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Mathematical Modeling and Design of a Rack and Pinion Steering Mechanism

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Abstract: *Steering is an important part of an automobile. It helps to change the directions of the automobile and also helps in the straight line stability of the vehicle. This paper focuses on mathematical modeling and design of a rack and pinion steering mechanism of a bio-hybrid vehicle. We have developed a set of mathematical equations that governs the complete design of steering and later prepared the CAD model. By solving these equations we can get different steering geometry parameters , by fixing some variables according to restriction and considering optimum steering geometry with respect to steering effort and percentage ackerman. This model can be used for ackerman as well as reverse ackerman steering geometry and further it can be used for two wheel steering as well as four wheel steering by applying this model on front and rear steering design.*

Keywords : Ackerman principle, rack and pinion, tie-rods, steering effort, steering ratio, toe-zero, bio-hybrid

Introduction:

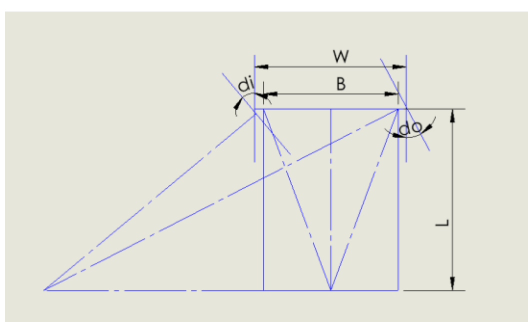
Steering mechanism is designed in a way that it meets the ackerman principle i.e. in order for a vehicle to perform a pure turning, the I-centers of all the wheels should meet at one single point.

We have achieved this condition using rack-and pinion gear box in our project. The pinion gear is rotated when the steering wheel is rotated. The output shaft from the steering wheel and the input shaft to the pinon gear are connected by a universal coupling.

The rotational motion of the pinion gear causes the rack to move transversally which in turn pushes the tie-rod and the tie-rod helps the wheels to turn by pushing the steering arm.

Ackerman condition:

Ackerman condition for a two wheeled steering is expressed as:



Here:

W = Track width

B = distance between left and right knuckles

L = Wheel base of the vehicle

do = outer wheel ange

di = inner wheel angle

Using the basic laws of trigonometry we arrive at the following equation:

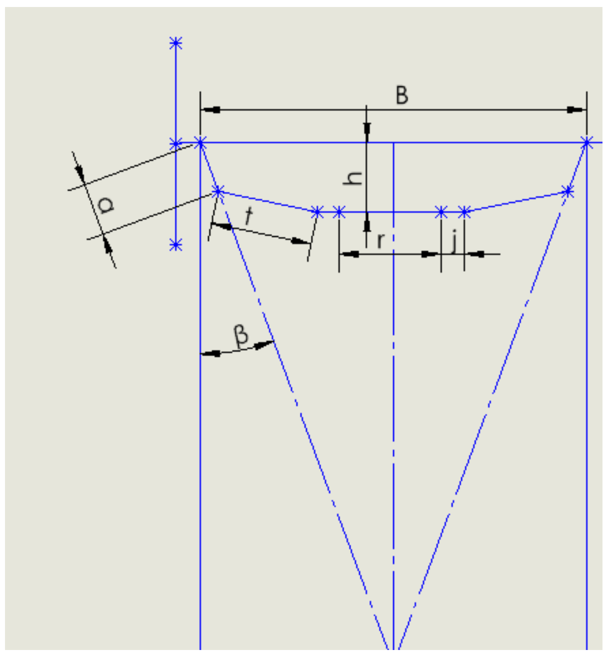
cotdo - cotdi = B/L ……………………………………….(1)

This is the ackerman condition for a two-wheeled steering.

When ackerman condition is satisfied in a steering mechanism the vehicle takes a turn. The inner wheel needs to be turned more than the outer wheel in order for the condition to be satisfied.

