Estimating Quadratic Cost Curve Coefficients of Thermal Power Plant Without Using Fuel Input Data

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Abstract - *Solving economic dispatch problems on a power system inevitably require the fuel cost curve coefficients. Therefore, accurate and frequent updating of these coefficients is very important to improve the accuracy of economic dispatch solutions. At times the data needed to compute the cost curve coefficients may not be available. This paper presents an approach to estimate the quadratic cost coefficients of a thermal power plant when the fuel input data is completely not available. The average heat rate at minimum operating capacity of the power plant is computed using the average heat rate at maximum capacity available on U. S. Energy Information Administration website and the type of turbine used. Intermediary heat rate values are computed using finite difference method. The fuel input corresponding to each heat rate and corresponding power output level is computed to generate the required fuel input versus power output data set. The cost curve coefficients are determined using least square estimation method. The proposal is demonstrated on some Nigeria thermal power plants. The coefficients obtained are compared with values available in recent literature. Coefficients obtained using this proposed method differ from the coefficients in recent literature by maximum of error of 27% over those in literature.*

Key Words: Cost curve coefficients, Economic Load Dispatch, Least Square Estimation, Finite difference method, Thermal Power Plant, Steam turbine valve point effect

1.INTRODUCTION

The input-output curve which is a plot of fuel input in British thermal unit per hour versus net real power output is a very important characteristics of a thermal power plant in economic operation perspective. There are other characteristic curves that indicate the economic performance of the thermal plant that can be derived from the input-output curve. They include fuel-cost curve which is the input-output curve scaled by the unit cost of fuel, heat-rate curve (ratio of the input fuel to the corresponding output power plotted against power output) and the incremental cost curve which plots the incremental cost as a function of power output. The incremental fuel cost can be obtained by differentiating the cost curve with respect to the power output [1].

The best polynomial of fit to the fuel cost curve is the fuel cost function and the coefficients of the polynomial are the fuel cost coefficients. The fuel cost coefficients are particularly important because the accuracy of economic dispatch problems is associated with the accuracy of the fuel cost curve coefficients. Therefore, determining or updating these coefficients is a very important issue to achieve overall accuracy of economic dispatch solutions. [2]The general mathematical model of the polynomial of fit to the cost curve of a power plant is;

$$F\left(P_g\right) = a_0 + \sum_{i=1}^{N} a_j P_g^j + r, \; j = 1, \dots, N \quad \dots \quad (1)$$

where $F(P_g)$is the fuel cost function of the plant, $P_{g\ is}$ the electrical power output of the plant in MW, a_j are the cost coefficients for the plant, j is the equation order (1 for linear, 2 for second-order, 3 for cubic and so on) and r is the error associated with the modelling. The coefficient a_0 represents the fixed costs which is made up of the capital investment, interest charged on the money borrowed, tax paid, labour charge, salary given to staff and any other non-fuel dependent expenses. During the course of plant operation, the re-estimation of cost coefficients is required due to fluctuations in various costs and ageing of the power unit. The higher the accuracy of the estimated coefficients, the more accurate the results obtained from the economic dispatch calculations. Several classical methods to calculate the input-output characteristics have appeared in literature, such as the quadratic least square regression method [3]. The cost curve coefficients estimation has also been formulated as an optimization problem where the estimation error is minimized. Heuristic methods such as particle swarm optimization algorithm [4], genetic algorithm [3, 5], Nelder-MEAD Local Search algorithm [6] and Cuckoo Search [7] have been employed to minimize the error associated with the estimated coefficients.

The major requirement for the estimation of the cost curve coefficients is a set of fuel input –power output pairs which are often derived from tests on the individual thermal generator. At times these data set may not be available. Probably it is the lack of data that motivated Andreas [8] to propose a technique for the approximation of the heat rate curve which can be applied for power plants in which data are available only at the maximum and minimum operating capacities of the power plant. This still required data pairs, though only two. The work proposed here is an attempt to estimate the quadratic cost curve coefficients when the fuel input data is completely unavailable. The fuel input for the maximum and minimum operating capacities are computed using the knowledge of the average heat rate at full capacity and the type of turbine used. Intermediary values of heat rate are determined using the method of finite difference. In this way the required input – output set of data is generated. The least square estimation method is then used to compute the cost curve coefficients.

