

# 2D localization systems of a mobile object based on the propagation time of signal, using synchronous and asynchronous distance measurement systems

Herinirina Rakotomanana<sup>1</sup>, Soloniaina Rakotomiraho<sup>2</sup>, Rivo Mahandrisoa Randriamaroson<sup>3</sup>

<sup>1</sup>Ph.D Student, ED-STII, Antananarivo University, Madagascar

<sup>2</sup>Professor, ED-STII, Antananarivo University, Madagascar

<sup>3</sup>Professor, ED-STII, Antananarivo University, Madagascar

**Abstract** - This paper presents the studies of localization system with a moving object in a delimited plane. The first work presents the trilateration system allowing the position of an object to be determined from known or more precisely measured distances. In this paper, the distance between the object and a sensor is evaluated using the measurement of the propagation time of a signal emitted by the object, here a sound waves. The definition of the departure time of the transmitted signal led to the study of two types distance measurement systems. The synchronous system of distance measurements, where the start time of the signal is known, and the asynchronous system where this time is unknown but an estimate is made to define it.

**Key Words:** Localization system, Plane, Trilateration, Distance, Sound wave, Synchronous system, Asynchronous system,

## 1 . INTRODUCTION

The use of robots, drones and ROVs are increasingly widespread in environments where humans do not have access. Robots, drones and ROVs are considered to be remotely piloted unmanned vehicles. Therefore, knowing the position is very useful for better controlling them remotely, especially when they are not within sight.

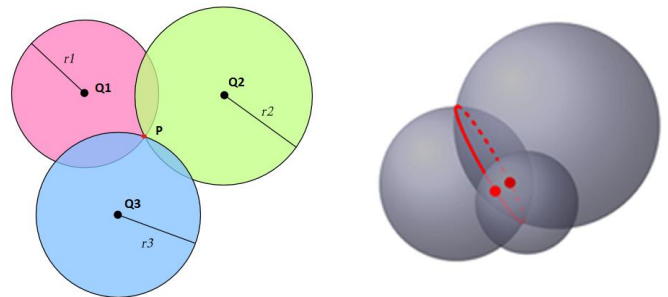
GPS is a location system in a fairly large reference frame, planetary scale, which does not have much precision, of the order of 5 to 15 m. The signals emitted by the GPS satellites do not arrive in the covered premises which could possibly be part of the exploration areas of unmanned vehicles. [1] [2] [3]

Thus, could we use a positioning system for a delimited space with an acceptable precision? In order to answer this problem, this work consists in studying a localization system using two complementary methods. The first, trilateration to determine the position of an object from the distances that separate it from fixed points. The second is the method using the propagation time of a signal to measure the distances between the object and the fixed points.

From this distance measurement system, we will see and compare two distance measurement methods depending on the knowledge of the start time of the measurement signal. Synchronous measurement consists of sending a signal to activate the measurement trigger of the sensors and the sending of the signal from the object, which allows them to be synchronized so as to obtain the start time of the signal. The asynchronous measurement consists in estimating the departure time of the signal using an additional sensor. The deduction of the distances between sensors and the object emitting the signal is thus done without knowing the exact moment of the signal departure.

## 2 . THE TRILATERATION METHOD

The trilateration method is a mathematical method for determining the relative position of a point P. It consists of using only the distances between fixed base points and the objects, and not the angles as by triangulation. [4]



**Fig -1 :** Illustration of the object P position determination by trilateration.

Knowing the positions of all bases Q and the distances between the object P and all bases. [5]

According to the illustration in Fig -1, the determination of the point P is obtained by the intersection of the 3 spheres of respective radius  $r_1, r_2, r_3$  which are the distances of the object compared to all bases points. The base points defined by the centers of the spheres ( $Q_1, Q_2, Q_3$ ) are fixed, and whose coordinates are known, [1][2][5]

$$Q_1 : (x_1, y_1, z_1), Q_2 : (x_2, y_2, z_2), Q_3 : (x_3, y_3, z_3).$$

Equation 1 gives the resolution by trilateration same as the intersection of three spheres in 3D:

$$\begin{cases} (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = (r_1)^2 \\ (x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 = (r_2)^2 \\ (x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 = (r_3)^2 \end{cases} \quad (1)$$

### 2.1 Trilateration reduced to two dimensions

As shown in Fig-2, in a 2D coordinate system (0xy), an object can be located using two known base points. This with the knowledge of the distances which separate these base points to the object (r1 and r2), then by seeking the intersection point of the two circles.