Mathematical model for the rack and pinion steering:

Here are the list of various steering parameters in the case of rack and pinion steering:



Where:

a = steering arm length

t = tie-rod length

B = Track width

j+2r = rack ball joint center to center length

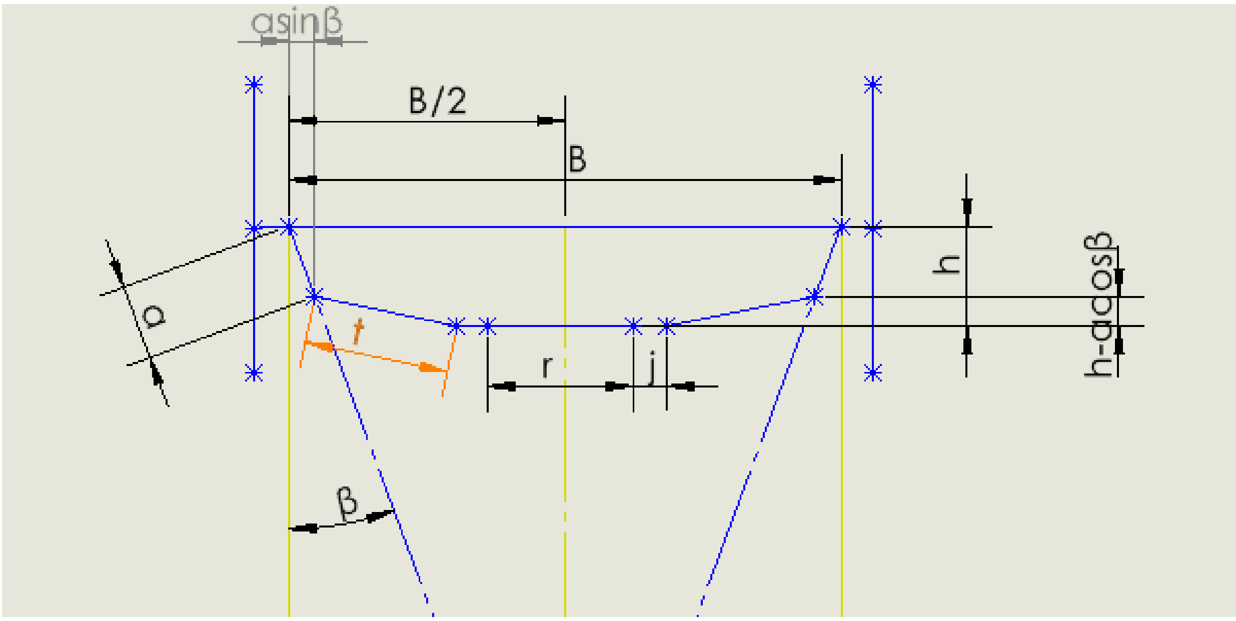
β = ackerman angle

h = distance between front axis and rack center axis

q = travel of rack

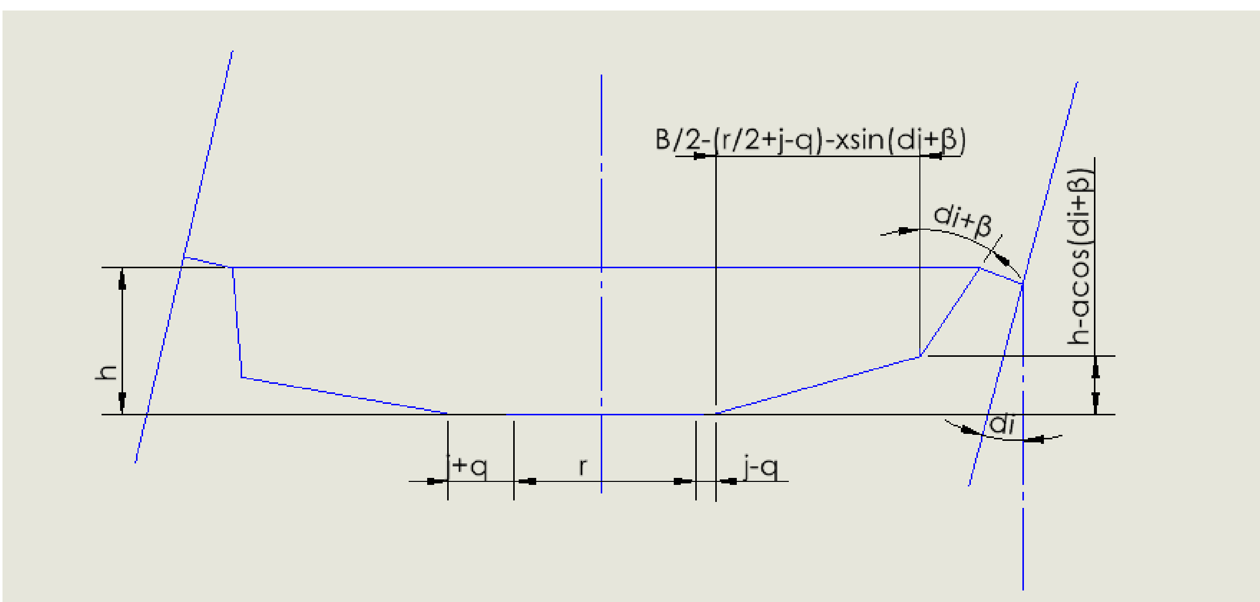
Now the mathematics involved in writing all the equations is just the pythagoras theorem