2. METHODOLOGY

2.1 Estimation of Fixed Cost

The fixed cost is made up of the capital investment, interest charged on money borrowed, tax paid, labour charge, salary given to staff and other expenses that continue irrespective of the load on the power plant. To derive the fixed cost, a detailed cost analysis must be prepared for the plant. The economic operation of an electrical power system can be achieved by minimizing the variable costs only while the personnel in charge of the plant operation have little control over the fixed costs. This work therefore does not consider the estimation of the fixed costs since they do not affect economic dispatch calculations.

2.2 Computation of Fuel Cost

The amount of fuel used to generate electricity depends on the efficiency or heat rate of the generator and the heat content of the fuel being used. Generator heat rate varies by type of generator prime mover, power plant emission controls, and other factors. The heat content of different fuels also varies.

A fundamental formula that can be used to calculate the amount of fuel used to generate a kilo watthour (kWh) of electricity is [9];

Amount of fuel used per kWh = Heat rate (in British Thermal Units (Btu) per kWh) divided by Fuel heat content (in Btu per physical unit) ... (2)

For natural gas, the Average Btu per cubic foot is 1000 per cubic foot so that the amount of fuel used is;

Amount of fuel used per kWh= $\frac{HR}{1000}$ cubic feet ... (3)

1cubic feet of natural gas [10] =1000 Btu ... (4)

The amount of fuel required is then,

Amount of fuel used = HR Btu per kWh ... (5)

If the power generation level (P) is given in MW, then;

Amount of fuel used=$HR * P * 10^3$ Btu per hour ... (6)

The average tested heat rates (HR_{fl}) in the year 2020 for full load conditions for different prime movers for natural gas are as given in Table 1

Table -1: Average Tested Heat Rates [11]

Prime mover type	Average tested heat rate (HR_{fl})
Steam generator	10347
Gas turbine	11,098
Internal combustion	8,899
Combined cycle	7633

The average tested heat rate HR_{fl} is reported at full load conditions *(P_{max})* so that the maximum fuel input (F_{max}) is;

$$F_{max} = HR_{fl} * P * 10^3 \text{ Btu per hour} \quad \text{... (7)}$$

The heat rate at minimum generation HR_{min} can be approximated with,

$$HR_{min} = \frac{P_{min}}{P_{max}} HR_{fl} \quad \text{... (8)}$$

Haven obtained the heat rate at maximum and minimum generation capacities, the one-dimensional

finite difference method [12] is applied to get the heat rate values for other levels of generation.

In applying the finite difference method, the interval HR_{min} to HR_{fl} is divided into N-1equal intervals and the step of approximation h, is calculated thus.

$$h = \frac{HR_{fl} - HR_{min}}{N-1} \quad \text{... (9)}$$

Then using finite differences, the approximate intermediary heat rate values can be obtained using [7];

$$HR_i = HR_{i-1} - \frac{HR_{i-1} - HR_{fl}}{h} \quad \text{... (10)}$$

Expanding equation (10) and rearranging gives

$$HR_i = A/c \quad \text{... (11)}$$

Where

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ -(\frac{h-1}{h}) & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & -(\frac{h-1}{h}) & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & -(\frac{h-1}{h}) & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{... (12)}$$

$$c = \begin{bmatrix} HR_{min} \\ \frac{HR_{fl}}{h} \\ \frac{HR_{fl}}{h} \\ \vdots \\ \frac{HR_{fl}}{h} \\ HR_{fl} \end{bmatrix} \quad \ldots \quad (13)$$

