A Settled Framework for Perishable Component with Lead Time and Price Dependent Condition

Lovekush[1], Lokendra Kumar[2]

[1]*M.Sc. Student, Monad University, Hapur*
[2]*Associate Professor, Department of Mathematics, Monad University ,Hapur*

Abstract: *In the enable paper, an effort has been created to develop a settled inventory model for biodegradable things with time interval and value dependent demand. Shortages area unit allowed and utterly backlogged. The matter of unsatisfactoriness or deterioration plays a crucial role within the field of internal control and management. The aim of our study is to reduce the whole variable inventory value throughout a given amount of your time. A numerical example is given to demonstrate the developed model.*

***Key-Words*:** Inventory, Deterioration, Lead-Time and Price-Dependent Demand

1. INTRODUCTION

Academicians likewise as industrialists have nice interest within the development of internal control and their uses. There area unit several merchandise that either deteriorate or become obsolete with passage of your time. For such destructible merchandise totally different modeling techniques area unit applied. Destructible inventory forms atiny low a part of total inventory and includes trendy clothes, electronic things, digital merchandise and periodicals. The destructible merchandise is classified supported two categories: (1) deterioration (2) degeneration. Deterioration is outlined as harm, decay or spoilage of the things that area unit keep for future use and that perpetually loose a part of their worth with passage of your time. Degeneration happens thanks to the arrival of latest and higher merchandise within the market.

In the existing literature, some inventory models that were developed by modern researchers considering some or all of the parameters associated with constant demand rate, increasing/ decreasing operate of your time, value and stock dependent are quoted. The demand of fresh arrived merchandise in market is influenced by their costs, as a result of the enticing costs or offers on the merchandise inspire the shoppers to shop for a lot of. This example will increase the order amount of the retailers or customers. In recent years some researchers additionally gave their attention towards a time dependent rate, as a result of the demand of fresh launched merchandise like trendy clothes, electronic things, motorcars, mobiles etc. will increase with time and later it becomes constant.

But within the real world there area unit several things within which these assumptions aren't valid like seasonal merchandise, work merchandise, electronic things and medicines. Some researches within the space area unit value mentioning. Goswami and Chaudhuri [1] developed an EOQ model for deteriorating things with linear trend in demand and shortages. Padmanabhan and Vrat [2] thought-about an EOQ model for destructible things with stock dependent commerce rate. Giri et al. [3] projected a list model for deteriorating things with stock dependent demand rate. Hargia [4] gave an EOQ model for deteriorating things with time variable demand. Giri and Chaudhuri [5] developed a settled inventory model for deteriorating things with non-linear holding value and stock dependent demand rate. Chang and Dye projected two inventory models [6] and [11]. The model [6] is an EOQ model for deteriorating things with time variable demand and partial backlogging. and also the model [11] is a list model for destructible things with permissible delay in payments and shortages. Chung et al. [7] gave a note on EOQ models for deteriorating things with stock dependent commerce rate. Lin et al. [8] projected AN EOQ model for deteriorating things with time variable demand and permitting shortages. Papachristos and Skouri developed two inventory models [9] and [12]. In model [9] they gave an optimum refilling policy for deteriorating things with exponential kind backlogging rate and time variable demand. The model [12] may be a continuous review inventory model for deteriorating things with time dependent demand and permitting shortages. Goyal and Giri developed two inventory models [10] and [15]. In model [10] they thought-about recent trends in modeling of deteriorating inventory. and also the model [15] may be a production inventory model with time variable demand, production and deterioration rate. Chinese [13] projected AN EOQ model for Weibull deteriorating things with time variable demand and permitting shortages. Wang [16] gave a note on EOQ model for destructible things with exponential distribution, deterioration and time dependent demand rate. They additionally thought-about shortages in their inventory model. Dye and Ouyang [17] developed an EOQ model for destructible merchandise with stock dependent commerce rate and permitting shortages. Sovereign [18] projected a list model for deteriorating things with continuance of cash and permissible delay in payments. She thought-about a finite coming up with horizon in her inventory model. Hou ANd sculptor [19] developed an EOQ model for deteriorating things with value and stock dependent commerce rate. They thought-about the impact of inflation and continuance of cash in their inventory model. Dye presented a joint rating and ordering policy for deteriorating things with partial backlogging. Roy et al. [21] given a list model for deteriorating things with stock dependent demand rate and fuzzy kind inflation. They additionally thought-about time discounting over a random coming up with horizon. Min and Chow dynasty [22] developed a list model for deteriorating things with stock dependent commerce rate and permitting shortages. Jain et al. [23] projected a list model for deteriorating things with fuzzy kind inflation and money discounting over random coming up with horizon. Panda et al. [24] developed a two warehouse inventory model for deteriorating things with fuzzy kind demand rate and interval. Roy [25] projected a fuzzy inventory model for deteriorating things with value dependent demand rate. Chaudhary and Sharma [26] given a list model for Weibull deteriorating things with value dependent demand rate beneath inflation. Maragatham and Palani [27] developed a list model for destructible things with interval, value dependent demand and permitting shortages

