

# Parametric amplification of optical phonons in magnetoactive III-V semiconductors

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**Abstract** - Using the hydrodynamic model of semiconductor plasmas and following the coupled mode approach, the parametric amplification of optical phonons is analytically investigated in magnetized doped III-V semiconductors. The origin of nonlinear interaction is assumed to lie in the complex effective second-order optical susceptibility arising from the nonlinear polarization created by induced current density and by interaction of the pump wave with molecular vibrations generated within the medium. Expressions are obtained for threshold pump amplitude for the onset of parametric process and parametric gain coefficient (well above the threshold pump field). Numerical analysis is made for a representative n-InSb crystal irradiated by 10.6  $\mu\text{m}$  pulsed  $\text{CO}_2$  laser. The proper selection of doping concentration and externally applied magnetic field (around resonance conditions) lowers the threshold pump amplitude and enhances the gain coefficient for the onset of parametric process. The analysis confirms the chosen nonlinear medium as a potential candidate material for the fabrication of parametric devices like parametric amplifiers and oscillators.

**Key Words:** Nonlinear optics, Optical parametric amplification, Optical phonons, III-V semiconductors

## 1. INTRODUCTION

Nonlinear optics is the branch of optics that describes the laser-matter interaction. As a consequence of this interaction, there arise phenomena such as parametric interactions, modulation interactions, stimulated scatterings etc [1, 2]. Among these, parametric interaction is the fundamental interaction. It is a second order nonlinear optical effect (i.e. the origin of this interaction lies in second-order optical susceptibility  $\chi^{(2)}$  of the medium). In this process, an intense laser beam (continuous or pulsed type), hereafter referred as 'pump', interacts with nonlinear medium and results into generation of waves at new frequencies [3]. This occurs due to mixing or controlled splitting of waves which may undergo amplification or attenuation depending on the material properties and geometry of externally applied electric and magnetic fields.

In a nonlinear medium, the dissection of superposition principle leads to interaction among waves of different frequencies. There exist a number of nonlinear interactions which can be allocated as parametric interaction of coupled modes. Parametric interaction of coupled modes are the

nonlinear optical phenomena in which pump field energy is transferred to the generated waves by a resonant mechanism under the condition that the pump field amplitude is large enough to cause the vibrations of certain physical parameters of the medium [4].

The phenomena of parametric interactions have played a distinctive role in nonlinear optics. Parametric processes have been widely used to generate tunable coherent radiation at a frequency that is not directly available from a laser source; these frequency conversion techniques provide an important means of extending the spectral range covered by coherent sources [5, 6]. Optical parametric amplifiers, optical parametric oscillators, optical phase conjugation, pulse narrowing, squeezed state generation etc. are some of the outcomes of optical parametric interactions in a nonlinear medium. Among these, optical parametric amplifiers are of special interest due to their vast applications in science and technology [7, 8].

While surveying ongoing global research activities on optical parametric amplification, it has been observed that the manipulations of threshold pump field and gain coefficient have been important issues to improve the efficiency and functionality of optical parametric amplifiers due to unavailability of desired nonlinear optical media. Among the various nonlinear optical materials, the doped III-V semiconductor crystals are advantageous hosts for fabrication of optical parametric amplifiers.

Up to now, the optical parametric amplification caused by optically excited coherent collective modes, in III-V semiconductor crystals have been reported by research groups of Singh et.al [12, 13], Bhan et.al. [14] and Ghosh et.al. [15]. It appears from available literature that no theoretical formulation has been developed till now to study the optical parametric amplification in magnetized doped III-V semiconductors like InSb, GaAs, GaSb, InAs etc. with optical phonons acting as the idler wave. Such crystal classes are usually partially ionic and, therefore, piezoelectric scattering is overshadowed by optical phonon scattering mechanisms [16]. The study of the propagation characteristics of coherent optical phonons are of significance important in the study of fundamental properties of crystals. The study of laser-longitudinal optical phonon interactions in III-V semiconductors is currently one of the most active fields of research due to its vast potentiality in fabrication of optoelectronic devices.

Keeping in view the possible impact of parametric interactions involving an optical phonon mode, we present here an analytical study of optical parametric amplification using the n-type doped III-V semiconductor crystals immersed in a large magnetostatic field following the hydrodynamic model for the semiconductor-plasma. The influences of material parameters and externally applied magnetostatic field on threshold pump amplitude and gain coefficient of parametric process have been studied in detail. Weakly polar n-type III-V semiconductor crystals have been chosen as nonlinear optical materials subjected to a pump wave. The pump photon energy is taken well below the band-gap energy of the sample. This allows the optical properties of the sample to be influenced considerably by free carriers and keeps it unaffected by the photo-induced inter-band transition mechanisms [17-19]. Using the coupled mode theory for semiconductor plasma, the complex effective second-order optical susceptibility  $\chi_{eff}^{(2)}$  of the crystal and consequent threshold pump amplitude  $E_{0,th}$  for the onset of parametric process and parametric gain coefficient  $g_{para}$  well above the threshold field ( $E_0 > E_{0,th}$ ) are obtained.