Equation for toe-zero condition:



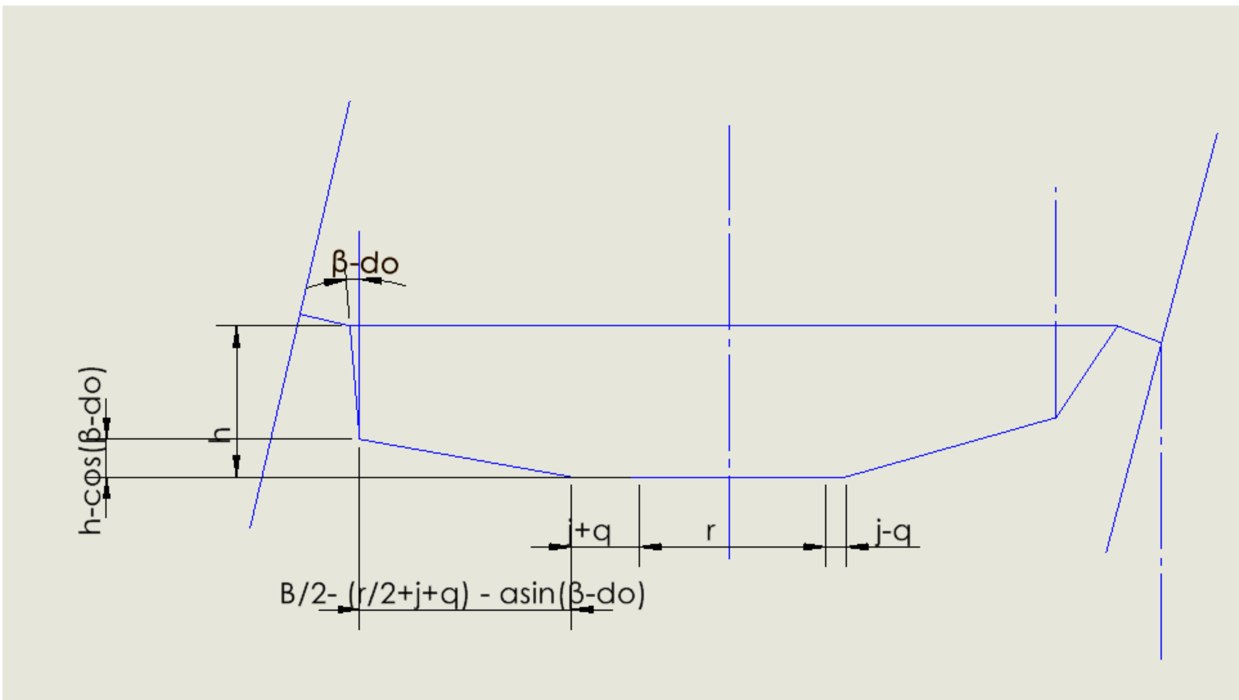
$t^2 = [\{B-(r+2j)\}/2 - a\sin\beta]^2 + [h-a\cos\beta]^2$ ……………………………………………………(2)

