Design of Bevel Gears for Automotive Differentials and a Brief Study

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Abstract - *An open differential achieves the speed difference between the two wheels by using a set of bevel gear train. It is crucial to design the bevel gears as they are responsible for power and motion transmission. A design approach and calculations are done considering that the spider gear drives the side gears by revolving around the axis of side gears. Direct equations for estimating the module and surface strength of bevel gears are derived using Lewis's equation and Buckingham equation. Design of bevel gears is done for a specific vehicle and FEA has been done on a geometry obtained from derived equations. A brief study about the power and torque split characteristics of an open differential and its comparison to locked differential/spool has been done.*

Key Words: **Bevel gear design, Differential, module, torque, power, transmission, surface hardness, open differential.**

1. INTRODUCTION

A differential is one of the most crucial elements of the drivetrain of a vehicle. Its fundamental purpose is to change the direction of power transmission towards both wheels and differentiate the speed of both wheels while turning. This mechanism is typically made of a set of bevel gears in order to make the speed differences possible. There are many types of automotive differentials, each with different torque and power split characteristics. They all serve their purpose according to the type of vehicle they are installed on. For example, a Limited Slip Differential would be best for a race-car that is designed to go faster on the corners. LSDs make it possible because of their distinct feature of transmitting different torque to each wheel according to the traction difference that arises as a consequence of the lateral weight transfer of a vehicle while turning. A locking differential is best suited for off-roading vehicles where a condition of almost zero traction at either wheel can arise. The type of automotive differential determines the longitudinal acceleration characteristics of a vehicle under unequal traction conditions.

While there are many types of differentials that may yield better performance than open differentials, the use of open differentials still prevails. In today's modern and extremely competitive automotive market, leading car manufacturing companies aim to reduce the cost of production to maximize their profit rather than simply increase the selling price. because there are always other rival companies that would offer the same value at a lesser price. This is why the most commonly used differential is an open differential, as it is extremely cost-effective, operates on a simple mechanism, and needs very little maintenance. It also makes efficient power transmission possible while differentiating the speeds of both wheels without making the tires slip while turning. An open differential also yields better fuel economy for the engine as compared to other differentials because it doesn't cause resistance to the motion of the wheel. This means lower service and maintenance costs, fewer tires that need to be replaced on a regular basis, and lower fuel costs for the owner.

An open differential could be used for FWD (front wheel drive) cars as well as RWD (rear wheel drive) cars. But it is much more efficient to use open differentials for the FWD vehicles as they have a shorter drivetrain and the weight distribution of the vehicle is more on the front side because of the engine and transmission, resulting in better traction at the front wheels and lesser rolling resistance from the rear wheels. This is one of the important factors to be considered while deciding the drivetrain of a vehicle. This is why most of the economy hatchbacks are designed with a front wheel drive drivetrain as they naturally have more weight distribution on the front. Better traction at driving wheels improves the overall stability and drivability of the vehicle as well.

Considering all the factors mentioned above, designing bevel gears for a differential is the most crucial part as the bevel gears are responsible for this power transmission.

2. OPEN DIFFERENTIAL

1.1 Mechanism

The open differential consists a set of bevel gears to differentiate the speeds between both driveshafts/wheels. It consists of two side gears which drive the driveshafts and a set of spider gears that drive the side gear by revolving around the axis of side gears. The spider gears also act like an idler gear for making the speed differences possible between the two side gears. The spider gears rotate around their own axis only when the side gears are subjected to rotate at different speeds. This happens in any case where both wheels are rotating at different speeds, example: turning and unequal traction condition at both wheels. The Ring gear is unibody with the differential housing. The spider gears are driven by the housing through a pin passing through the spider and housing. The spider always stays in mesh with both side gears simultaneously.

1.2 Power and Torque split characteristic of an Open Differential

The torque split of an open differential is 50–50% to both wheels in all cases. This is simply because the spider gears that are in mesh with both side gears simultaneously apply equal forces to both side gears even when the spider rotates around itself. The distinction is in power split, or power at both wheels. The power at each wheel depends on traction or tractive force at each wheel.

The wheel with more traction requires more tractive torque or tractive effort to rotate, while the wheel with less traction requires less tractive effort. In a case where there is an extensive traction difference and one of the wheels has almost no traction, the vehicle with an open differential may not be able to achieve longitudinal acceleration. In this case, power at the wheel with almost no traction will be greater because the tractive effort for this wheel is only due to its own inertia and the very little traction offered by the wheels. This puts this wheel in motion very easily, and the power at this wheel rises rapidly as the speed of the wheel increases. Since input power is limited at the source, all the power will be consumed by the wheel with no traction. The wheel with good traction, which can be responsible for achieving longitudinal acceleration, however, doesn't get enough power. The above-mentioned case, however, involves a vehicle designed for roads being driven in extreme terrain where there is a possibility of one of the wheels losing contact with the ground. One of the things that can happen on good roads as well is a mild to moderate traction difference between the two wheels. This traction difference occurs because the traction is dependent on the normal force, and the normal force on each wheel can change as a result of the lateral weight transfer of a vehicle while turning. During high-speed turning, the outer wheel has more traction and is therefore more responsible for causing longitudinal acceleration. However, because the inner wheel consumes more power, longitudinal acceleration during turning is significantly less than straight-line acceleration.