## 2. THEORETICAL FORMULATIONS

Let us consider the hydrodynamic model of an n-type semiconductor plasma. This model proves to be suitable for the present study as it simplifies our analysis, without any loss of significant information, by replacing the streaming electrons with an electron fluid described by a few macroscopic parameters like average carrier density, average velocity, etc. However, it restricts our analysis to be valid only in the limit ( $k_{op}l \ll 1$ ;  $k_{op}$  the optical phonon wave number, and  $l$  the carrier mean free path).

In order to obtain an expression for  $\chi_{eff}^{(2)}$ , the three-wave coupled mode scheme has been employed [20]. The origin of  $\chi_{eff}^{(2)}$  lies in coupling between the pump and signal waves via density perturbations in the crystal. Let us consider the parametric coupling among three waves: (i) the input strong pump wave  $E_0(x, t) = E_0 \exp [i(k_0x - \omega_0t)]$ , (ii) the induced optical phonon mode (idler)  $u(x, t) = u_0 [i(k_{op} - \omega_0t)]$ , and (iii) the scattered Stokes component of pump electromagnetic wave (signal)  $E_s(x, t) = E_s \exp [i(k_sx - \omega_s t)]$ .

The momentum and energy conservation relations for these modes should satisfy the phase matching conditions:  $\hbar\vec{k}_0 = \hbar\vec{k}_s + \hbar\vec{k}_{op}$  and  $\hbar\omega_0 = \hbar\omega_s + \hbar\omega_{op}$ . We consider the semiconductor crystal to be immersed in a transverse static magnetic field  $\vec{B}_0 = \hat{z}B_0$  (i.e. perpendicular to the direction of input pump beam).

In a weakly polar III-V semiconductor, the scattering of high frequency pump wave is enhanced due to excitation of a

normal vibrational (optical phonon) mode. We consider that the semiconductor medium consists of  $N$  harmonic oscillators per unit volume; each oscillator being characterized by its position  $x$ , molecular weight  $M$  and normal vibrational coordinates  $u(x, t)$ .

The equation of motion for a single oscillator (optical phonon) is given by [21]

$$\frac{\partial^2 u}{\partial t^2} + \Gamma \frac{\partial u}{\partial t} + \omega_l^2 u = \frac{F}{M}, \quad (1a)$$

where  $\Gamma$  is the damping constant equal to the phenomenological phonon-collision frequency ( $\sim 10^{-2} \omega_l$ ) [22];  $\omega_l$  being the un-damped molecular vibrational frequency and is taken to be equal to the transverse optical phonon frequency.  $F$  is the driving force per unit volume experienced by the medium can be put forward as:  $F = F^{(1)} + F^{(2)}$ , where  $F^{(1)} = q_s E$  and  $F^{(2)} = 0.5\epsilon\alpha_u \bar{E}^2(x, t)$  represent the forces arising due to Szigei effective charge  $q_s$  and differential polarizability  $\alpha_u = (\partial\alpha/\partial u)_0$  (say), respectively.  $\epsilon = \epsilon_0\epsilon_\infty$ ;  $\epsilon_0$  and  $\epsilon_\infty$  are the absolute and high frequencies permittivities, respectively. After substituting the value of  $F$ , the modified equation of motion for  $u(x, t)$  of molecular vibrations in a semiconductor crystal is given by

$$\frac{\partial^2 u}{\partial t^2} + \Gamma \frac{\partial u}{\partial t} + \omega_l^2 u = \frac{1}{M} \left[ q_s E + \frac{1}{2} \epsilon\alpha_u \bar{E}^2(x, t) \right]. \quad (1b)$$

The other basic equations in the formulation of  $\chi_{eff}^{(2)}$  are:

$$\frac{\partial \vec{v}_0}{\partial t} + \vec{v}_0 \cdot \nabla \vec{v}_0 = -\frac{e}{m} [\vec{E}_0 + (\vec{v}_0 \times \vec{B}_0)] = -\frac{e}{m} \vec{E}_{eff} \quad (2)$$

$$\frac{\partial \vec{v}_1}{\partial t} + \vec{v}_1 \cdot \nabla \vec{v}_1 + \left( \vec{v}_0 \cdot \frac{\partial}{\partial x} \right) \vec{v}_1 = -\frac{e}{m} [\vec{E}_1 + (\vec{v}_1 \times \vec{B}_1)] \quad (3)$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} + n_1 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial n_1}{\partial x} = 0 \quad (4)$$

$$\dot{P}_{mv} = \epsilon N \alpha_u u^* \dot{E}_{eff} \quad (5)$$

$$\frac{\partial E_{1x}}{\partial x} + \frac{1}{\epsilon} \frac{\partial}{\partial x} \left( \dot{P}_{mv} \right) = -\frac{n_1 e}{\epsilon} \quad (6)$$

These equations are well described in Ref. [12-15]. The molecular vibrations at frequency  $\omega_{op}$  causes a modulation of the dielectric constant of the medium leading to an exchange of energy between the electromagnetic fields separated in frequency by multiples of  $\omega_{op}$  (i.e.,  $(\omega_0 \pm p\omega_{op})$ , where  $p = 1, 2, 3, \dots$ ). The modes at frequencies  $\omega_0 + p\omega_{op}$  are known as anti-Stokes modes; while those at  $\omega_0 - p\omega_{op}$  are Stokes modes. In the forthcoming formulation, we will consider only the first-order Stokes component of the back-scattered electromagnetic wave.

