Automata Theory Models and Algorithms for Controlling an Aircraft Group in Aviation sector

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Abstract - *Models and algorithms have been designed to monitor the landing phase when the location of the aircraft in space is unknown due to undesirable, unforeseen external influences. The dispatcher's command execution time is regarded as the objective feature, and criteria for aircraft safety are added as constraints. An approach has been developed based on the developed formalism of generalized synchronization of linear automata, which allows this problem to be reduced to an integer linear programming problem. The use of established models and methods to monitor the aircraft landing process has been considered as an example.*

***Key Words*: synchronisation, generalised state machine, essential combinations of events, aircraft, aviation, linear automata, finite state machine**

1.INTRODUCTION

In the real-world activity of complex systems, unfavourable environmental conditions contribute to uncertainty in the location of their components in space and time. When the activities of device components become disjointed, incidents that aren't harmful under normal circumstances can combine to cause accidents and disasters.

The problem in aviation transportation systems is to restore a stable cooperative arrangement of aircraft after an ambiguity arises due to unexpected external factors, such as climatic or radio-electronic influences. For pilots and dispatchers, solving this issue is followed by a high degree of informational and psychological tension in real time.

It is proposed that the established mathematical models, methods, and algorithms for generalised synchronisation of finite automata be used to assist the dispatcher in making decisions that will improve the efficiency of controlling a group of aircraft in adverse environmental conditions.

2. PROBLEM SETTING

Assume that a group of aircraft is now in a potentially dangerous situation as a result of a short-term loss of contact caused by environmental effects or other means.

We need to get the group of aircraft out of the potentially dangerous situation as soon as possible after contact is restored. The knowledge and advisory system produces and presents to the dispatcher a number of solutions to the issue. Based on his experience, professionalism, and common sense, the dispatcher selects the safest option from among them.

The following is the formal problem statement. A group of aircraft planning to land in the airport's area of responsibility is denoted by the letter A. Each aircraft's state as it enters A is defined by its location at some assumed altitude level.

The state of this group is determined by the combination of aircraft states from group A, which can be formalized with a vector whose coordinates correspond to the conditions of individual vessels. A set of corresponding vectors can thus be used to define the set of possible states for a group. J represents the set of possible states for the group.

A group of aircraft from the set J0 J is in the s(0) state as a result of a temporary lack of contact with the dispatcher. The set J0 is considered to include states that are listed as hazardous, such as states with dangerous aircraft proximity. As a result, the current state of a group of planes is regarded as potentially hazardous and undesirable.

A subset Js J of safe conditions for a group of aircraft also exists. The aim is to create methods, models, and algorithms that allow for the discovery of a sequence of signals u(t1),..., u(tk) from the set of admissible control actions U in the time interval [t0, t1] allotted for solving this problem, as a result of which a group of aircraft can move into some state from any initial state from the set J0, for all admissible environmental influences x(t) X(t).

The equation for the above scenario has been given below:-

$$W = \sum_{i=1}^{k} F(u(t_i), x(t_i), t_i) \to \min, \quad (1)$$

where F(u(ti), x(ti), ti) is a given function that describes the dispatcher's command execution period.

3. THE PROBLEM SOLVING MODEL, PROCESS, AND APPROACH

Since the number of aircraft and the possibilities for their position at various flight levels are restricted, it is proposed that the problem be solved using methods from the theory of finite automata.

The theory of finite automata has a formalism that is both rigorous and straightforward in describing its solutions, as well as effective in terms of implementing the results in terms of time and number of computational operations.

The vector designating the position of aircraft at flight levels 1, 2,...,n is called the condition of the finite state machine in the problem at hand (FSM). The input signal of the system is a series of dispatcher instructions given to aircraft at these levels for a limited period of time.

The concept of automata synchronization can be used to solve the problem. A set of input signals for an FSM u(0),..., u(k 1) is said to be synchronizing if it holds for all initial states s(0),s (0).

$$f(\mathbf{s}(0), \mathbf{u}(0), \ldots, \mathbf{u}(k-1)) = f(\mathbf{s}'(0), \mathbf{u}(0), \ldots, \mathbf{u}(k-1)),$$

where f(s(0), u(0),..., u(k 1)) is the state through which the machine transfers from state s(0) under the control of the input sequence u(0),..., u(k 1), i.e., the synchronizing sequence enables the FSM to be transferred into a predefined final state under unknown initial conditions.

The dispatcher will ensure a known configuration of the aircraft arrangement under conditions of uncertainty about their initial state by selecting the shortest synchronizing sequence and supplying the shortest sequence of commands.