Equation from inner wheel geometry



$t^2 = [B/2-(r/2+j-q) - a\sin(di+\beta)]^2 + [h-a\cos(di+\beta)]^2$……………………………………………(3)

Equation from outer wheel geometry:

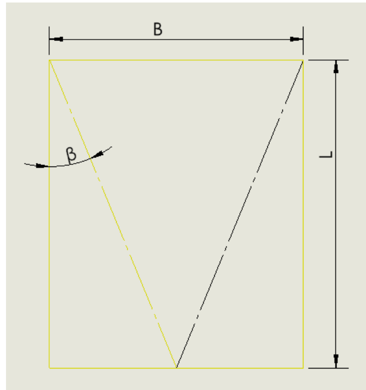


$t^2 = [B/2-(r/2+j+q)-a\sin(\beta-do)]^2 + [h-\cos(\beta-do)]^2$………………………………………………..(4)

Equations (2),(3) and (4) denote the governing equations of the steering mechanism

Methodology:

1. The value of B is fixed by the track width of the vehicle and knuckle to knuckle distance.
2. For a given rack and pinion mechanism, the value of r and j is known
3. Ackerman angle value is fixed by the wheelbase(L) and the track width(B).



4. For any given value of inner angle turn(di) the value of outer angle turn can be calculated as:

   cotdo - cotdi = B/L

do = cot^{-1}(B/L+cotdi)...........................(5)

5. Off all the variables that have appeared in the above equations, we have to calculate the value of a,h,t and q. We have three equations and four variables so we have to fix one in order to obtain the other.

6. Here the value of the steering arm can be fixed as it depends upon the knuckle and frame design of the vehicle.

7. Now, we have 3 equations and 3 unknowns. We can obtain all the required values.

Experimental setup:

The vehicle for which we are designing the steering is a bio-hybrid vehicle. The specifications of it are:

1. Track width(W) = 33inches = 83.82 cm
2. Knuckle to knuckle width(B) = 25inches = 63.5 cm
3. Wheel base(L) = 48inches = 121.92cm
4. Ackerman angle(β) = 14.6 degrees
5. Length of rack(r) = 6inches = 15.24cm
6. Radius of ball joints(j) = 1inch = 2.54 cm
7. Steering arm(a) = 2inches = 5.08cm

We are designing the steering for an inner wheel turn of 49 degrees and outer wheel turn of 33 degrees as given by the ackerman condition.

Putting these values in the above equations, we are left with the following equations:

$t^2 = [21.6 - 0.25a]^2 + [h - 0.96a]^2$..(6)

$t^2 = [21.6+q-0.89a]^2 + [h-0.44a]^2$..(7)

$t^2 = [21.6\text{-}q+0.32a]^2 + [h\text{-}0.95a]^2$……………………………………………………….(8)

Upon solving these three equations, we get the following results:

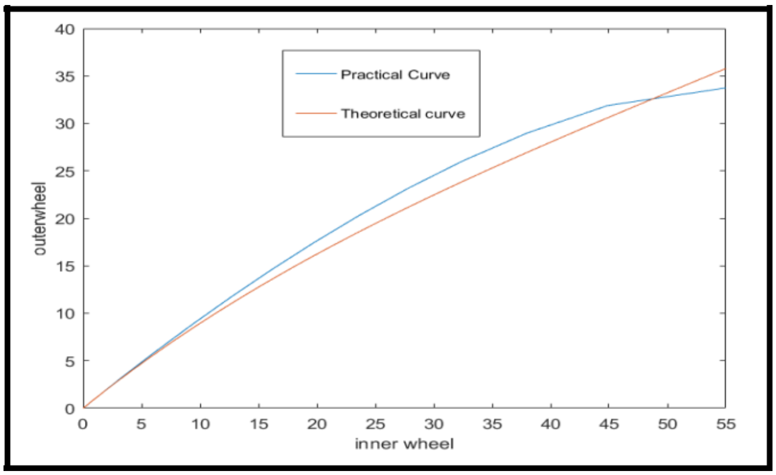
Tie-rod length(t) = 8inches = 20.32cm

Rack travel(q) = 1.2 inch = 3.048cm

Distance from wheel axis to tie rod(h) =2.5inches = 6.25cm

In order to check weather our steering have satisfied the ackerman criteria or not, we plot the theoretical graph and compare it with the graph obtained from our calculation.