The high frequency pump field gives rise to a carrier density perturbation, which in turn derives an electron-plasma wave and induces current density in the semiconductor medium. Now the perturbed electron density ( $n_{op}$ ) of the semiconductor medium due to molecular vibrations can be deduced from equations (1) – (6) as:

$$n_{1op} = \frac{2iMk_{op}(\omega_l^2 - \omega_{op}^2 + i\omega_{op}\Gamma)u^*}{e\alpha_u(E_{eff})_x^*} - \frac{i\epsilon Nk_{op}u^*}{e\alpha_u(E_{eff})_x^*} \left\{ \frac{2q_s\alpha_u}{\epsilon} - \alpha_u^2 |(E_{eff})_x|^2 \right\}. \quad (7)$$

The density perturbation associated with the molecular vibrations at frequency  $\omega_{op}$  beats with the pump at frequency  $\omega_0$  and produces fast components of density perturbations. The Stokes mode of this component at frequency  $\omega_s = \omega_0 - \omega_{op}$  is obtained as:

$$n_{1s} = \frac{ie(k_0 - k_{op})(E_{eff})_x}{m(\Omega_{rs}^2 - i\omega_s)} n_{1op}^*. \quad (8)$$

In Eq. (8),  $\Omega_{rs}^2 = \bar{\omega}_r^2 - \omega_s^2$ , where

$$\bar{\omega}_r^2 = \omega_r^2 \left( \frac{v^2 + \omega_{cx}^2}{v^2 + \omega_c^2} \right), \text{ in which } \omega_{cx,z} \left( = \frac{e}{m} B_{sx,z} \right),$$

$$\omega_r^2 = \frac{\omega_p^2 \omega_l^2}{\omega_l^2}, \omega_p = \left( \frac{n_e e^2}{m\epsilon_0 \epsilon_L} \right)^{1/2}, \text{ and } \frac{\omega_l}{\omega_l} = \left( \frac{\epsilon_L}{\epsilon_\infty} \right)^{1/2}.$$

$\omega_l$  is the longitudinal optical phonon frequency and is given by  $\omega_l = k_B \theta_D / \hbar$ , where  $k_B$  and  $\theta_D$  are Boltzmann constant and Debye temperature of the lattice, respectively.  $\epsilon_L$  is the lattice dielectric constant.

The components of oscillatory electron fluid velocity in the presence of pump and the magnetostatic fields are obtained from Eq. (2) as:

$$v_{0x} = \frac{\bar{E}}{v - i\omega_0}, \quad (9a)$$

and

$$v_{0y} = \frac{(e/m)[\omega_{cz}E_{0x} + (v - i\omega_0)E_{0x}]}{[\omega_{cz}^2 + (v - i\omega_0)^2]}. \quad (9b)$$

Now the resonant Stokes component of the current density due to finite nonlinear polarization of the medium has been deduced by neglecting the transient dipole moment, which can be represented as:

$$J_{cd}(\omega_s) = n_{1s}^* e v_{0x}$$

$$= \frac{\epsilon k_{op}(k_0 - k_{op})|\bar{E}_0|E_{1x}}{(\Omega_{rs}^2 + i\omega_s)(v - i\omega_s)} \times \left[ 1 - \frac{\epsilon N}{2M(\Omega_{rop}^2 + i\Gamma\omega_{op})} \left\{ \frac{2q_s\alpha_u}{\epsilon} - \alpha_u^2 |(E_{eff})_x|^2 \right\} \right] \quad (10)$$

where  $\Omega_{rop}^2 = \bar{\omega}_r^2 - \omega_{op}^2$ .

The time integral of induced current density yields nonlinear induced polarization as

$$P_{cd}(\omega_s) = \int J_{cd}(\omega_s) dt = \frac{\epsilon_\infty e^2 k_{op}(k_0 - k_{op})|\bar{E}_0|E_{1x}}{m^2 \omega_0 \omega_s (\Omega_{rs}^2 + i\omega_s)} \times \left[ 1 - \frac{\epsilon N}{2M(\Omega_{rop}^2 + i\Gamma\omega_{op})} \left\{ \frac{2q_s\alpha_u}{\epsilon} - \alpha_u^2 |(E_{eff})_x|^2 \right\} \right]. \quad (11)$$