2. ASSUMPTIONS NOTATIONS

We consider the following assumptions and notations

The demand rate is $R(p) = a p^{-b}$, $a,b > 0$

Here p is the selling price.

The deterioration rate is taken as $\theta(t) = \theta t$

O_C is the ordering cost per order.

h_c is the holding cost per unit time.

s_c is the shortage cost per unit time.

p_c is the purchase cost per unit time.

T is the replenishment cycle length.

$I(t)$ is the inventory level at any time t in $[0,T]$.

T_1 is the time at which inventory level becomes zero.

$TC(L,T_1,T)$ is the total variable inventory cost per cycle.

The replenishment rate is infinite.

The lead time is L.

There is no repair or replacement of the deteriorated items

3. MATHEMATICAL FORMULATION

Suppose a list system contains the utmost inventory level letter $R_{(p)}$ within the starting of every cycle, wherever $R_{(p)}$ is that the worth dependent demand. Throughout the interval L,T_1, the inventory level decreases because of each demand and deterioration and it becomes zero at t, T_1. Throughout the shortage interval $[T_1,T]$ the demand is unhappy. The instant inventory level at any time t in L, T is given by the subsequent differential equations:

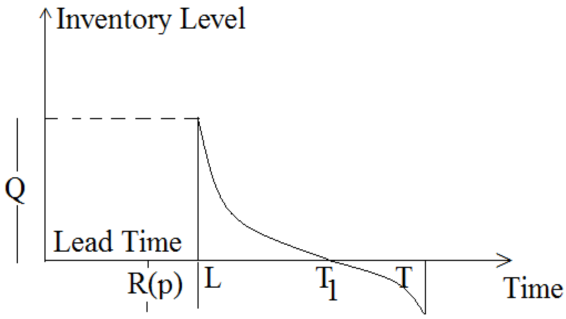

Figure 1, Inventory Model

$$\frac{dI}{dt} + \theta t I = -a p^{-b}, \qquad L \le t \le T_1$$

$$\frac{dI}{dt} = -a p^{-b}, \qquad T_1 \le t \le T$$

Boundary condition $I(T_1) = 0$ is taken in both equations.

The solutions of the above equations are given by the following equations . By considering the first degree terms in θ , we have

$$I = a p^{-b}\left[T_1 - t + \frac{\theta}{6}T_1^3 + \frac{\theta}{3}t^3 - \frac{\theta}{2}T_1 t^2\right]$$

$$I = a p^{-b}[T_1 - t]$$

The maximum inventory level is obtained by putting $t = L$ in equation (3), so

$$Q = a p^{-b}\left[T_1 - L + \frac{\theta}{6}T_1^3 + \frac{\theta}{3}L^3 - \frac{\theta}{2}T_1 L^2\right]$$

The quantity $Q = LD(p)$ is ordered in the beginning of each cycle. The maximum back ordered quantity I_B is obtained by putting $t = T$ in equation (4). Therefore

$$I_B = a p^{-b}[T_1 - T]$$

The ordering cost per cycle is $O_C = O_C$

The holding cost per cycle is

$$H_C = h_C \int_L^{T_1} I(t)\,dt$$

or

$$H_C = a\,h_C\,p^{-b}\begin{bmatrix}\frac{1}{2}T_1^2 - LT_1 + \frac{1}{2}L^2 + \frac{\theta}{12}T_1^4 \\ -\frac{\theta}{12}L^4 + \frac{\theta}{6}T_1L^3\end{bmatrix}$$

