consistent with the observed data structure. In this study, the calculations used for bottom-up approaches are;

$\hat{y}_{Q1,h} = \hat{y}_{M1,h} + \hat{y}_{M2,h} + \hat{y}_{M3,h}$

$\hat{y}_{Q2,h} = \hat{y}_{M4,h} + \hat{y}_{M5,h} + \hat{y}_{M6,h}$

$\hat{y}_{Q3,h} = \hat{y}_{M7,h} + \hat{y}_{M8,h} + \hat{y}_{M9,h}$

$\hat{y}_{Q4,h} = \hat{y}_{M10,h} + \hat{y}_{M11,h} + \hat{y}_{M12,h}$

$\hat{y}_{H1,h} = \hat{y}_{M1,h} + \hat{y}_{M2,h} + \hat{y}_{M3,h} + \hat{y}_{M4,h} + \hat{y}_{M5,h} + \hat{y}_{M6,h}$

$\hat{y}_{H2,h} = \hat{y}_{M7,h} + \hat{y}_{M8,h} + \hat{y}_{M9,h} + \hat{y}_{M10,h} + \hat{y}_{M11,h} + \hat{y}_{M12,h}$

$\hat{y}_{A,h} = \hat{y}_{M1,h} + \hat{y}_{M2,h} + \hat{y}_{M3,h} + \hat{y}_{M4,h} + \hat{y}_{M5,h} + \hat{y}_{M6,h} + \hat{y}_{M7,h} + \hat{y}_{M8,h} + \hat{y}_{M9,h} + \hat{y}_{M10,h} + \hat{y}_{M11,h} + \hat{y}_{M12,h}$

The top-down approach aims to perform a forecast for the top level of the hierarchy ($\hat{y}_h$), and then disaggregate it to the different nodes by using a predefined proportion. The most common approach is the use of the average for each node, j, relative to its "parent" node as a proportion. The equation for selecting the proportion (p_j) is given by;

For the hierarchy shown in Fig. 3, the predictions for the different nodes based on $\hat{y}_h$ are given by:

$$p_j = \frac{1}{T}\sum_{t=1}^{T}\frac{y_{j,t}}{y_t}$$

$\hat{y}_{H1,h} = p_{H1} \cdot \hat{y}_{A,h}$

$\hat{y}_{H2,h} = p_{H2} \cdot \hat{y}_{A,h}$

$\hat{y}_{Q1,h} = p_{Q1} \cdot \hat{y}_{A,h}$

$\hat{y}_{Q2,h} = p_{Q2} \cdot \hat{y}_{A,h}$

$\hat{y}_{Q3,h} = p_{Q3} \cdot \hat{y}_{A,h}$

$\hat{y}_{Q4,h} = p_{Q4} \cdot \hat{y}_{A,h}$

$\hat{y}_{M1,h} = p_{M1} \cdot \hat{y}_{A,h}$

$\hat{y}_{M2,h} = p_{M2} \cdot \hat{y}_{A,h}$

$\hat{y}_{M3,h} = p_{M3} \cdot \hat{y}_{A,h}$

$\hat{y}_{M4,h} = p_{M4} \cdot \hat{y}_{A,h}$

$\hat{y}_{M5,h} = p_{M5} \cdot \hat{y}_{A,h}$

$\hat{y}_{M6,h} = p_{M6} \cdot \hat{y}_{A,h}$

$\hat{y}_{M7,h} = p_{M7} \cdot \hat{y}_{A,h}$

$\hat{y}_{M8,h} = p_{M8} \cdot \hat{y}_{A,h}$

$\hat{y}_{M9,h} = p_{M9} \cdot \hat{y}_{A,h}$

$\hat{y}_{M10,h} = p_{M10} \cdot \hat{y}_{A,h}$

$\hat{y}_{M11,h} = p_{M11} \cdot \hat{y}_{A,h}$

$\hat{y}_{M12,h} = p_{M12} \cdot \hat{y}_{A,h}$

Using these equations, corresponding forecasts are calculated.