Using the relation  $P_{cd}(\omega_s) = \epsilon_0 \chi_{cd}^{(2)} |\bar{E}_0|E_{1x}$  and equation (11), the second-order optical susceptibility  $\chi_{cd}^{(2)}$  due to induced current density is given by

$$\chi_{cd}^{(2)} = \frac{\epsilon_\infty e^2 k_{op}(k_0 - k_{op})}{\epsilon_0 m^2 \omega_0 \omega_s (\Omega_{rs}^2 + i\omega_s)} \times \left[ 1 - \frac{\epsilon N}{2M(\Omega_{rop}^2 + i\Gamma\omega_{op})} \left\{ \frac{2q_s\alpha_u}{\epsilon} - \alpha_u^2 |(E_{eff})_x|^2 \right\} \right]. \quad (12)$$

Here, it is worth pointing out that in addition to the polarization  $P_{cd}(\omega_s)$ , the system also possesses a polarization created by the interaction of the pump wave with the molecular vibrations generated within the medium, obtained from equations (1) and (5) as:

$$P_{mv}(\omega_s) = \frac{\epsilon^2 \omega_0^2 N \alpha_u}{2M(\Omega_{rop}^2 + i\Gamma\omega_{op})} |E_0|E_{1x}. \quad (13)$$

Using the relation  $P_{mv}(\omega_s) = \epsilon_0 \chi_{mv}^{(2)} |E_0|E_{1x}$  and equation (13), the second-order optical susceptibility  $\chi_{mv}^{(2)}$  due to electrostrictive polarization is given by

$$\chi_{mv}^{(2)} = \frac{\epsilon^2 \omega_0^2 N \alpha_u}{2\epsilon_0 M (\Omega_{rop}^2 + i\Gamma\omega_{op})}. \quad (14)$$

The effective second-order optical susceptibility at Stokes frequency in a weakly polar III-V semiconductor crystal due to nonlinear current density and molecular vibrations is given by

$$\chi_{eff}^{(2)} = \chi_{mv}^{(2)} + \chi_{cd}^{(2)}$$

$$\begin{aligned}
 &= \frac{\epsilon^2 \omega_0^2 N \alpha_u}{2 \epsilon_0 M (\Omega_{rop}^2 + i \Gamma \omega_{op})} \\
 &+ \frac{\epsilon_{xz} e^2 k_{op} (k_0 - k_{op})}{\epsilon_0 m^2 \omega_0 \omega_s (\Omega_{rs}^2 + i \nu \omega_s)} \\
 &\times \left[ 1 - \frac{\epsilon N}{2 M (\Omega_{rop}^2 + i \Gamma \omega_{op})} \left\{ \frac{2 q_s \alpha_u}{\epsilon} - \alpha_u^2 |(E_{eff})_x|^2 \right\} \right] \\
 &= [\chi_{eff}^{(2)}]_r + [\chi_{eff}^{(2)}]_i, \tag{15}
 \end{aligned}$$

where  $[\chi_{eff}^{(2)}]_r$  and  $[\chi_{eff}^{(2)}]_i$  represent the real and imaginary parts of complex  $\chi_{eff}^{(2)}$ .

Eq. (15) reveal that  $[\chi_{eff}^{(2)}]_r$  and  $[\chi_{eff}^{(2)}]_i$  are influenced by differential polarizability  $\alpha_u$ , szigeti effective charge  $q_s$ , externally applied transverse magnetostatic field  $B_0$  (via parameter  $\omega_c$  and hence  $\Omega_{rs}^2$ ) and doping concentration  $n_0$  (via parameter  $\omega_p$  and hence  $\Omega_{rs}^2$ ).

Here it should be worth pointing out that  $[\chi_{eff}^{(2)}]_r$  is responsible for parametric dispersion while  $[\chi_{eff}^{(2)}]_i$  give rise to parametric amplification/attenuation and oscillation. The present paper deals with study of parametric amplification of optical phonons in transversely magnetized doped weakly polar semiconductor crystals only. As is well known, parametric amplification can be achieved at excitation intensities above a certain threshold value. This threshold nature can be obtained by setting  $[\chi_{eff}^{(2)}]_i = 0$ . This condition yields

$$E_{0,th} = \frac{m \Omega_{rs} \Omega_{rop} (\omega_0^2 - \omega_c^2)}{e k_{op} [(\omega_0^2 - \omega_{cx}^2) + \nu \omega_{cz}]}. \tag{16}$$

In order to obtain the three-wave parametric amplification/gain coefficient  $\alpha_{para}$  in a magnetized doped semiconductor crystal, we employ the relation [12]:

$$g_{para} = \frac{\omega_s}{\eta c} [\chi_{eff}^{(2)}]_i. \tag{17}$$

The nonlinear parametric gain of the signal as well as the idler waves can be possible only if  $\alpha_{para}$  is negative for pump field  $|E_0| > |E_{0,th}|$ .

### 3. RESULTS AND DISCUSSION

To have a numerical appreciation of the results, the semiconductor crystal is assumed to be irradiated by 10.6  $\mu\text{m}$  pulsed CO<sub>2</sub> laser. The other parameters are given in Ref. [19].

The nature of dependence of the threshold pump electric field  $E_{0,th}$  necessary for the onset of parametric process on different parameters such as wave number  $k_{op}$ , externally applied magnetostatic field  $B_0$ , doping concentration  $n_0$  etc. may be studied from equation (16). The results are plotted in Figs. 1 and 2.