The problem of finding synchronization sequences for general-purpose automata reduces to finding solutions for synchronizing trees, which in the worst case can exceed the height O(n3), where n is the number of states in the machine (it can be calculated by O(n2) according to the Cerny hypothesis). In most cases, there can only be one synchronization sequence.

In general, if the requisite final set does not have to be a single state, it is a good idea to think about synchronization in a broad sense. The generalized state is a set of states defined as if the states of the system are represented by vectors of elements of a certain set.

$$\bar{s} = \{s \in J | [s]_v = [\bar{s}]_v\},$$

where [s]v is a vector of dimension v that characterizes this generalized state s, and [s]v is a vector with coordinates of the vector S with indices 1,..., v. A generalized state is, in essence, a subset of the set of all possible states J for which those coordinates are equal to given ones.

If an input sequence takes an automaton to a finite generalized state that is independent of its initial state, it is called generalized synchronizing. Even if a synchronizing sequence of the general form does not exist, a generic synchronization sequence for an FSM may exist.

It has been shown that the existence of a generalized synchronization sequence of length k for an FSM with a linear transition function implies the existence of all sequences of length at least k, and that finding the synchronization sequence in this case reduces to solving a system of log2 N linear algebraic equations, where N is the number of states in the FSM.

The representation of an FSM in the form of an automaton with a linear transition function has been considered, as well as algorithms for solving this problem.

The following steps of problem-solving have been suggested.

- Construct the FSM model and create a relationship between state machine components, aircraft system state, and dispatcher commands.
- Ascertain that the resulting FSM is linear or linearizable.
- If the model is linear, find synchronizing sequences using generalized synchronization for linear FSMs; if the model is general type, use the corresponding synchronization formalism.
- Analyze the data and come up with dispatcher directives to solve the problem.

4. THE PROBLEM'S SOLUTION

Consider the situation where the presence of four aircraft at four different altitude levels determines the state s(t) of the device. If si(t) = 1, one aircraft is on level I and if si(t)=0, one or more aircraft have moved to a single level.

0-1 vectors of dimension 4 are often used to describe dispatcher commands associated with FSM input signals. If ui(t) = 1, the aircraft on level I must change levels, and if ui(t)=0, the aircraft on level I receives no order.

Consider the linear case, where the state of the aircraft group s(t + 1) is determined by the relation after executing the command corresponding to a signal u(t).

$$\mathbf{s}(t+1) = A\mathbf{s}(t) + B\mathbf{u}(t), \qquad (2)$$

The characteristic matrices are defined as follows:

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and all operations are carried out on the GF sector (2).

The safe states (1, 1, 1, 1)T and (1, 1, 1, 0)T can be interpreted as a generalized state s = (1, 1, 1,...)T, where the last coordinate can take the values 0 or 1.

The number of "drops," or pairs of different neighboring coordinates of input vectors, is used as a measure of control efficiency. Then condition (1) takes the following shape:

$$W = \sum_{i=1}^{4k-1} G(\bar{U}_i, \bar{U}_{i+1}) \to \min, \quad (3)$$

where U I is the coordinate of the united vector of input actions, G(U i,U i+1) = 0 if U I = U i+1 and G(U i,U i+1) = 1 if U I = U i+1, and k is the length of the desired sequence of input actions (it is assumed that this number should be as small as possible, since each new command takes time from the controller and the system of altitude levels to complete the landing).

The degree of the main characteristic matrix A that appears in (2) has been found to verify the generalized synchronization condition:

$$A^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad A^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

Specifically, [A3]3 = [0], implying that this FSM is generalized synchronically in the first three coordinates and that all sequences of length three or more are generalized synchronizing.

The system's transformation from any initial state to the generalized state s has the form according to the complete transition formula for a linear automaton.

$$Q\bar{U} = [\bar{s}]_3 mod2, \quad (4)$$

In which

$$Q = \left[A^2B, AB, B\right]_3 = \begin{pmatrix} 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&0&0&0&1&0&0 \\ 1&0&0&0&1&1&0&0&0&0&1&0 \end{pmatrix},$$

where U = (u1(0),..., u4(0), u1(1),..., u4(1), u1(2),..., u4(2), u1(2),..., u4(2), u1(2),..., u4(2)) T. Using continuous numbering in order to represent the coordinates of vector U, condition (4) can be written as

$$\begin{cases} \bar{U}_9 - 2d_1 = 1 \\ \bar{U}_5 + \bar{U}_{10} - 2d_2 = 1 \\ \bar{U}_1 + \bar{U}_5 + \bar{U}_6 + \bar{U}_{11} - 2d_3 = 1, \end{cases}$$

where U I 0, 1, dj are non-negative integers, 1 I 12, 1 j 3 are non-negative integers. Notice that when the constraints on the coordinates of U are taken into account, the equations d1 = d2 = 0 and d3 = 1 follow immediately.