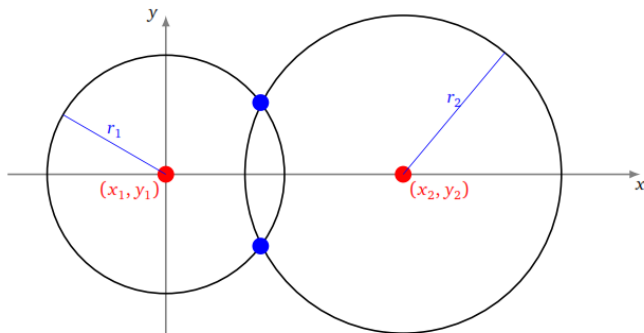


Fig -2 : Graphic representation of two circles intersection [6]

Equation 2 is used to determine the intersection points of these two circles, depending on the value of ρ.

$$\rho = r_1^2 - \left( \frac{r_1^2 - r_2^2 + x_2^2}{2x_2} \right)^2 \quad (2)$$

Three values of ρ are possible, and the intersection point P of the two circles depend on these values. These intersection points are defined in equations 3, 4 and 5.

if  $\rho < 0$  then the equation has no solution. The two circles do not intersect. [6] (3)

if  $\rho = 0$  The system admits a unique solution. The two circles intersect at a single point P, [6]

$$\text{With } P = \left( \frac{r_1^2 - r_2^2 + x_2^2}{2x_2}, 0 \right). \quad (4)$$

if  $\rho > 0$  The system admits two solutions, The two circles intersect in two points, P1 and P2. [6]

$$P1 = \left( \frac{r_1^2 - r_2^2 + x_2^2}{2x_2}, +\sqrt{\rho} \right)$$

$$\text{and } P2 = \left( \frac{r_1^2 - r_2^2 + x_2^2}{2x_2}, -\sqrt{\rho} \right) \quad (5)$$

The space of this study being a defined and limited space, and that the orientation of the devices can also be defined, for equation 5, only one value of P must be chosen. In this work, the values of P1 that lie in the positive y-axis were taken.

### 3. DISTANCE MEASUREMENT USING WAVE PROPAGATION TIME.

Positioning using the trilateration method requires measurements of distances between the object and the base points. The distance measurement method using the wave propagation time is studied for the measurement. This method is based on the knowledge of the wave propagation speed, and as well as its departure time. The GPS system uses this method, a radio signal or a sound signal can be used as a measurement signal.

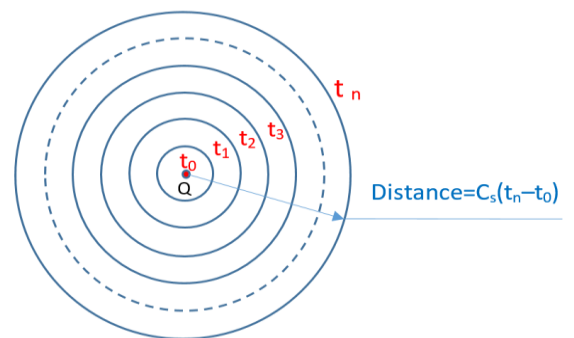


Fig -3 : Representation of the position of the signal emitted from point Q at each instant t and the distance traveled

By knowing the speed of the signal  $C_s$ , the distance d traveled by the signal between two times  $t_0$  and t can be deduced by the formula of equation 6.

$$d = (t - t_0)C_s \quad (6)$$

The Fig-3 illustrates the radiation in the circles form described by the distances traveled at time  $t_n$ , by a wave emitted from a point Q.