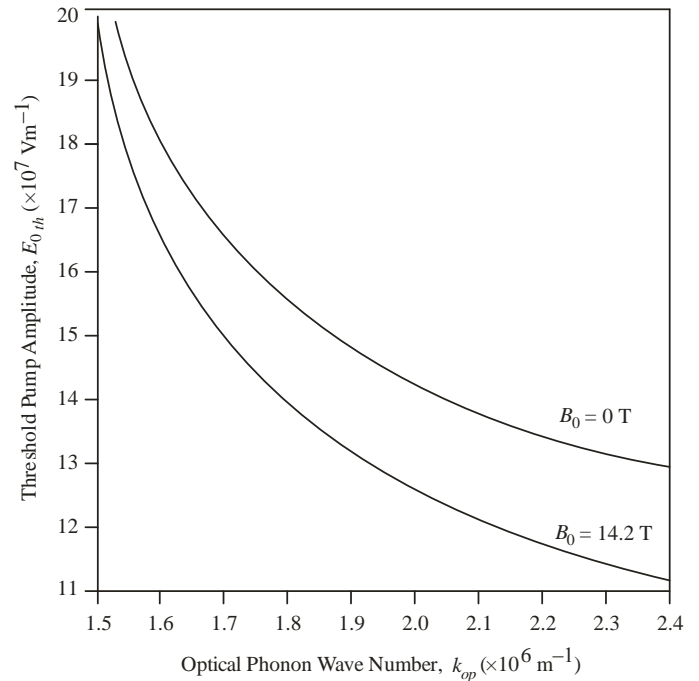
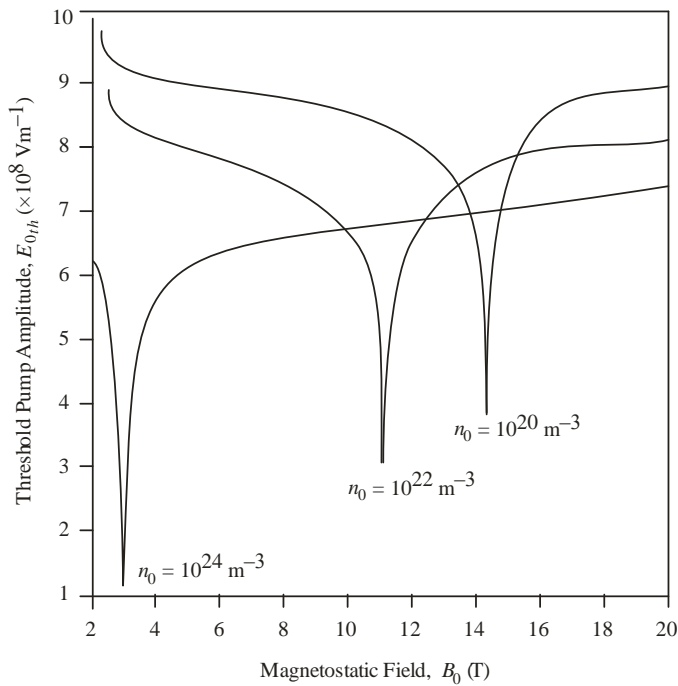


Fig -1: Variation of threshold pump amplitude  $E_{0,th}$  with optical phonon wave number  $k_{op}$  in the absence ( $B_0 = 0\text{T}$ ) and presence of magnetostatic field ( $B_0 = 14.2\text{T}$ ) with  $n_0 = 10^{20}\text{m}^{-3}$ .

Fig. 1 shows the variation of threshold pump amplitude  $E_{0,th}$  with wave number  $k_{op}$  in the absence ( $B_0 = 0\text{T}$ ) and presence of magnetostatic field ( $B_0 = 14.2\text{T}$ ) with  $n_0 = 10^{20} \text{ m}^{-3}$ . It can be observed that in both the cases,  $E_{0,th}$  is comparatively larger for  $k_{op} = 1.5 \times 10^6 \text{ m}^{-3}$ . With increasing  $k_{op}$ ,  $E_{0,th}$  decreases parabolically. This behaviour may be attributed to the fact that  $E_{0,th} \propto k_{op}^{-1}$  as suggested from equation (16). A comparison between the two cases reveals that for the plotted regime of  $k_{op}$ , for  $B_0 = 14.2 \text{ T}$ ,  $E_{0,th}$  is comparatively smaller than that for  $B_0 = 0 \text{ T}$ . This is due to the fact that around  $B_0 = 14.2 \text{ T}$ ,  $\omega_c^2 \sim \omega_0^2$  and  $(\omega_0^2 - \omega_c^2) \rightarrow 0$  [Eq. (16)], thus lowering the value of  $E_{0,th}$ .