As a result of the above notation, the efficiency criterion (3) takes the following form:

$$\bar{W} = \sum_{i=1}^{11} G(\bar{U}_i, \bar{U}_{i+1}) \to \min \quad (5)$$

within the bounds U¯i ∈ {0, 1}, 1 i 12, of the form's variables As a result of the above notation, the efficiency criterion (3) takes the following

$$\begin{cases} \bar{U}_9 = 1 \\ \bar{U}_5 + \bar{U}_{10} = 1 \\ \bar{U}_1 + \bar{U}_5 + \bar{U}_6 + \bar{U}_{11} - 2d_3 = 1. \end{cases} \quad (6)$$

Notice that we can choose sets of consecutive vector U coordinates that are not included in the scheme of constraints (6): U 2, U 3, U 4, U 7, U 8, and separately the coordinate U12.

The structure of criterion (3) implies that the coordinates in the described sets should be equal to each other and to the nearest coordinate included in the constraints in order to ensure a minimum number of "drops" of coordinates in the appropriate vector U. (6). We may assume, for example, that these coordinates are independent.

$$\begin{aligned} &\bar{U}_2^* = \bar{U}_3^* = \bar{U}_4^* = \bar{U}_1^*, \\ &\bar{U}_7^* = \bar{U}_8^* = \bar{U}_9^* = 1, \\ &\bar{U}_{12}^* = \bar{U}_{11}^* \end{aligned} \quad (7)$$

Criteria (5) can be rewritten as

$$W = G(\bar{U}_1, \bar{U}_5) + G(\bar{U}_5, \bar{U}_6) + G(\bar{U}_6, \bar{U}_9) + G(\bar{U}_9, \bar{U}_{10}) + G(\bar{U}_{10}, \bar{U}_{11}) \to \min. \quad (8)$$

The problem was attempted to be solved using a well-known separation and estimation method, which is part of the branch-and-bound method family commonly used in integer programming.

Conditions (6) imply that U 5 = U 10, implying that among pairs (U 5,U 6),(U 6,U 9),(U 9,U 10) there is at least one "fall" and W(U) 1. It would be the one we are searching for if there is a value of U for which equality is achieved. Let's see if we can find this value U.
The vector d3 is used to divide the set of admissible values U. U 1 + U 5 + U 6 + U 11 = 3 for d3 = 1, implying that one of these numbers is 0 and the others are 1. If U 1 = U 5 at the same time, there are at least two "drops" among pairs (U 1,U 5),(U 5,U 6),(U 6,U 9) and (U 9,U 10); therefore, we consider the case U 1 = U 5.

Then U 1 = U 5 = 1, U 10 = 0, and U 6 = U 11. If U10 = U11, there are at least two "drops" between pairs (U1,U5),(U5,U6),(U6,U9),(U9,U10),(U10,U11), and when U10 = U11, we get U6 = 1, U11 = 0, and there is no "drop" between pairs (U1,U5), U5,U6), (U6,U9), (U9,U10), (U10,U11).

This value corresponds to the achievement of the minimal approximation W(U) 1 and allows one of the original problem solutions to be constructed using relations (7):

$$\bar{U}^* = (1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0)^{\mathrm{T}},$$

which is the same as the input sequence

$$\mathbf{u}(0) = (1,1,1,1)^{\mathrm{T}},$$
$$\mathbf{u}(1) = (1,1,1,1)^{\mathrm{T}}, \quad (9)$$
$$\mathbf{u}(2) = (1,0,0,0)^{\mathrm{T}},$$

and the system transitions to the necessary generalized state (1, 1, 1,...)T after this input series. Other problems have similar solutions.

As shown in the example below, the established model can be interpreted as a formalized explanation of the control process for a group of aircraft during their landing in the airport's zone of responsibility at stages such as entry into the zone of responsibility, vectoring, flight in the waiting area, and so on.

5. EXAMPLE

The dispatcher chooses logical and secure variants from the sequences ensuring that the device reaches the safe collection of states Js, which would avoid the dangerous proximity of different aircraft.

A sequence of directives is built based on these choices. The phases of landing an aircraft are as follows: flight to the entrance to the waiting area; flight in the waiting area; departure from the waiting area; transition to the first turning point; flight in a circle (on a "box"); pre-landing; alignment; standing; touch and run; taxi, according to aviation regulatory documents.

In a real case, the air traffic control centre is likely to be dealing with a large number of aircraft performing manoeuvres as they approach the airport's area of responsibility for landing.