3.3 Reconciliation

Reconciliation is the process of making the forecasts coherent. In this study, the reconciliation on hierarchical forecast is done using OLS (Ordinary least square) method. Reconciliation to hierarchical forecasting gives optimal combination approach. The OLS reconciliation technique is introduced by using programming language of python.

The convenient general matrix representation is;

$$Y_t = Sb_t$$

where S is a "summing matrix" of order m × n which aggregates the bottom level series to the series at aggregation levels above. For this study, the matrix representation is;

$$\begin{pmatrix} Ya \\ Yh1 \\ Yh2 \\ Yq1 \\ Yq2 \\ Yq3 \\ Yq4 \\ Ym1 \\ Ym2 \\ Ym3 \\ Ym4 \\ Ym5 \\ Ym6 \\ Ym7 \\ Ym8 \\ Ym9 \\ Ym10 \\ Ym11 \\ Ym12 \end{pmatrix} = \begin{matrix} 1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1&1&1&1&1 \\ 1&1&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&1&1 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{matrix} \begin{pmatrix} Ym1 \\ Ym2 \\ Ym3 \\ Ym4 \\ Ym5 \\ Ym6 \\ Ym7 \\ Ym8 \\ Ym9 \\ Ym10 \\ Ym11 \\ Ym12 \end{pmatrix}$$

Here the summing matrix in in the order of 19 x 12. The reconciliation is conducted by introducing a mapping matrix (P) in to the equation.

The hierarchical forecasting with reconciliation can then be written as,

$$\tilde{Y}_{n(h)} = SP\hat{Y}_{n(h)}$$

For some appropriately chosen matrix P. That is, existing methods involve linear combinations of the base forecasts. These linear combinations are "reconciled" in the sense that lower-level forecasts sum to give higher level forecasts. The effect of the P matrix is to extract and combine the relevant elements of the base forecasts $\hat{Y}_{n(h)}$, which are then summed by S to give the final revised hierarchical forecasts, $\tilde{Y}_{n(h)}$.

For OLS reconciliation;

$$P = (S'S)^{-'} S'$$

These equations are programmed in python. The hierarchical forecasts of each level are reconciled using python software. The reconciled forecasts then used for comparison to find optimum strategy.

Mean absolute percentage error (MAPE) is the average of absolute percentage error. Assume that the validation data has n observations and the forecasting is carried out on these n observations. The mean absolute percentage error is given by;

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n} \frac{|Yt-Ft|}{Yt} *100$$

The equation is used for measuring the accuracy of the forecast. The forecast with minimum MAPE value is taken as best forecasting method. The MAPE value of each level are used for comparing the forecasting techniques and to develop a set of aggregation strategy. The calculations of MAPE of each level in each forecasting methods and approaches are tabulated in appendices.

4.RESULT AND DISCUSSION

Over 10-year data of foreign tourists visited in Kerala are taken for this study. The data divided in to training data set of 6 year and a test data set of 4 year. The test data set is used for validation. Corresponding MAPE values for each approach for the test data set were calculated. The approach with minimum MAPE value is taken as optimum approach.

Individual forecast of each node in each level are calculated and the corresponding MAPE are measured. The comparison of each forecasting techniques using MAPE value is discussed in Table 2.

Table 2: Base forecasts of all levels

LEVELS	SES	DES	TES
Level 3	7.82628	7.2704	**6.10305**
Level 2	6.87958	6.15453	**6.12763**
Level 1	6.4065	6.0496	**3.787**
Level 0	5.7073	**5.236**	

Where, SES – Single exponential smoothing, DES – Double exponential smoothing, and TES – Triple exponential smoothing.

From the study, the best forecasting technique for each level is identified and it is listed below:

- For base forecast of level 3, Triple exponential smoothing is the best forecasting technique.
- For base forecast of level 2, Triple exponential smoothing is the best forecasting technique.
- For base forecast of level 1, Triple exponential smoothing is the best forecasting technique.
- For base forecast of level 0, Double exponential smoothing is the best forecasting technique.