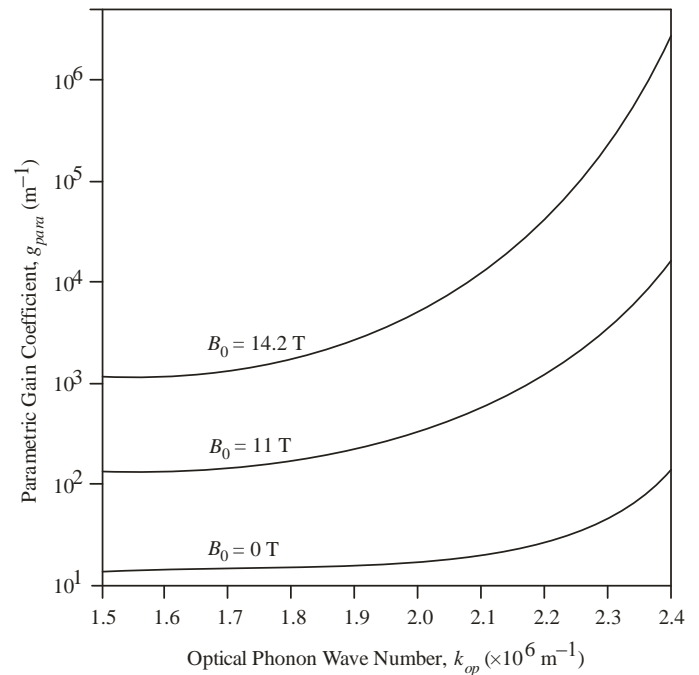


**Fig -2:** Variation of threshold pump amplitude  $E_{0,th}$  with magnetostatic field  $B_0$  for three different values of doping concentration  $n_0$ .

Fig. 2 shows the variation of threshold pump amplitude  $E_{0,th}$  with magnetostatic field  $B_0$  for three different values of doping concentration  $n_0$ . It can be observed that in all the three cases,  $E_{0,th}$  shows a dip at a particular value of  $B_0$  (i.e. 14.2 T for  $n_0 = 10^{20} \text{ m}^{-3}$ , 11 T for  $n_0 = 10^{22} \text{ m}^{-3}$ , 3 T for  $n_0 = 10^{24} \text{ m}^{-3}$ ). This behaviour can be explained as follows: (i) For  $n_0 = 10^{20} \text{ m}^{-3}$ , the dip at  $B_0 = 14.2 \text{ T}$  (corresponding  $\omega_c \sim \omega_0$ ) is due to the factor  $(\omega_0^2 - \omega_c^2) \rightarrow 0$  [Eq. (16)]. (ii) For  $n_0 = 10^{22} \text{ m}^{-3}$ , the dip at  $B_0 = 11 \text{ T}$  is due to factor  $\Omega_{rs}$  (i.e.  $\omega_c^2 \sim \omega_s^2$ ) [Eq. (16)]. (iii) For  $n_0 = 10^{24} \text{ m}^{-3}$ , the dip at  $B_0 = 3 \text{ T}$  is due to parameter  $\Omega_{rop}$  (i.e.  $\omega_c^2 \sim \omega_{op}^2$ ) [Eq. (16)]. A comparison among the three cases reveals that with increasing  $n_0$ , the dip in the value of  $E_{0,th}$  becomes more deeper and shifts towards lower values of  $B_0$ . Hence, we conclude from this figure that externally applied magnetostatic field plays an important role in lowering the threshold pump amplitude for the onset of optical parametric amplification in III-V semiconductors. The increase in doping concentration further lowers the threshold pump amplitude and shifts the dip towards lower values of magnetostatic field.

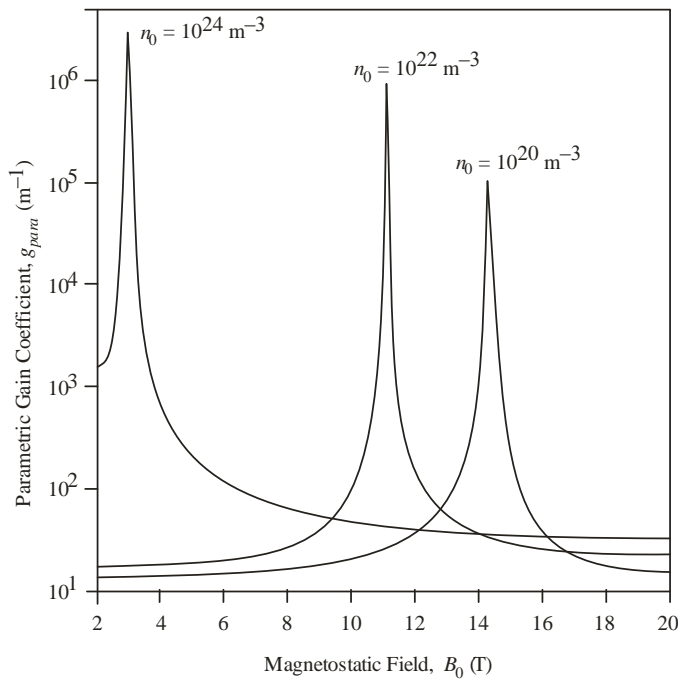
Using the material parameters (for n-Insb) given above, the nature of dependence of parametric gain coefficient  $g_{para}$  on different parameters such as wave number  $k_{op}$ , externally applied magnetostatic field  $B_0$ , doping concentration  $n_0$ , pump electric field  $E_0$  etc. well above the threshold pump

electric field may be studied from equation (17). The results are plotted in Figs. 3 – 6.