1.3 Comparison to Locked differential/spool and it's torque and power split characteristics

Both the wheels always rotate at same speeds in any case. The torque and power can change proportionately, but their ratio remains constant. In locked differentials/spools, the torque and power split both vary depending on traction at both wheels. The torque is split in such a way that the wheel with greater traction gets greater torque. And the power is transmitted proportionately. The wheel with greater traction will have more tortional load, and the per the theory of torsion, torsional sheer stress is greater towards the wheel with greater traction. This means torque at this wheel is greater. If one of the wheels has no traction and loses contact with ground, the torsional load on the side of this wheel is now negligible. Hence a very less torque is split to this wheel, typically enough torque just to keep it rotating with the same speed of the wheel with greater traction.

Even if this is the desired character which can achieve greater longitudinal acceleration during yaw moment, this type of differential is not very practical to use in commercial cars. Not only does it cause excessive tire wear and increase repetitive expense on tire change, but also cause a vehicle to understeer which affects the handling maneuverability in a negative way. It also causes lot of tire screeching noise. Comparatively lesser longitudinal acceleration during turning in open differentials is compromised for all its positive features that are a remedy to all the problems in a locked differential/spool.

3. DESIGN APPROACH OF BEVEL GEARS

A pair of bevel gears is designed for it to have enough beam strength and surface strength to embrace and endure the forces acting on gear tooth and angular speeds at which it is operating. Estimating module and magnitude of surface hardness is the end goal of the design procedure. The equation for the estimation of module is derived from the standard Lewis Equation, various force equations, and effective loading on a gear tooth. Once the module is obtained, the dimensions are finalised according to a standard parametric geometry as per the mechanical design standards that are used world-wide. Final pitch line velocity is then calculated using available dimensions and known input speeds. The input speeds depend on various vehicle dynamics factors such as track width, turning radius, and the maximum speed of a vehicle while turning. The case where there is maximum rotational speed of a spider gear is considered for the surface strength calculations.

3.1 Derivation for Estimating Module

$$Sb = m\, b\, \sigma b\, Y\left(1 - \frac{b}{Ao}\right)$$

$$Pt = \left(\frac{2Mt}{Dp}\right)$$

$$Peff = \frac{Cs}{Cv}(Pt) = \frac{Cs}{Cv}\left(\frac{2Mt}{Dp}\right)$$

$$Fs = \frac{Sb}{Peff}$$

$$Fs = \frac{m\, b\, \sigma b\, Y\left(1 - \frac{b}{Ao}\right)}{\frac{Cs}{Cv}\left(\frac{2Mt}{Dp}\right)}$$

$$Fs = \frac{m\, b\, \sigma b\, Y\left(1 - \frac{b}{Ao}\right) Cv\, Dp}{Cs(2Mt)}$$

$$b = k \times m$$

$$k = \left(\frac{b}{m}\right) assumed$$

$$Dp = m \times Zp$$

$$Fs = \frac{m^3 k\, \sigma b\, Y \left(1 - \frac{b}{Ao}\right) Cv\, Zp}{Cs(2Mt)}$$

$$\sigma b = \left(\frac{Sut}{3}\right)$$

$$\boxed{\therefore m = \sqrt[3]{\frac{Cs\ Fs\ (2Mt)}{k\left(\frac{Sut}{3}\right) Y \left(1 - \frac{b}{Ao}\right) Cv\ Zp}}}$$

Cs= Service factor

Fs= Factor of Safety

Mt= Torque transmitted

b= face width

m= module

k= assumed b/m ratio

Sut= Ultimate tensile stress

Ao = Cone length

Cv= Velocity factor

Zp= Number of teeth on pinion

3.2 Mathematical Model of calculations

Data	Value	Unit
Torque transmitted	1300000	N-mm
No. of spider gears	2	
No. of pairs in mesh	4	
Mt=Torque transmitted for one pair	325000	N-mm
Zp (spider)	10	
Zg (side)	16	
Zg/Zp	1.6	
Assumptions/Requirements	Value	Unit
Fs	1.35	
v (pitch-line velocity)	0	
Cv	1	
Cs (service factor)	1.25	
b/m	4	
b/A0	0.33333	
pressure angle	0.34907	
Material Properties		
Sut	2400	MPa
σ bending (Permissible)	1250	MPa
Young's Modulus	210000	Mpa
Poission's ratio (-)	0.3	
Sheer Modulus	80000	Mpa