Single exponential smoothing, Double exponential smoothing, and Triple exponential smoothing techniques of forecasting are used in Bottom-up, Top-down, and Optimal combination approaches. The minimum MAPE value among three techniques are used for comparison.

The comparison of each forecasting approaches for all levels of hierarchy using MAPE value is discussed in Table 3.

Table 3: MAPE comparison of all levels of hierarchy

FORECASTING APPROACH	Level 3	Level 2	Level 1	Level 0
BU	**5.63623**	**4.8478**	**5.3737**	3.5254
TD	9.04322	5.4986	5.4414	5.2360
OC	8.03957	5.58133	5.3777	**3.4781**

Where; BU – Bottom-up, TD – Top-down, and OC – Optimal combination.

From the study, the best forecasting technique for each level is identified and it is listed below:

- For level 3, Bottom-up approach is the best forecasting strategy.
- For level 2, Bottom-up approach is the best forecasting strategy.
- For level 1, Bottom-up approach is the best forecasting strategy.
- For level 0, Optimal combination approach is the best forecasting strategy.

For identifying the best strategy, the MAPE value is used.

The study helps to develop a set of aggregation strategy to forecast the number of foreign tourists visited in Kerala for the period of 2016 to 2019 in hierarchies based on temporal (time) aggregation corresponding to a specific class of data. Monthly, Quarterly, Half yearly, and Yearly forecasts are calculated using the developed model for the test period and it is validated using actual data.

5. CONCLUSIONS

The main objective of the study was to develop a set of aggregation strategy to forecast the number of foreign tourists visited in Kerala in hierarchies based on temporal (time) aggregation corresponding to a specific class of data. In this study, the test data set is from 2016 to 2019. For the developed hierarchy, the best forecasting methods are identified in each node of each level and the best forecasting strategy are identified in each level. That is, the optimum forecast and strategy for all months, all quarters, all half years, and all years of test data set are developed. The approaches used are Bottom-up, Top-down and Optimal combination. The forecasting techniques used are single exponential smoothing, double exponential smoothing, and triple exponential smoothing. MAPE (error measure) is used for finding the best forecasting approach and best forecasting technique.

From the study, for the tourist visit data set the suitable forecasting technique is triple exponential smoothing. Triple exponential smoothing technique is suited for seasonal data set. The yearly tourists visit data shows only a trend. So, double exponential smoothing is more appropriate for yearly tourists visit data. The study concludes that the suitable forecast aggregation strategy for forecasting temporally aggregated hierarchical data series when the base level data exhibits a seasonal pattern is bottom-up approach. Bottom-up approach outperform all top-down approaches and optimal combination approaches on average and across all levels.

There are different forecasting techniques like AR, MA, ARMA, ARIMA, Single exponential smoothing, double exponential smoothing, triple exponential smoothing, etc. This study focused on the exponential smoothing methods according to the literature. It is one of the limitations of the study. Similarly, there are different reconciliation techniques like OLS, WLS, MinTrace, etc. For this study, the OLS method of reconciliation is used. It can also be a limitation of the study. In addition to Bottom-up and Top-down approaches there are middle-out and optimal combination approach are there. In this study, the middle-out and optimal combination approaches are not used. It is also a limitation to the study. Forecasting in temporal hierarchy needs long period of data. The data taken for the study is only over 10 – years.

The future work of the study relies on different areas. Forecasting for intermittent demand data and forecasting for seasonal data using SARIMA models can be included in this work in future. Covid pandemic period plays badly on tourism sector. So, the forecasting will be effective in 3 different time periods. That is, pre-covid period (up to 2019), Pandemic period (from 2020), and a post-covid period. The forecasting on any product demand will have an effect due to the covid pandemic period. For future works, the time period of data collection is very important.

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