The deterioration cost per cycle is

$$D_C = d_C\left[Q - \int_L^{T_1} R(t)dt\right]$$

Or

$$D_C = a\,d_C\,p^{-b}\left[\frac{\theta}{6}T_1^3 + \frac{\theta}{3}L^3 - \frac{\theta}{2}T_1L^2\right]$$

The shortage cost per cycle is

$$S_C = -s_C\int_{T_1}^{T} I(t)dt$$

Or

$$S_C = a\,s_C\,p^{-b}\left[\frac{1}{2}T_1^2 + \frac{1}{2}T^2 - TT_1\right]$$

The purchase cost per cycle is

$$P_C = p_C[Q + I_B]$$

Or

$$P_C = a\,p_C\,p^{-b}\left[2T_1 - T - L + \frac{\theta}{6}T_1^3 + \frac{\theta}{3}L^3 - \frac{\theta}{2}T_1L^2\right]$$

The total variable inventory cost per cycle is

$$TC(L,T_1,T) = \frac{1}{T}[O_C + H_C + D_C + S_C + P_C]$$

Putting the values of O_C, H_C, D_C, S_C and P_C in above equation , we obtain

$$TC(L,T_1,T) = \frac{1}{T}\Big[o_C + a\,p^{-b}\Big\{2p_CT_1 - p_CL - p_CT + \frac{(h_C+s_C)}{2}T_1^2$$

$$\frac{h_C}{2}L^2 - h_CLT_1 + \frac{s_C}{2}T^2 - s_CTT_1$$

$$+\frac{\theta(d_C+p_C)}{6}T_1^3 + \frac{\theta(d_C+p_C)}{3}L^3 - \frac{\theta(d_C+p_C)}{2}T_1L^2$$

$$+\frac{\theta h_C}{12}T_1^4 - \frac{\theta h_C}{12}L^4 + \frac{\theta h_C}{6}T_1L^3\Big\}\Big]$$

The necessary conditions for $TC(L,T_1,T)$ to be minimum are

$$\frac{\partial TC(L,T_1,T)}{\partial L} = 0, \quad \frac{\partial TC(L,T_1,T)}{\partial T_1} = 0 \text{ and } \frac{\partial TC(L,T_1,T)}{\partial T} = 0.$$

On solving these equations, we find the optimum values of L, T1 and T for which the total variable inventory cost is minimum The sufficient conditions for TC(L,T1,T) to be minimum are that the principal minors of Hessian matrix or H matrix are positive definite. The Hessian matrix is defined as follows:

$$H = \begin{bmatrix} \frac{\partial^2 TC(L,T_1,T)}{\partial L^2} & \frac{\partial^2 TC(L,T_1,T)}{\partial L\partial T_1} & \frac{\partial^2 TC(L,T_1,T)}{\partial L\partial T} \\ \frac{\partial^2 TC(L,T_1,T)}{\partial T_1\partial L} & \frac{\partial^2 TC(L,T_1,T)}{\partial T_1^2} & \frac{\partial^2 TC(L,T_1,T)}{\partial T_1\partial T} \\ \frac{\partial^2 TC(L,T_1,T)}{\partial T\partial L} & \frac{\partial^2 TC(L,T_1,T)}{\partial T\partial T_1} & \frac{\partial^2 TC(L,T_1,T)}{\partial T^2} \end{bmatrix}$$

