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Random Walks in Statistical Theory of Communication

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Abstract - *In a statistical theory of communication, the probabilistic approach helps the random process to achieve some specific objectives. A random walk is an example of a random process. A random walk is a random process that describes a path that consists of a succession of random steps in some mathematical space. In statistical theory, the random walks are of two types namely, Discrete type random walks and continuous-time random walks. The continuous-type random walk is an example of a 'Brownian motion'. In this article, we aim to provide a comprehensive review of classical random walks. We first review the knowledge of classical random walks and Brownian motion, including basic concepts and some typical algorithms. Then we introduce the actual representation of random walks in 2-D and 3-D spaces. This study aims to contribute to this growing area of research by exploring random walks and their applications.*

Key Words: **Random walks, Statistical theory, Brownian motion, Communication, Random process, Bernoulli trials**

1. INTRODUCTION

A random walk is a mathematical object, known as a stochastic or random process, that describes a path that consists of a succession of random steps in some mathematical space. The concept of random walks is purely based on the probabilistic approach in a statistical theory. Many real-life phenomena can be modelled quite faithfully using a random walk. The motion of gas molecules in a diffusion process, thermal noise phenomena, and the stock value variations of a particular stock are supposed to vary in consequence of successive collisions/occurrences of some sort of random impulses [1]. In computer networks, random walks can model the number of transmission packets buffered at a server. In the same field, random walks are used for fog computing using a load balancer [3]. In image segmentation, random walks are used to determine the labels (i.e., "object" or "background") to associate with each pixel [4]. Random walks have a wide range of applications in the medical field. In population genetics, a random walk describes the statistical properties of genetic drift. In brain research, random walks and reinforced random walks are used to model cascades of neuron firing in the brain [6]. Random walks have also been used to sample massive online graphs such as online social networks [5]. By using different types of continuous and discrete type random variables the probability of the random walks can be calculated. The probability theory is the backbone of random walks and it enables the process simplicity to measure or to control complex engineering and scientific problems. In particular, this random walk model will enable us to study the long-time behavior of a prolonged series of individual observations.

2. Methodology

Consider a sequence of independent random variables that assume values +1 and -1 with probabilities 'p' and 'q = 1 – p', respectively. A natural example is the sequence of bernoulli trials $X_1, X_2, \ldots\ldots X_n$ with probability of success equal to 'p' in each trial. where X_k = + 1 if the kth trial results in a success and X_k = -1 otherwise. Let Sn denote the partial sum

$$s_n = x_1 + x_2 + \cdots + x_n \qquad (2.1)$$

that represents the accumulated positive or negative excess at the nth trial. In a random walk model, the particle takes a unit step up or down at regular intervals, and Sn represents the position of the particle at the nth step. The figure 2.1 describes the basic operation of random walks.

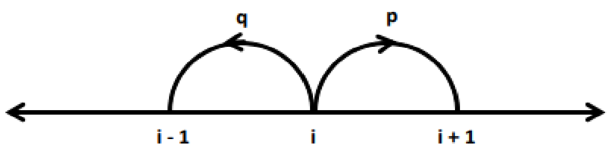

Fig 2.1: Basic random walk

In n successive steps, 'return to the origin (or zero)' that represents the return of the random walk to the starting point is a noteworthy event since the process starts all over again from that point onward.

To compute the probability of this event, let {Sn = r} represent the event at stage n, the particle is at the point r and Pn,r its probability.

$$p_{n,r} \triangleq P\{s_n = r\} = \binom{n}{k} p^k q^{n-k} \qquad (2.2)$$

where k represents the number of successes in n trials and n - k the number of failures. The net gain r = k – (n-k) = 2k – n. Therefore equation (2.2) results in

$$p_{n,r} = \binom{n}{(n+r)/2} p^{(n+r)/2} q^{(n-r)/2} \tag{2.3}$$

where the binomial coefficient is understood to be zero unless (n + r) /2 is an integer between 0 and n, both inclusive.

2.1 Return to the origin case

If the accumulated number of successes and failures are equal at stage n, then Sn= 0, and the random walk has returned to the origin. In that case, r = 0 or n=2k so that n is necessarily even, and the probability of return at the '2n'th trail is given by

$$P\{s_n = 0\} = \binom{2n}{n} (pq)^n \triangleq u_{2n} \tag{2.1.1}$$

On solving the combination part of the equation (2.1.1), the actual or simplified equation for the return to the origin case is given as

$$u_{2n} = (-1)^n \binom{-1/2}{n} (4pq)^n \tag{2.1.2}$$

3. RESULTS AND DISCUSSION

In random walks the particle motion is purely dependent on the probability of success. If the event results in the success, then particle will move to pre-defined path. The generation of random probabilities is the important operation in the execution of random walk model.