#### 3.1 . Application

For application of these two methods of positioning and distance measurement, the system studied in this paper consists of a device which emits the signal and by sensors which measure the time of arrival of the signal. These sensors constitute the fixed base points for the trilateration method.

Knowing the start time of the signal as well as the times of its arrival on the sensors, equation 6 can be directly applied. Here, the question that arises is how to let the positioning system know the start time of a signal received at a time  $t$  on one of the sensors?

Two synchronization systems have been studied so that the system can determine this starting time. The **synchronous system** seen in paragraph 4 as well as the **asynchronous system** seen in paragraph 5.

#### 4 . SYNCHRONOUS SYSTEM

The system is composed of an object that emits the signal as a measurement base and two sensors that detect the arrival of the signal. The calculation module, made up of a microcontroller, determines the difference between the start time and the reception times of the signal emitted by the object to deduce the durations of the propagation of the signal. These times are used to determine the distances between the object and the sensors.

The system is defined as synchronous, because the object emits a control signal other than the positioning signal which allows the system to know the moment of emission of the positioning signal. This control signal has a higher speed than that of the measurement signal.

For our system described in Fig-4, the measurement signal is a 40 kHz ultrasound signal with a speed of 0.34 km/s, and the control signal is an electromagnetic wave with a speed of 300,000 km/s. The received data is transferred to a computer through a microcontroller, to then be interpreted.

$t_0$  is the departure time of the signal which is synchronized with the measurement system of the arrival times of the measurement signal on the sensors.  $t_1$  and  $t_2$  are respectively the times when the signal arrives at sensors 1 and sensors 2.

The positions of the sensors are fixed and immobile and used as a reference base for positioning.

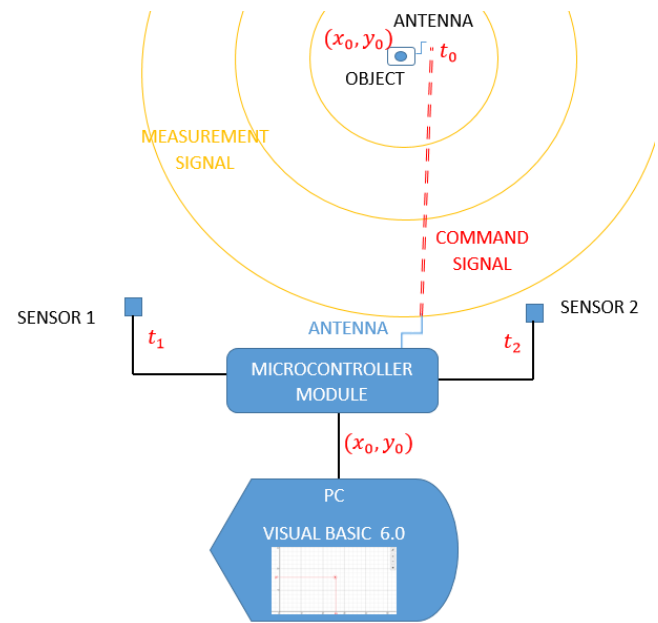


Fig -4 : Block diagram of the localisation system and the signal with synchronous measurement

#### 4.1 System geometry, calculation and measurement

Figure 5 presents the system geometry. Made up of the two sensors and the object, the whole forms a triangle. The determination of the coordinates  $x$  and  $y$  is obtained by intersection of the two circles of respective radius  $d_1$  and  $d_2$  centered on the positions of the sensors.