The fuel input for every heat rate value is computed as;

$$F = HR * P * 10^3 \quad \ldots \quad (14)$$

2.3 Treatment of Steam Turbine Plants

For steam turbine generator, the input-output curve is non-linear and also non convex when it has multiple admission valves. [4]. For such generators, the input–output characteristics are not always smooth because the steam admission valves are opened in sequence for more steam intake thereby producing ripples in the input–output characteristics. In this situation the fuel input function is modeled by adding a sinusoid term as in equation (15)

$$F_i(P_{gi}) = \left[a_{0i} + \sum_{i=1}^{N} a_{ji} P_{gi}^{j} + r_i\right] + \left|e_i \sin(f_i(P_{i\,min} - P_i))\right| \quad \ldots \quad (15)$$

Where a_{oi}, a_{ji} are the fuel cost coefficients of the i^{th} unit, e_i and f_i are the fuel cost coefficients of the i^{th} unit valve-point effects [4,13,14]. Therefore, for steam turbine generators, after getting the fuel input and power output pairs using equations (13) and (14), the results have to be refined using equation (15) to take into account the valve point effect. If the average cost rate of natural gas is, say, R $ per million British thermal units (mmBtu), then the fuel inputs are converted to fuel costs (CF) as in equation (16).

$$CF_i = R * F_i \quad \ldots \quad (16)$$

The coefficients of quadratic polynomial of best fit to the fuel costs function are determined using method of least square estimation.

2.4 The Least Square Estimation (LSE)

The least square estimation LSE is a method used to perform curve fitting on input-output data. The polynomial of fit can be any order and the major task is to determine the coefficients of the polynomial of fit. The quadratic cost curve, for example, is a second order polynomial so the three coefficients a_0, a_1 and a_2 need to be determined. LSE requires three or more data points and is based on minimizing the sum of the squared errors between the data points and those on the polynomial of fit. For the quadratic curve fitting, it is intended to determine a_0, a_1 and a_2 such that $f(x) = a_0 + a_1 x + a_2 x^2$ fits the data y_i in the sense of least square. The least square estimates a_0, a_1 and a_2 are those for which the predicted values of the curve minimize the sum of the squared deviations given by;

$$F(a_0, a_1, a_2) = \sum_{i=1}^{N} (a_0 + a_1 x + a_2 x^2 - y_i)^2 \quad \ldots \quad (17)$$

Finding the minimizing values a_0, a_1 and a_2 in equation (17) is to solve the equations resulting from setting,

$$\frac{\partial F}{\partial a_0} = \frac{\partial F}{\partial a_1} = \frac{\partial F}{\partial a_2} = 0 \quad \ldots \quad (18)$$

Solving equation (18) implies solving equation (19) to determine a_0, a_1 and a_2 [15].

$$\begin{bmatrix} N & \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i^3 \\ \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i^3 & \sum_{i=1}^{N} x_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} y_i \\ \sum_{i=1}^{N} y_i x_i \\ \sum_{i=1}^{N} y_i x_i^2 \end{bmatrix} \quad \ldots \quad (19)$$

The LSE can be implemented in MATLAB environment using *polyfit* command. The usage form of the *polyfit* command is as follows.

$$x = polyfit(P, F, j); \quad \ldots \quad (20)$$

where P is a row vector of power outputs, F is a row vector of actual cost while j is power of polynomial to be fit for P and F. The function returns x which is a row vector of length *j* containing the polynomial coefficients in descending powers of P.

2.5 Case Study

Four Nigerian thermal power plants namely Egbin ST2-5 (Combine cycle), Delta II-III, Afam IV-V and Sapele (steam) power plants are used as case study. Their generation limits are [16], 118MW≤ P_{Egbin} ≤1100MW, 10MW ≤P_{Delta} ≤110MW, 24MW≤ P_{Afam} ≤543MW and 33MW ≤P_{Sapele} ≤223MW. The cost rate of natural gas in May 2019 which is 5.65 $ per million British thermal unit (mmBtu) [18] is used.