Calculations	Formula	Value	Unit
γ side+ γ spider		1.5708	Rad
γ spider		0.66896	Rad
γ side		0.90183	Rad
Z'p (spider)		12.7475	-
Z'g (side)		25.7992	-
Y for p (lewis form factor)		0.25886	-
Y for g (lewis form factor)		0.37276	-
Estimated module based on assumed Fs			
module	((Fs*Cs*2*Mt)/(b/m)*σb*Y*(1-(b/Ao))*Zp*Cv))^1/3	***5.03***	mm
Selected module		**5**	
Pt	2*Mt/Dp	13000	N
Sb	m*b*σb*Y*(1-(b/Ao))	21571.6	N
Peff	Cs*2*Mt/Dp	16250	N
Fs NEW	Sb/Peff	**1.32748**	-
Dimensions			
Dp	m*Zp	50	mm
Dg	m*Zg	80	mm
b	(b/m)*m	20	mm

4. PARAMETRIC CAD MODELING OF BEVEL GEAR

The parametric cad modeling has been done in solidworks based on the dimensions obtained from design procedure. The rest dimensions of gear tooth profile and blank has been coded through parametric equations such that the module, face width and teeth ratios are the parameters. This is done by writing equations through defining certain global variables in solidworks. By doing this, it becomes easy to assign dimension to a specific curve.

Equations, Global Variables, and Dimensions

Name	Value / Equation	Evaluates to	Comments
Global Variables			
"dp"	= 50	50	pitch diameter
"Zp"	= 10	10	number of teeth
"a"	= 20	20	pressure angle
"gamma"	= atn ("Zp" / "Zg")	32.0054	pitch angle
"L"	= 20	20	face width
"m"	= "dp" / "Zp"	5	Module
"tha"	= atn (2 * sin ("gamma") / "Zp")	6.05075	addendum angle
"thf"	= atn (2 * 1.25 * sin ("gamma") / "Zp")	7.54771	deddendum angle
"dpv"	= "dp" / cos ("gamma")	58.9624	virtual pitch diameter
"dfv"	= "dpv" - (2 * 1.25 * "m")	46.4624mm	virtual root diameter
"dav"	= ("dp" + 2 * "m" * cos ("gamma")) / cos ("gamma")	68.9624	virtual addendum diameter
"dfvu"	= IIF ("dbv" > "dfv" , "dfv" , "dbv" * 0.99)	46.4624	dfv undercut
"psi"	= 360 / (4 * "Zp" / cos ("gamma"))	7.63198	half tooth angle
"inva"	= tan ("a") * 180 / pi - "a"	0.853958	involute alpha
"t"	= sqr ("dav" ^ 2 / "dbv" ^ 2 - 1) * 1.01	1.00179	max involute para
"dbv"	= "dfv" + (2 * 0.25 * "m")	48.9624mm	Base circle
"Zg"	= 16	16	gear teeth
"P"	= "Zp" / "dp"	0.2	Diametral pitch
"c"	= 0.157 / "P"	0.785mm	teeth thickness
"ctt"	= 3.142 / (2 * "P")	7.855mm	circular tooth thickness
Add global varia			
Features			

Fig 4.1- Parametric dimensions of Spider gear

Equations, Global Variables, and Dimensions

Name	Value / Equation	Evaluates to	Comments
Global Variable			
"dp"	= 80	80	pitch diameter
"Zp"	= 10	10	number of teeth
"a"	= 20	20	pressure angle
"gamma"	= atn ("Zg" / "Zp")	57.9946	pitch angle
"L"	= 20	20	face width
"m"	= "dp" / "Zg"	5	Module
"tha"	= atn (2 * sin ("gamma") / "Zg")	6.05075	addendum angle
"thf"	= atn (2 * 1.25 * sin ("gamma") / "Zg")	7.54771	deddendum angle
"dpv"	= "dp" / cos ("gamma")	150.944	virtual pitch diameter
"dfv"	= "dpv" - (2 * 1.25 * "m")	138.444mm	virtual root diameter
"dav"	= ("dp" + 2 * "m" * cos ("gamma")) / cos ("gamma")	160.944	virtual addendum diameter
"dfvu"	= IIF ("dbv" > "dfv" , "dfv" , "dbv" * 0.99)	138.444	dfv undercut
"psi"	= 360 / (4 * "Zg" / cos ("gamma"))	2.98124	half tooth angle
"inva"	= tan ("a") * 180 / pi - "a"	0.853958	involute alpha
"t"	= sqr ("dav" ^ 2 / "dbv" ^ 2 - 1) * 1.01	0.556818	max involute para
"dbv"	= "dfv" + (2 * 0.25 * "m")	140.944mm	Base circle
"Zg"	= 16	16	gear teeth
"P"	= "Zg" / "dp"	0.2	Diametral pitch
"c"	= 0.157 / "P"	0.785mm	teeth thickness
"ctt"	= 3.142 / (2 * "P")	7.855mm	circular tooth thickness
Add global va			