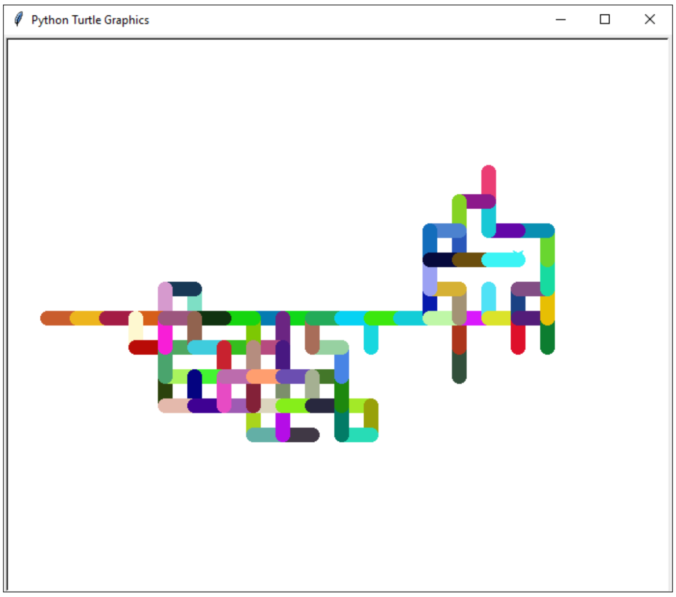

Fig 3.1: 2-D Random walk

In 2-D random walk, the particle motion is decided by the directions [0 = Right, 180 = Left, 90 = Up, 270 = Down]. The particle randomly chooses any direction to complete the random walk. Figure 3.1 shows the 2-D random walks in turtle graphics.

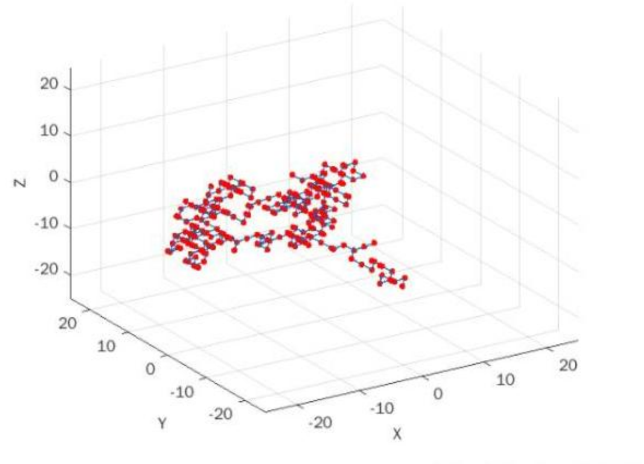

Fig 3.2: 3-D Random walk

The coverage of random walks is not limited only to the 2-D. For various applications of computing the dimensions are extended to 3-D. Figure 3.2 described the particle motion representing random walk in a 3-D space.

When a continuous type random variable distributions are considered to generate a random walk, it is also known as 'Brownian Motion'. The brownian motion model is used in real time applications for comparison of different processes. Figure 3.3 indicates the 3-D brownian motion for different type of processes.

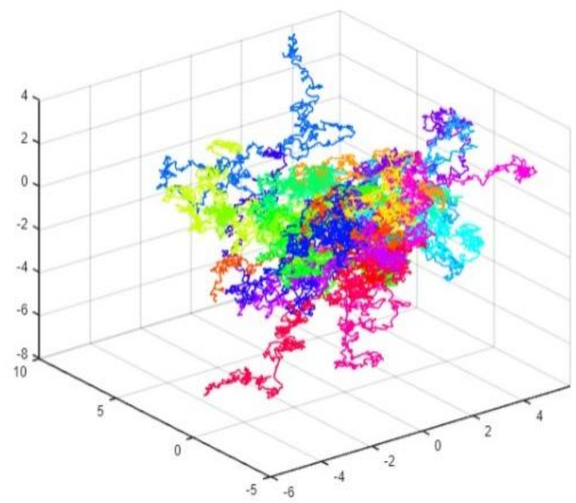

Fig 3.3: 3-D Brownian motion

4. CONCLUSION

In this article, we have designed a random walk model that generates a random motion of the particle in 2-D and 3-D space. Bernoulli trial approach is suitable to assign a value to random variables depending upon the direction of the motion. 'Return to the origin' is important case for particle tracking. The probability of such event can be calculated with a binomial distribution in random walk domain. Brownian motion can be achieved by replacing discrete type random variable distributions with the continuous one. The fundamental approach of random walk model can be used to solve complex problems in science, engineering and medical fields.

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