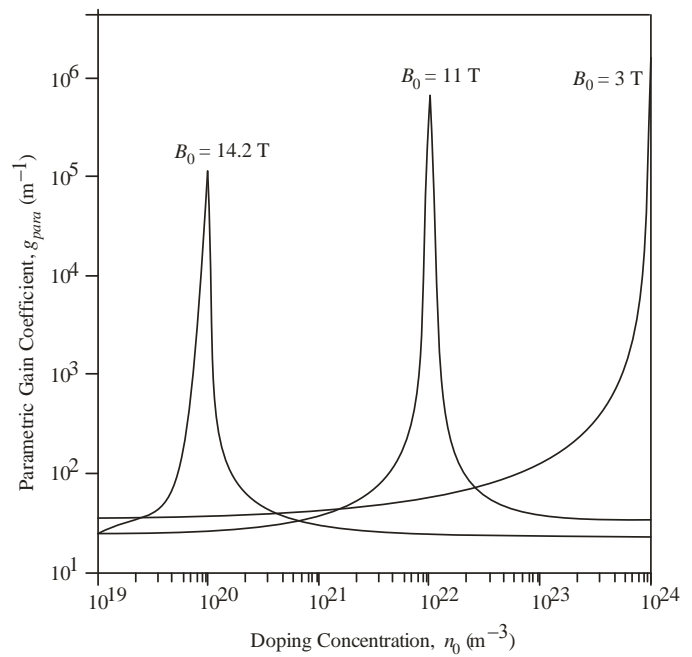
**Fig -3:** Nature of dependence of parametric gain coefficient  $g_{para}$  on optical phonon wave number  $k_{op}$  for three different cases, viz. (i)  $B_0 = 0 \text{ T}$ , (ii)  $B_0 = 11 \text{ T}$ , (iii)  $B_0 = 14.2 \text{ T}$  for  $n_0 = 10^{20} \text{ m}^{-3}$  and  $E_0 = 12.5 \times 10^8 \text{ Vm}^{-1}$ .

Fig. 3 shows the nature of dependence of parametric gain coefficient  $g_{para}$  on wave number  $k_{op}$  for the cases, viz. absence of magnetostatic field ( $B_0 = 0 \text{ T}$ ) and presence of magnetostatic field ( $B_0 = 11, 14.2 \text{ T}$ ) for  $n_0 = 10^{20} \text{ m}^{-3}$  and  $E_0 = 12.5 \times 10^8 \text{ Vm}^{-1}$  ( $> E_{0,th}$ ). It can be observed that in the absence of magnetostatic field ( $B_0 = 0 \text{ T}$ ),  $g_{para}$  remains constant for  $k_{op} \leq 2 \times 10^6 \text{ m}^{-1}$  and increases quadratically for  $k_{op} > 2 \times 10^6 \text{ m}^{-1}$ . In the presence of magnetostatic field ( $B_0 = 11, 14.2 \text{ T}$ ),  $g_{para}$  increases quadratically for the plotted regime of  $k_{op}$ . A comparison among all the above three cases reveal that the gain coefficient satisfies the inequality condition:  $(g_{para})_{B_0=14.2 \text{ T}} > (g_{para})_{B_0=11 \text{ T}} > (g_{para})_{B_0=0 \text{ T}}$ . Hence we conclude from this figure that the parametric gain coefficient can be enhanced by increasing the wave number and simultaneous application of externally applied magnetostatic field.

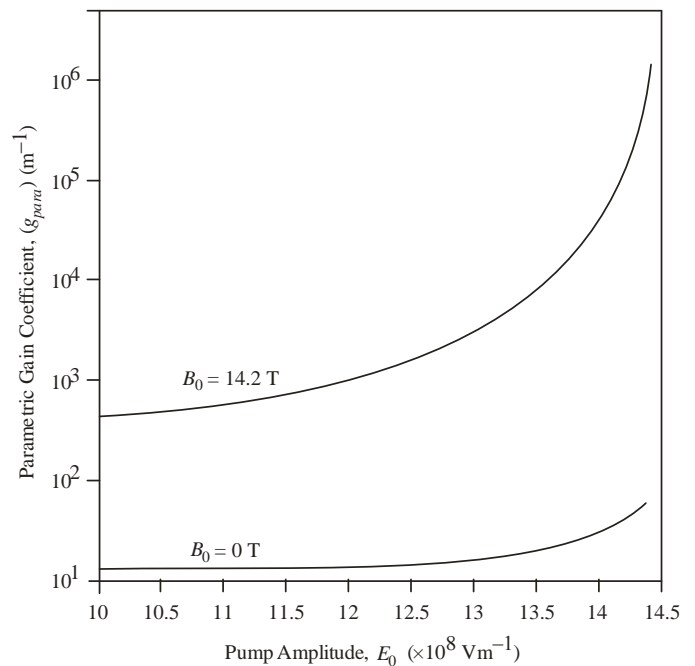


**Fig -4:** Nature of dependence of parametric gain coefficient  $g_{para}$  on magnetostatic field  $B_0$  for three different values of doping concentration  $n_0$  with  $E_0 = 12.5 \times 10^8 \text{ Vm}^{-1}$ .