Partially differentiating equation (13), we have

$$\frac{\partial TC(L,T_1,T)}{\partial L} = \frac{a\,p^{b}}{T}\Big[-p_C + h_CL - h_CT_1 + \theta(d_C +$$

$$p_C)L^2 - \theta(d_C+p_C)T_1L - \frac{\theta h_C}{3}L^3 + \frac{\theta h_C}{2}T_1L^2\Big]$$

$$\frac{\partial TC(L,T_1,T)}{\partial T_1} = \frac{a\,p^{-b}}{T}\Big[2p_C + (h_C+s_C)T_1 - h_CL - s_CT$$

$$+\frac{\theta(d_C+p_C)}{2}T_1^2 - \frac{\theta(d_C+p_C)}{2}L^2 + \frac{\theta h_C}{3}T_1^3$$

$$+\frac{\theta h_C}{6}L^3\Big]$$

$$\frac{\partial TC(L,T_1,T)}{\partial T} = \frac{a\,p^{-b}}{T}[-p_C + s_CT - s_CT_1] - \frac{1}{T^2}\Big[o_C$$

$$+a\,p^{-b}\left\{2p_C T_1 - p_C L - p_C T + \frac{(h_C+s_C)}{2}T_1^2\right.$$

$$+\frac{h_C}{2}L^2 - h_C L T_1 + \frac{s_C}{2}T^2 - s_C T T_1 + \frac{\theta(d_C+p_C)}{6}T_1^3$$

$$+\frac{\theta(d_C+p_C)}{3}L^3 - \frac{\theta(d_C+p_C)}{2}T_1 L^2$$

$$\left.+\frac{\theta h_C}{12}T_1^2 - \frac{\theta h_C}{12}L^4 + \frac{\theta h_C}{6}T_1 L^3\right\}\Bigg]$$

$$\frac{\partial^2 TC(L,T_1,T)}{\partial L^2} = \frac{a\,p^{-b}}{T}\left[h_C + 2\theta(d_C+p_C)L - \theta\right.$$

$$\frac{\partial^2 TC(L,T_1,L)}{\partial T_1^2} = \frac{a\,p^{-b}}{T}\left[(h_C+s_C)+\theta(d_C+p_C)T_1+\theta h_C T_1^2\right]$$

$$\frac{\partial^2 TC(L,T_1,)}{\partial L\partial T_1} = \frac{a\,p^{-b}}{T}\left[-h_C - \theta(d_C+p_C)L + \frac{\theta h_C}{2}L^2\right]$$

$$\frac{\partial^2 TC(L,T_1,T)}{\partial L\partial T} = -\frac{a\,p^{-b}}{T^2}\left[-p_C + h_C L - h_C T_1 + \theta(d_C+p_C)\right.$$

$$\left.L^2 - \theta(d_C+p_C)T_1 L - \frac{\theta h_C}{3}L^3 + \frac{\theta h_C}{2}T_1 L^2\right]$$

$$\frac{\partial^2 TC(L,T_1,T)}{\partial T_1\partial L} = \frac{a\,p^{-b}}{T}\left[-h_C - \theta(d_C+p_C)L + \frac{\theta h_C}{2}L^2\right]$$

$$\frac{\partial^2 TC(L,T_1,T)}{\partial T_1\partial T} = -\frac{a\,p^{-b}s_C}{T} - \frac{a\,p^{-b}}{T^2}\left[2p_C - \frac{\theta(d_C+p_C)}{2}L^2\right.$$

$$+(h_C+s_C)T_1 - h_C L - s_C T + \frac{\theta(d_C+p_C)}{2}T_1^2$$

$$\left.+\frac{\theta h_C}{3}T_1^2 + \frac{\theta h_C}{6}L^3\right]$$

$$\frac{\partial^2 TC(L,T_1,T)}{\partial T\partial L} = -\frac{a\,p^{-b}}{T^2}\left[-p_C + h_C L - h_C T_1 + \theta(d_C+p_C)L^2\right.$$

$$\frac{\partial^2 TC(L,T_1,T)}{\partial T\partial T_1} = -\frac{a\,p^{-b}s_C}{T} - \frac{a\,p^{-b}}{T^2}\left[2p_C + (h_C +\right.$$

$$\left.p_C)T_1 - h_C L - s_C T + \frac{\theta(d_C+p_C)}{2}T_1^2\right.$$

$$\left.-\frac{\theta(d_C+p_C)}{2}L^2 + \frac{\theta h_C}{3}T_1^3 + \frac{\theta h_C}{6}L^3\right]$$

Numerically, the Hessian matrix or H matrix is given by

$$H = \begin{bmatrix} -17.8959 & 17.1981 & 0.0131 \\ 17.1981 & 29.5256 & 13.8449 \\ 0.0131 & -14.2443 & 9.2057 \end{bmatrix}$$