3. RESULTS AND DISCUSSION

Nine incremental steps are taken to get ten power and fuel input sample points. The fuel input in $ per hour at each sample point for each of the gas power plants are calculated using equation (12). For the steam plant equation (12) is used and refined using equation (15) so that the valve point effect is taken into consideration. The valve point effect coefficients for Sapele power plant are $e_i = 200$ and $f_i = 0.0042$ [16]. The power output and the computed fuel input at each sample point are given in Table 2. The quadratic cost curve coefficients obtained using the method of least square estimation and the exact values available in literature [16] are presented in Table 3. Percentage error between the estimated coefficients (*estimated*) and those in literature (*literature value*) are computed as in equation (21) and are also presented in Table 3.

$$Error = \frac{estimated - literature\ value}{literature\ value} \quad \dots \quad (21)$$

Table -2: Computed fuel input and corresponding generation level

Power plant	Egbin		Delta		Afam		Sapele	
Turbine Type	Combine Cycle		Simple Cycle		Simple Cycle		Steam	
Generation level and fuel input	Generation Level (MW)	Fuel cost ($/h)	Generation Level (MW)	Fuel cost ($/h)	Generation Level (MW)	Fuel cost ($/h)	Generation Level (MW)	Fuel cost ($/h)
	118.00	528	10.00	551	24.00	77	33.00	276
	227.10	2198	21.11	2791	71.67	807	54.11	817
	336.20	5010	32.22	6753	119.33	2306	75.22	1643
	445.30	8965	43.33	12438	167.00	4575	96.33	2755
	554.40	14063	54.44	19846	214.67	7615	117.44	4154
	663.60	20303	65.56	28977	262.33	11424	138.56	5838
	772.70	27686	76.67	39831	310.00	16003	159.67	7809
	881.80	36211	87.78	52407	357.67	21352	180.78	10065
	990.90	45879	98.89	66706	405.33	27471	201.89	12608
	1100.00	56690	110.00	82728	453.00	34360	223.00	15437

Table -3: Thermal plants' quadratic cost coefficients

Power Plant	Cost Coefficients [Proposed method]				Cost Coefficients in literature [16]		Generation limits [17]	
	Linear Coefficients (a_1)		Quadratic Coefficients (a_2)		Linear Coefficients (a_1)	Quadratic Coefficients (a_2)	P_G^{MIN}	P_G^{MAX}
	(a_1)	Error (%)	(a_2)	Error (%)				
Egbin	0.0234	17.61	2.93	25.26	0.0284	3.92	118	1100
Afam	0.0264	8.65	1.48	27.09	0.0289	2.03	24	453
Delta	0.0291	10.74	7.17	10.82	0.0326	6.47	10	110
Sapele	0.0177	21.68	6.82	15.80	0.0226	8.10	33	223

Cost of generating electricity is not the same for all generating plants. It depends upon the fuel consumption rate or operating efficiency, age of the plant and fuel cost rate. Cost curve coefficients are also affected by these factors, therefore methods of cost curve estimation should not be evaluated by comparing its results with results of similar work in literature except if the plants fuel consumption rate, operating efficiency and age are equal which hardly happens. This is why, for example, the cost coefficients in [16] are quite different from those in [19] even though they are for the same power plants. Evaluation of this work should therefore be based on the objectivity of the procedure. Nevertheless, comparison have been done here and the results in this work differ from results available in literature by a maximum of 27%. These results are therefore accurate enough to be used in solving economic dispatch problems.

5. CONCLUSION

A simple to apply method of estimating quadratic cost curve coefficients of thermal power plants when fuel input data is completely not available has been developed and presented in this work. The fuel input corresponding to a particular level of power output is computed using the heat content of fuel and the type of power plant in use. In this way, fuel input versus power output set of data are generated. Method of least square estimation is used to compute the fuel cost coefficients The proposal is demonstrated on some Nigerian thermal power plants. The coefficients obtained are compared with values available in recent literature for the same power plants. Results obtained show that coefficients obtained by this proposed method differ from those available in recent literature by maximum of 27% over those in literature.

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