Fig. 4 shows the nature of dependence of parametric gain coefficient  $g_{para}$  on magnetostatic field  $B_0$  for three different values of doping concentration  $n_0$ . It can be observed that in all the three cases,  $g_{para}$  shows a sharp peak at a particular value of  $B_0$  (i.e. 14.2 T for  $n_0 = 10^{20} \text{ m}^{-3}$ , 11 T for  $n_0 = 10^{22} \text{ m}^{-3}$ , 3 T for  $n_0 = 10^{24} \text{ m}^{-3}$ ). This behaviour can be explained as follows: (i) For  $n_0 = 10^{20} \text{ m}^{-3}$ , the peak at  $B_0 = 14.2$  T (corresponding  $\omega_c \sim \omega_0$ ) is due to the factor  $(\omega_0^2 - \omega_c^2) \rightarrow 0$  [Eq. (17)]. (ii) For  $n_0 = 10^{22} \text{ m}^{-3}$ , the peak at  $B_0 = 11$  T is due to factor  $\Omega_{rs}$  (i.e.  $\omega_c^2 \sim \omega_s^2$ ) [Eq. (17)]. (iii) For  $n_0 = 10^{24} \text{ m}^{-3}$ , the peak at  $B_0 = 3$  T is due to parameter  $\Omega_{rop}$  (i.e.  $\omega_c^2 \sim \omega_{op}^2$ ) [Eq. (17)]. A comparison among the three cases reveals that with increasing  $n_0$ , the peak in the value of  $g_{para}$  becomes more higher and shifts towards lower values of  $B_0$ . Hence, we conclude from this figure that externally applied magnetostatic field plays an important role in enhancing the parametric gain coefficient for the onset of optical parametric amplification in III-V semiconductors. The increase in doping concentration further enhances the gain coefficient and shifts the peak towards lower values of magnetostatic field.



**Fig -5:** Nature of dependence of parametric gain coefficient  $g_{para}$  on doping concentration  $n_0$  for three different values of magnetostatic field  $B_0$  with  $E_0 = 12.5 \times 10^8 \text{ Vm}^{-1}$ .



**Fig -6:** Nature of dependence of parametric gain coefficient  $g_{para}$  on pump amplitude  $E_0$  in the absence ( $B_0 = 0\text{T}$ ) and presence of magnetostatic field ( $B_0 = 14.2\text{T}$ ) for  $n_0 = 10^{22} \text{ m}^{-3}$ .

Fig. 5 shows the nature of dependence of parametric gain coefficient  $g_{para}$  on doping concentration  $n_0$  for three

different values of magnetostatic field  $B_0$ . The results of Fig. 5 support results of Fig. 4.

Fig. 6 shows the nature of dependence of parametric gain coefficient  $g_{para}$  on pump amplitude  $E_0 (> E_{0,th})$  for the cases, viz. absence of magnetostatic field ( $B_0 = 0$  T) and presence of magnetostatic field ( $B_0 = 14.2$  T) for  $n_0 = 10^{20} \text{ m}^{-3}$ . We observed that in both the cases  $g_{para}$  increases quadratically with respect to  $E_0$ . Thus, higher pump field yield higher parametric gain coefficient.

#### 4. CONCLUSIONS

In the present study, a detailed numerical analysis of parametric amplification of optical phonons in magnetized doped III-V semiconductors has been undertaken. The hydrodynamic model of semiconductor-plasma has been successfully applied to study the effects of externally applied magnetostatic field and doping concentration on threshold pump amplitude and gain coefficient for the onset of parametric process in III-V semiconductor crystals duly irradiated by slightly off-resonant not too high power pulsed lasers with pulse duration sufficiently larger than the optical phonon lifetime. The threshold pump amplitude can be reduced while parametric gain coefficient can be enhanced by increasing the wave number and simultaneous application of externally applied magnetostatic field. The threshold pump amplitude can be reduced by proper selection of magnetostatic field (around resonance conditions). The increase in doping concentration further lower the threshold pump amplitude and shifts the dip towards lower values of magnetostatic field. The parametric gain coefficient can be enhanced by proper selection of magnetostatic field (around resonance conditions). The increase in doping concentration further enhances the gain coefficient and shifts the peak towards lower values of magnetostatic field. Moreover, higher pump field yield higher parametric gain coefficient. The technological potentiality of a transversely magnetized weakly polar doped semiconductor plasma as the hosts for parametric devices like parametric amplifiers and oscillators are established. In III-V semiconductor crystals, parametric amplification and oscillation in the infrared regime appears quite promising under the resonance conditions and replaces the conventional idea of using high power pulsed lasers.

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