The figure employs the following notation: c1,...,ck1—entry into the waiting area; d1,...,dk2—waiting area altitude levels; e1,...,ek3—transitions between altitude levels; f1,...,fk4—waiting area exits; RW—runway 1a, 2a, 3a, 4a,..., 1n, 2n, 3n, 4n—RWa,..., RWn turning points.

The table contains a model example of the directives chosen by a dispatcher of the "Bangalore-Approach" centre based on the values of the input signals in the generalised synchronization series, based on the above approach and materials from [16]. (9).

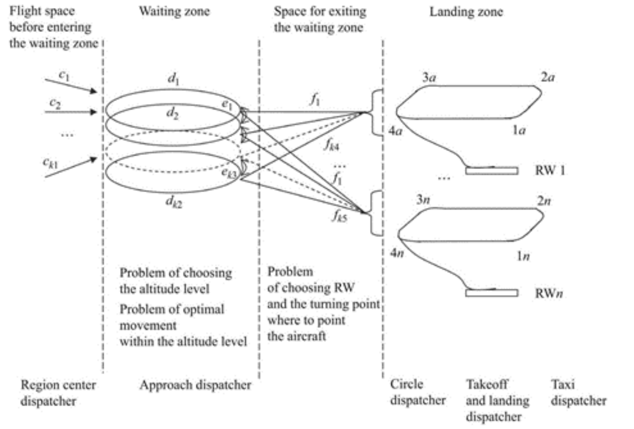

Fig -1: The general plan for the routine phase of aircraft entry into the airport's control area, including landing.

6. CONCLUSION

When landing aircraft in uncertain states, a method for constructing dispatcher directives has been proposed. The method is based on a formalism for generalised synchronization of FSMs with a linear transition function that has been developed.

The dispatcher produces landing directives from the shortest generalised synchronizing input sequences presented to him by the information and advisory framework. The findings of the study can be used to build and improve simulators, as well as for operational management of aircraft landings in the future.

The table displaying a real time example for better understanding of the situation is as follows:-

Table -1: Centered on the signals from the generalised synchronization series, a model example of the "Approach" dispatcher instructions

Input signal of the FSM	Flight	Dispatching point number	Directive
U1(0)	6E 477	"Bangalore-VOBL Approach"	Descend 2400 to Kilo
U2(0)	6E 342	"Bangalore-VOBL Approach"	On Bravo, stay at 5700
U3(0)	AI 804	"Bangalore-VOBL Approach"	Descend 1800 by Charlie
U4(0)	AI 127	"Bangalore-VOBL Approach"	Wait for the vector on RW 29 right
U1(1)	6E 477	"Bangalore-VOBL Approach"	Expect vectoring on RW 09 left

U3(1)	AI 804	"Bangalore-VOBL Approach"	Distance 40, on 1800 work with the circle 120.8
U4(1)	AI 127	"Bangalore-VOBL Approach"	Alpha 210, distance 125, RW 07, descend to 3rd
U1(2)	6E 477	"Bangalore-VOBL Approach"	Distance 80, for 2000 work with the circle 120.8
U2(2)	6E 342	"Bangalore-VOBL Approach"	Expect vectoring on RW 09 left
U3(2)	AI 804	"Bangalore-VOBL Approach"	Alpha 160, removal 100, RW 29, descend to the 2nd
U4(2)	AI 127	"Bangalore-VOBL Approach"	Alpha 210, removal 125, RW 07, descend to the 3rd

REFERENCES

[1] Lal, R., McKinnis, A., Hauptman, D., Keshmiri, S., & Prabhakar, P. (2021, July). Formally Verified Switching Logic for Recoverability of Aircraft Controller. In International Conference on Computer Aided Verification (pp. 566-579). Springer, Cham.M. Young, The Technical Writer's Handbook. Mill Valley, CA: University Science, 1989.

[2] Messaoud, M. B. (2021). A thorough review of aircraft landing operation from practical and theoretical standpoints at an airport which may include a single or multiple runways. Applied Soft Computing, 98, 106853.K. Elissa, "Title of paper if known," unpublished.

[3] Enayatollahi, F., Atashgah, M. A., Malaek, S. M. B., & Thulasiraman, P. (2021). PBN-based time-optimal terminal air traffic control using cellular automata. IEEE Transactions on Aerospace and Electronic Systems, 57(3), 1513-1523.

[4] Abbass, H. A. A., & Tang, J. T. (2018). Behavioral learning of aircraft landing sequencing using a society of Probabilistic Finite state Machines.

[5] Herasimov, S., MaksimPavlenko, E., & Lytvynenko, M. (2020). Aircraft flight route search method with the use of cellular automata. International Journal, 9(4).