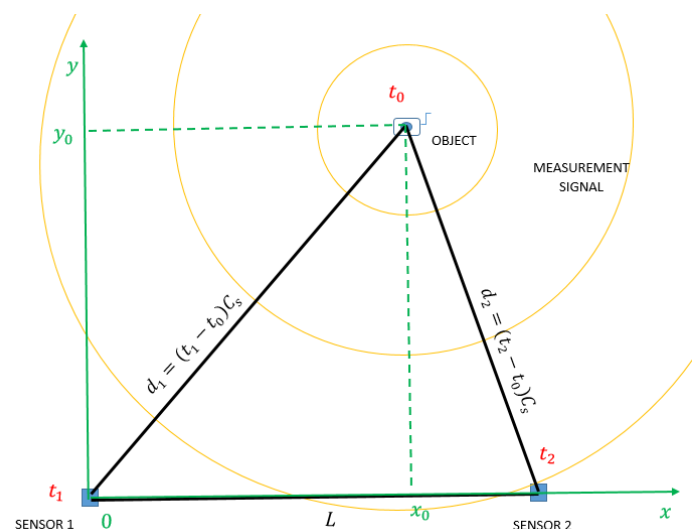


Fig -5 : Geometric representation of the synchronous system with the known signal start time.

The position P of the object defined by  $P = (x_0; y_0)$ , is given by the equations 7 and 8.

$$x_0 = \frac{((t_1 - t_0)C_s)^2 - ((t_2 - t_0)C_s)^2 + L^2}{2L} \tag{7}$$

$$y_0 = \sqrt{((t_1 - t_0)C_s)^2 - \left(\frac{((t_1 - t_0)C_s)^2 - ((t_2 - t_0)C_s)^2 + L^2}{2L}\right)^2} \tag{8}$$

### 5 . ASYNCHRONOUS SYSTEM

For the asynchronous system the control signal is removed from the transmitter object. Therefore, the start time of the signal is unknown. In order to compensate for the ignorance of this starting instant, a third sensor is added to the system which gives another time t3 to solve the trilateration equation which is a system of three (3) equations with three (3) unknowns.

The system is therefore composed of the transmitter object and also of a microcontroller module which calculates the difference between the signal reception times on each of the sensors (figure 6). This gives an estimate of the durations of the propagation of the signal from the object to the sensors. The time differences between the arrivals on the different sensors are used to deduce the position of the object.

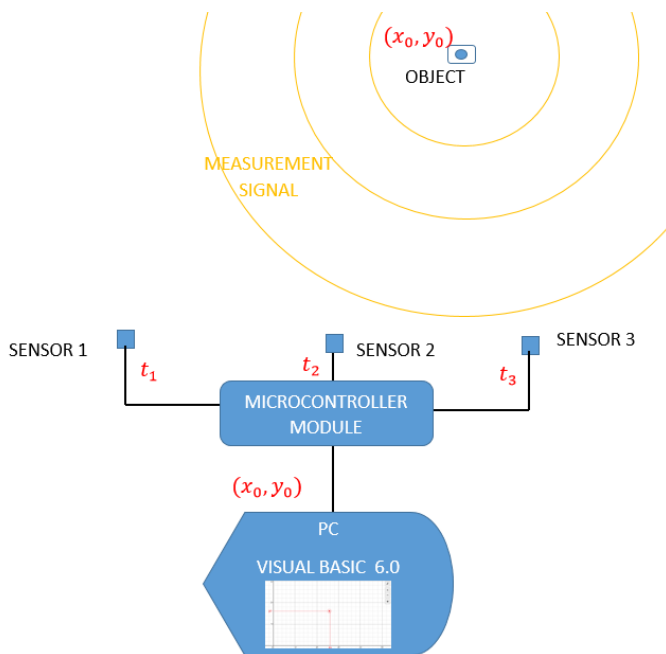


Fig -6 : Block diagram of the localisation system and the signal with asynchronous measurement

### 5.1 System geometry, Calculation and measurement

Figure 7 gives the system geometry to deduce the necessary equations to determine the position of the object. The system equation for trilateration is given by equation 9.