4. NUMERICAL EXAMPLE

Let us consider the following data for parameters in the appropriate units as follows

$a=300,\ b=1,\ o_C=100,\ h_C=5,\ d_C=2,$

$s_C=8,\ p_C=10,\ p=25,\ \theta=0.05$

?	L	T_1	T	$TC(L,T_1,T)$
0.05	15.2579	6.0163	10.4284	303.5640
0.10	12.4212	4.4046	8.8553	307.2574
0.15	11.2630	3.7017	8.4324	334.1419
0.20	10.6132	3.2903	8.3558	366.2981
0.25	10.1905	3.0164	8.4296	399.6628

Table 1, variation in total inventory cost with respect to a

From the table 1, we see that if we increase the deterioration parameter ? then the values of L, T_1 and T are decreased, but thevalues of $TC(L,T_1,T)$ get increased.

a	L	T_1	T	$TC(L,T_1,T)$
300	15.2579	6.0163	10.4284	303.5640
400	15.2507	6.0088	10.3936	401.0250
500	15.2449	6.0028	10.3660	498.3465
600	15.2423	6.0000	10.3531	595.7901
700	15.2404	5.9981	10.3439	693.2439

Table 2, variation in total inventory cost with respect to a

From this table, we see that if we increase the demand parameter a, then the values of values of TC(L, T_1, T) get increased. L, T_1 and T are decreased, but the values of $TC(L, T_1, T)$ get increased.

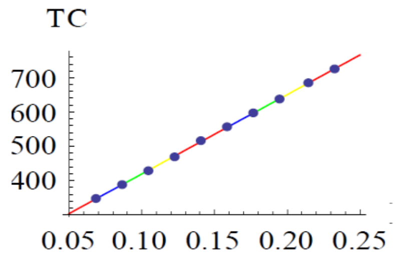

Figure 2, variation in TC with respect to θ ,

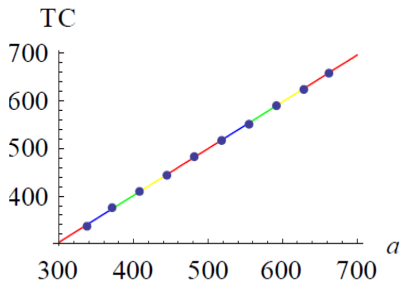

Figure 3, variation in TC with respect to a

b	L	T_1	T	$TC(L,T_1,T)$
1	15.2579	6.0163	10.4284	303.5640
2	15.8578	6.6203	13.4213	21.3155
3	19.9962	10.1909	40.6606	4.5005
4	30.4007	17.8134	183.5960	1.0255

Table 3, variation in total inventory cost with respect to b

From this table, we see that if we increase the demand parameter b, then the values of L, T_1 and T are increased, but the values of TC(L, T_1, T) get decreased.

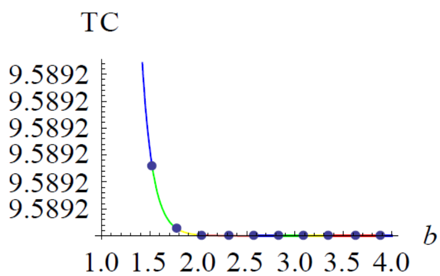

Figure 4, variation in TC with respect to b

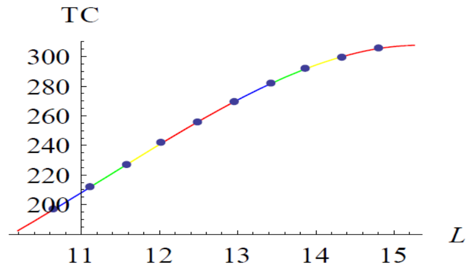

Figure 5, variation in TC with respect to L

5. CONCLUSION

The results of the proposed model show that the total variable inventory cost is deeply impacted by the parameters a and b in comparison with the parameter □ . This is due to the reason that the newly arrived goods/products in the super market increase the demand. The cycle length and lead time are main components for optimizing the cost/profit of an organization. The products such as vegetables, milk, bakery products and news papers are necessarily to be sold in the market as the cycle length decreases.

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