Theoretical graph can be plotted using (5) and we can use (3) and (4) in order to plot the practical graph after all the values have been obtained.



This is the comparison of our steering mechanism with theoretical ackerman

Calculation and selection of other steering parameters:

A pinion diameter of 42 mm and module 2 is selected and then other calculations are performed:

1. Steering ratio:

   This is defined as the ratio of total angle turned by steering(left + right) wheel in lock positions to the sum of total steering angle of tires.

   Angle turned by steering wheels = {360/(**π***pinion diameter)}*q = (360/(**π***4.2))*360

   = 166 degrees

Steering ratio = Angle turned by steering wheel/(total angle turned by wheels)

= 166/(49+33)

= 2.028 : 1

2. Number of teeth on rack and pinion :

For pinion
Number of teeth = circumference/module = diameter/module = 42/2 = 21

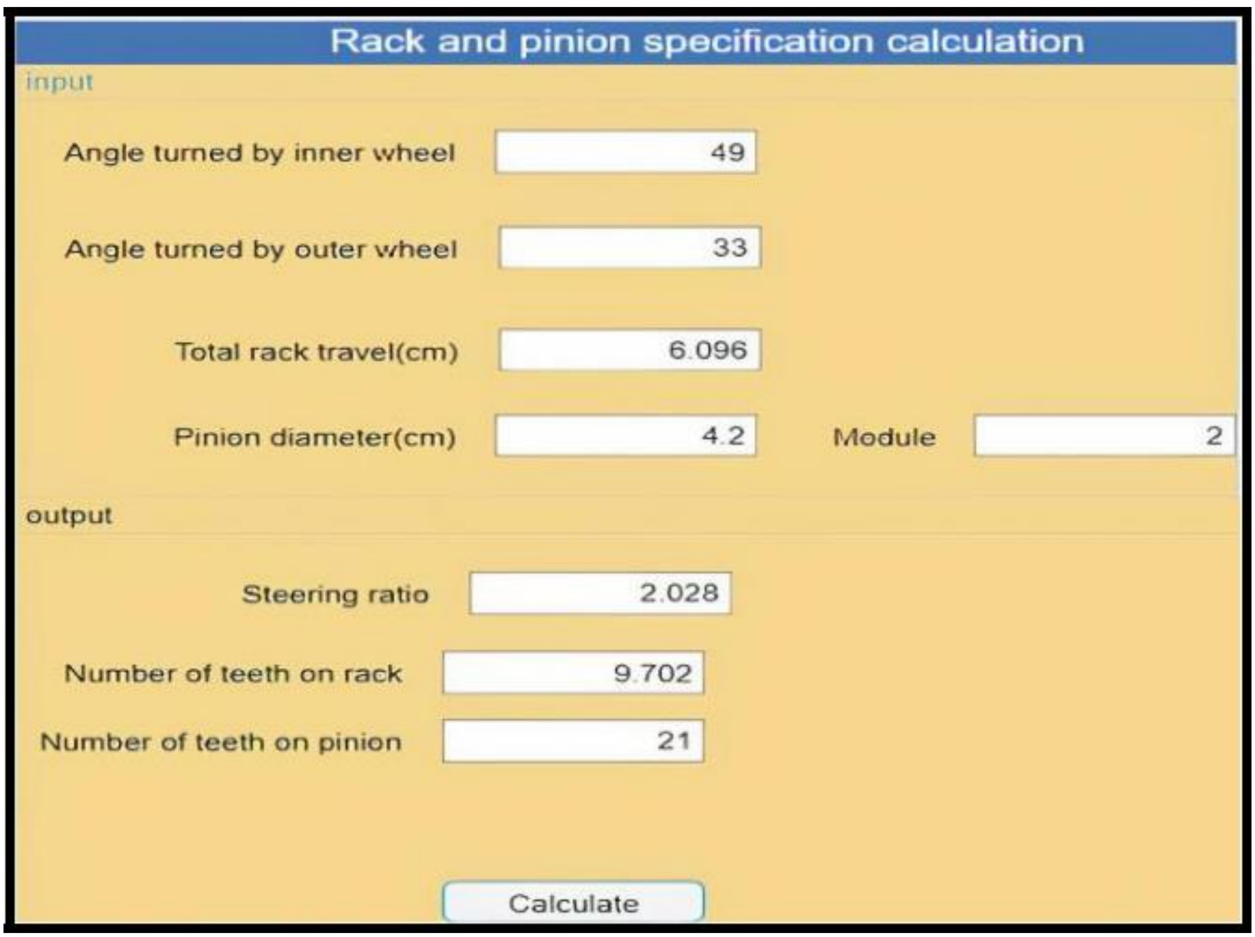
For rack

Number of teeth = (number of teeth on pinion/360)*angle turned by the steering wheel

= (21/360)*166

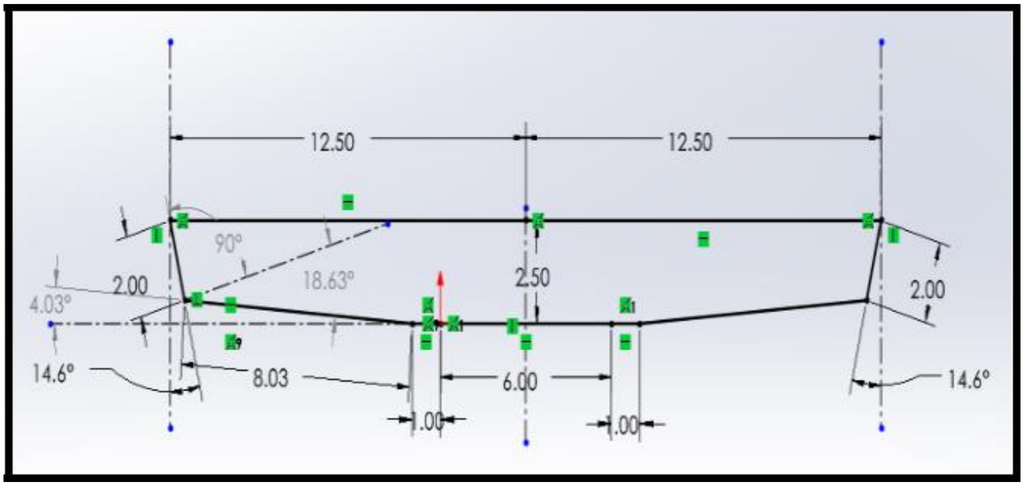
= 9.68 = 10 teeth

A GUI was also developed on MATLAB for the calculation of these parameters



3. Calculation of steering effort:

Steering effort is defined as the effort to be made by the driver in turning the steering wheel. This can be calculated in either static condition i.e. when the vehicle is stationary and in dynamic conditions. Steering effort is maximum when the vehicle is stationary.



The above diagram(all dimensions in inches) could be used for the calculation of steering effort.

Mass of vehicle = 142+20 (payload of 20kg)+95 (with driver of 95kg) = 257kg

Lets suppose the weight of the vehicle is equally distributed in all four wheels

Reaction at each tyres = 257/4 = 64.25kg = 629.65N

Frictional force at the tyres = **μ***R = .6*629.65 = 377.79N[taking **μ** = .6]

We need a force of 377.79N on the steering arm to turn the wheels.

Force on the tie-rod = 377.79/cos(18.4)

= 398.18N

Force on the rack = 398.18/cos(4.03) = 402.2N

This force has to be transmitted by the pinion through the steering wheel to the rack.

Torque on steering wheel = Torque on rack = Force on rack*radius of pinion = 402.2*0.021

= 8.4462Nm

Fig: fully assembled steering(designed on SOLIDWORKS)

Conclusion:

The paper above presents the complete design of a rack and pinion steering mechanism for an bio-hybrid vehicle. The mathematical model presented can also be used to design steering for rear wheels. We can vary the value of the steering arm and perform many iterations in order to optimize the steering geometry and make it as close to the ideal ackerman graph. We used softwares like MATLAB, FUSION360 and SOLIDWORKS for the project.

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