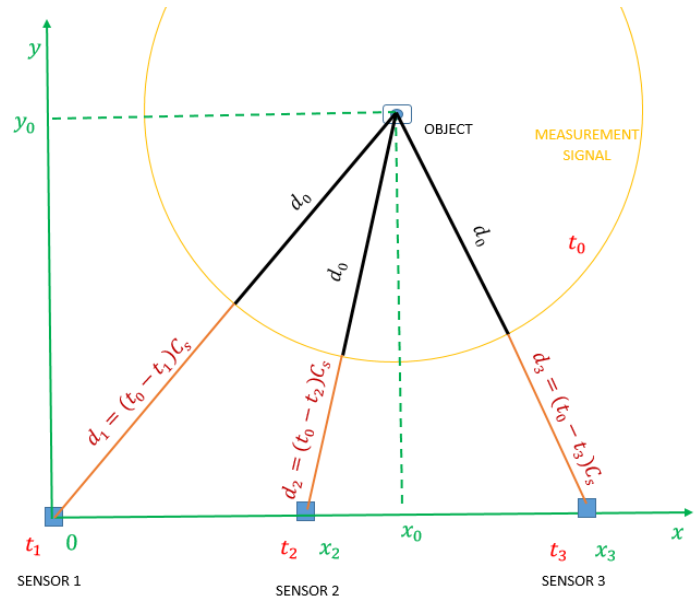


Fig -7 : Geometric representation of the asynchronous system with 3 sensors.

$$\begin{cases} x^2 + y^2 = D_1^2 \\ (x - x_2)^2 + y^2 = D_2^2 \\ (x - x_3)^2 + y^2 = D_3^2 \end{cases} \text{ with } \begin{cases} D_1 = d_1 + d_0 \\ D_2 = d_2 + d_0 \\ D_3 = d_3 + d_0 \end{cases} \tag{9}$$

With equation 9,  $d_0$  can be deduced and is given by the equation 10.

$$d_0 = \frac{x_3(d_1^2 - d_2^2 + x_2^2) - x_2(d_1^2 - d_3^2 + x_3^2)}{2(x_3(d_2 - d_1) - x_2(d_3 - d_1))} \tag{10}$$

Hence, the object position,  $P = (x_0; y_0)$  is given by the equation 11.

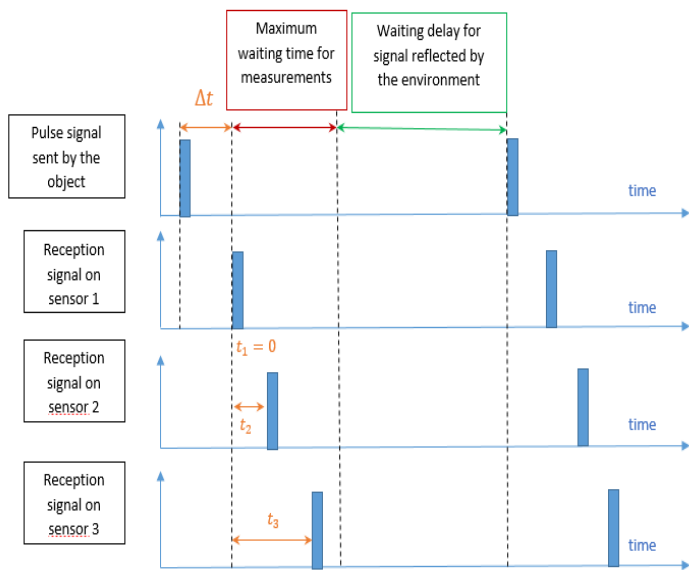
$$x_0 = \frac{D_1^2 - D_3^2 + x_3^2}{2x_3} \text{ and } y_0 = \sqrt{D_1^2 - \left(\frac{D_1^2 - D_3^2 + x_3^2}{2x_3}\right)^2} \tag{11}$$

### 5.2 Measurement time management

The measurement time management is very important to have a good measurement result, because it is necessary to

take into account the delays to wait until all the disturbing signals have passed before making a new measurement. It is also necessary to take into account the maximum waiting time for the measurements. Figure 8 gives the chronology of events within the asynchronous system.

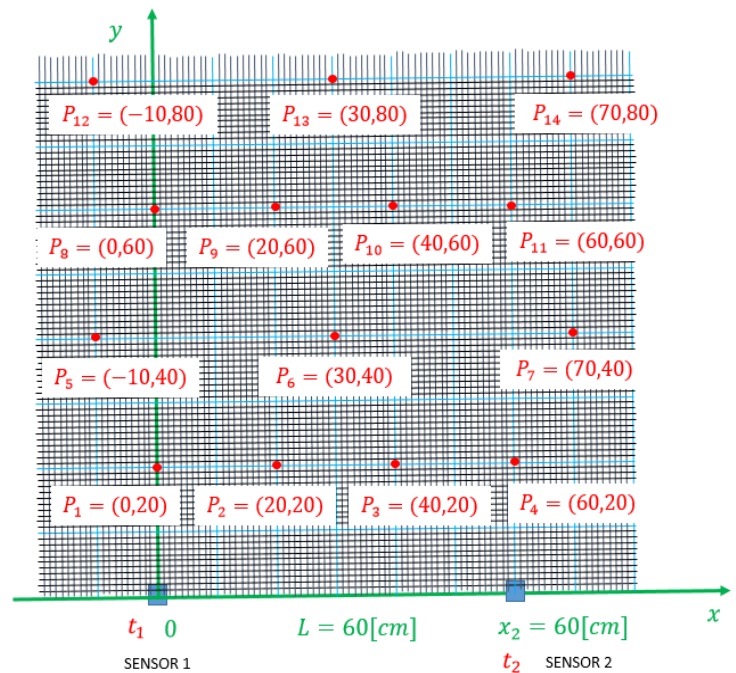
The instant  $t_0$  is taken as being the instant when the signal is first detected on one of the sensors. The measurements of the arrival times of the signal on the other sensors are defined from this moment. Also from this moment begins the counting of the measurement time interval, which is defined as being shorter than the time between the emission of two (2) successive signals.



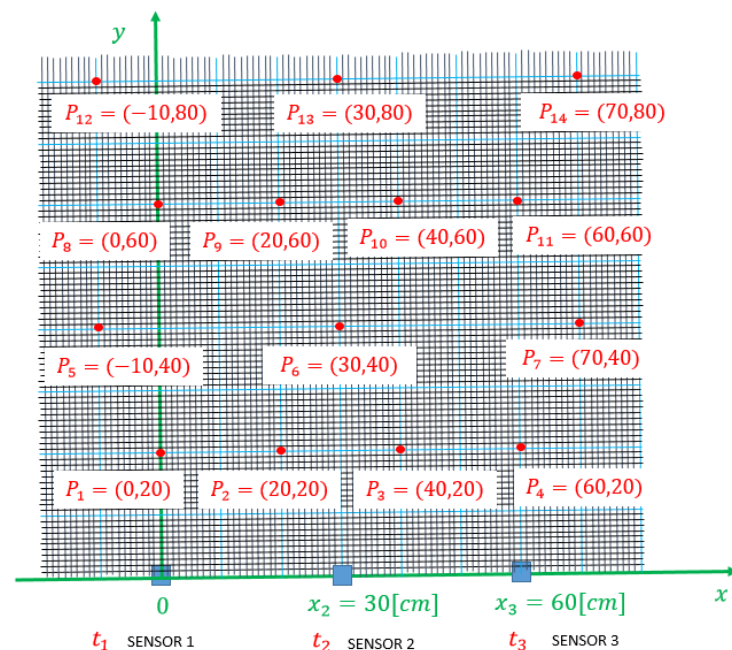
**Fig -8 :** Chronology of the sending of the signal by the object and reception of the signal by the sensors, illustration of an example where sensor 1 receives the signal first and from this moment the other times are measured.

### 6 . MEASUREMENT BENCH FIELD AND REFERENCE POINTS

Figures 9 and 10 show the field and all the reference points used for the measurements in the tests, and respectively give the positions of the sensors on the measurement bench for the synchronous and asynchronous systems. Figure 11 illustrates the measurement bench, the calculation system as well as the emitting object (black box).



**Fig -9 :** Illustration of the field and the positions where the object is placed during the measurements on a synchronous system with 2 sensors



**Fig -10 :** Illustration of the field and the positions where the objects are placed during the measurements on an asynchronous system with 3 sensors.

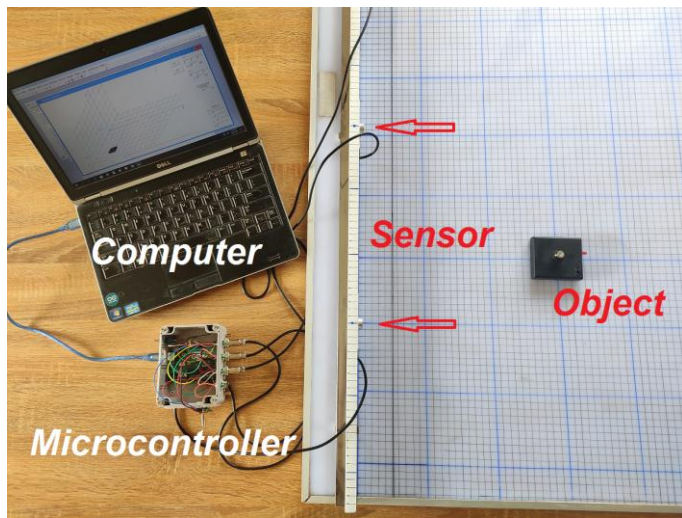


Fig-11 :The measurement bench system.

### 7 . RESULTS

Table 1 show the times of arrival of the signals on the sensors for the synchronous and asynchronous systems from the object, and this for each position of the object. Table 2 represents the actual positions as well as the measured positions of the object for the two measurement systems.

Table -1: Time measurement on a synchronous two-sensor system and on an asynchronous three-sensor system with ultrasonic signal as measuring signal.

POINT	MOBILE OBJECT POSITION		TIMES MEASURED SYNCHRONOUS SYSTEM		TIMES MEASURED ASYNCHRONOUS SYSTEM		
			2 SENSORS		3 SENSORS		
P	X[cm]	Y[cm]	T1 [μs] SENSOR 1	T2 [μs] SENSOR 2	T1 [μs] SENSOR 1	T1 [μs] SENSOR 1	T1 [μs] SENSOR 1
1	0	20	589	1853	0	476	1271
2	20	20	824	1309	170	0	656
3	40	20	1311	827	655	0	171
4	60	20	1868	589	1270	476	0
5	-10	40	1221	2368	0	461	1167
6	30	40	1465	1470	288	0	288
7	70	40	2370	1220	1152	447	0
8	0	60	1759	2485	0	211	735
9	20	60	1854	2118	65	0	338
10	40	60	2116	1854	338	0	64
11	60	60	2485	1764	735	209	0
12	-10	80	2367	3132	0	268	767
13	30	80	2500	2506	153	0	152
14	70	80	3132	2369	765	269	0

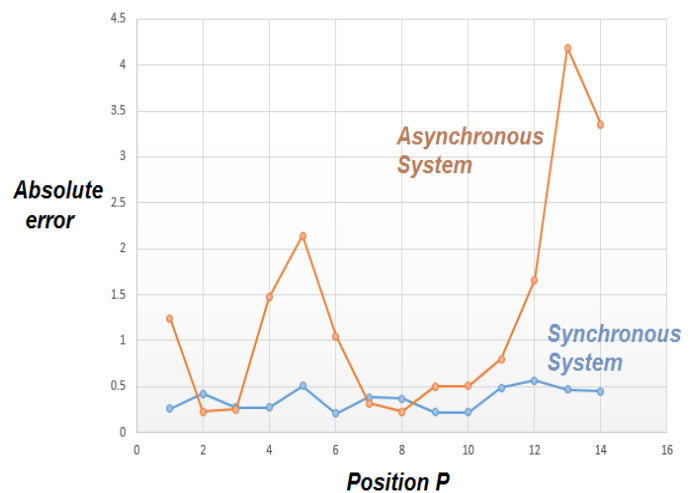
**Table -2:** Positions calculated using the trilateration method from the times measured on a synchronous system with two sensors and on an asynchronous system with 3 sensors with speed of sound 340[m/s]

POINT	MOBILE OBJECT POSITION [cm]		SYNCHRONOUS SYSTEM 2 SENSORS COORDINATES CALCULATED BY TRILATERATION		ASYNCHRONOUS SYSTEM 3 SENSORS COORDINATES CALCULATED BY TRILATERATION	
	X[cm]	Y[cm]	X[cm]	Y[cm]	X[cm]	Y[cm]
P						
1	0	20	0.26	20.02	-0.71	21.02
2	20	20	20.03	19.58	19.91	20.21
3	40	20	39.96	19.73	40.05	20.25
4	60	20	60.27	20.02	60.82	21.22
5	-10	40	-9.65	40.37	-11.58	41.45
6	30	40	29.85	39.86	30.00	41.05
7	70	40	69.77	40.31	69.86	40.29
8	0	60	0.31	59.8	-0.22	60.06
9	20	60	19.89	59.81	19.55	59.78
10	40	60	40.02	59.78	40.50	59.92
11	60	60	59.51	59.97	59.86	59.21
12	-10	80	-10.52	79.78	-11.25	81.09
13	30	80	29.71	79.63	30.05	84.19
14	70	80	70.43	79.86	71.88	82.77

### 8 . INTERPRETATIONS

Chart-1 show the graph of absolute errors during measurements of the position of the object with the two measurement systems.

Therefore, we can see the synchronous system presents good measurement results with an average error of 0.37 cm. The asynchronous system, on the other hand, presents a large variation in errors with an average of 1.28 cm.



**Chart -1:** Graph of the absolute error, difference between the measured value and the real position, of the asynchronous system and synchronous system depending on the position.

### 3. CONCLUSIONS

This work has shown the adaptation of the trilateration method for determining the position of an object in the plane of a delimited space. For the synchronous system of distance measurements, the errors due to the processing delay of the trigger signal are less important than those of the asynchronous system. The source of error of the asynchronous system being the estimation of the departure time of the signal from the source.

To further reduce errors on these systems, reviewing the Time to Digital Converter (TDC) system might help. A research subject has just emerged from this work, the search for the departure time of the signal for the asynchronous system. The asynchronous system is the most suitable for positioning an unmanned vehicle, because it makes it possible to reduce the payload of the vehicle and thus improve its autonomy.

### REFERENCES

- [1] F. Zafari, A. Gkelias, and K. Leung. "A survey of indoor localization systems and technologies." <https://arxiv.org/abs/1709.01015>, 2017.
- [2] A. Mukhopadhyay; A. Malliscry, "TELLIL: A Trilateration and Edge Learning based Indoor Localization Technique for Emergency Scenarios", IEEE, 2018.
- [3] S. Sadowski; P. Spachos, "RSSI-Based Indoor Localization With the Internet of Things", IEEE, 2018
- [4] Geoffrey Blewitt, "Basics of the GPS Technique: Observation Equations", 2000.
- [5] X. Yan, Q. Luo, Y. Yang, S. Liu, H. Li, C. Hu, "ITL-MEPOSA: Improved Trilateration Localization with Minimum Uncertainty Propagation and Optimized Selection of Anchor Nodes for Wireless Sensor Networks", IEEE Access, 2019.
- [6] A. Bodin, L. Blanc-Centi, N. Borne, B. Boutin, L. Desideri, P. Romon, "GEOMETRIE", <http://exo7.emath.fr/cours/livre-geometrie.pdf>, 2016,