Fig 4.2- Parametric dimensions of Side gear

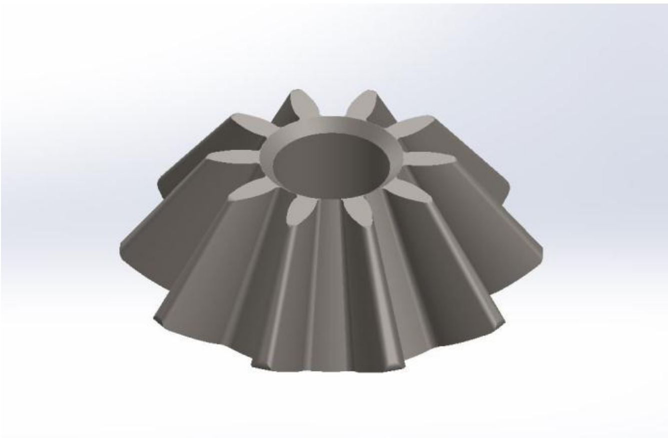

Fig 4.3 – Z=10 Spider Gear

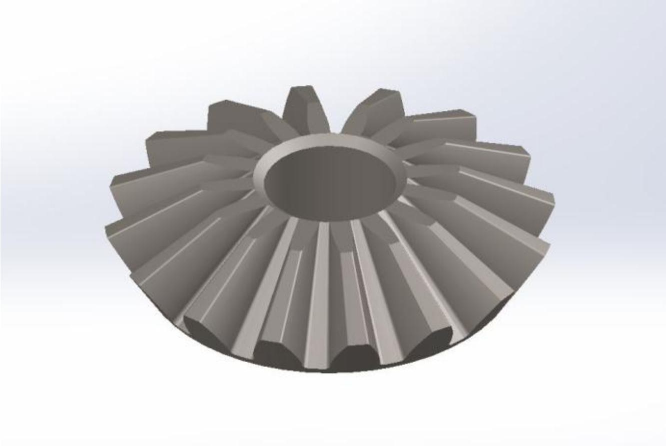

Fig 4.4- Z=16 Side gear

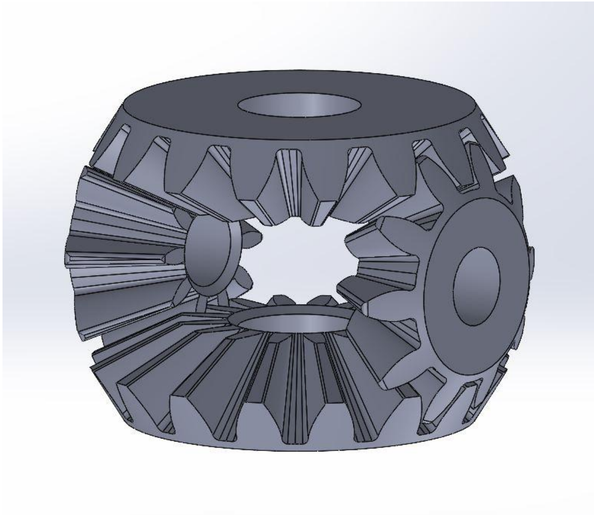

Fig 4.5- Meshing of all gears

5. ESTIMATING SURFACE STRENGTH

5.1 Derivation for estimating surface hardness

$$Sw = \frac{0.75\ b\ Q\ Dp\ K}{\cos(\gamma)}$$

$$Pd = \frac{21\ V\ (\ C\ e\ b + Pt)}{21V + \sqrt[2]{C\ e\ b + Pt}}$$

$$Sw = \frac{0.75\ b\ Q\ Dp\ (0.27\ \times 9.81(BHN)^2 \sin(\alpha)\cos(\alpha)\left(\frac{1}{Ep}+\frac{1}{Eg}\right)}{\cos(\gamma) \times 1.4}$$

$$Fs = \frac{Sw}{Pd}$$

$$Fs = \frac{0.75\ b\ Q\ Dp\ (0.27\ \times 9.81(BHN)^2 \sin(\alpha)\cos(\alpha)\left(\frac{1}{Ep}+\frac{1}{Eg}\right)(21V + \sqrt[2]{C\ e\ b + Pt})}{\cos(\gamma) \times 1.4\ \times 21\ V\ (\ C\ e\ b + Pt)}$$

$$\therefore BHN = \sqrt[2]{\frac{Fs\ \cos(\gamma) \times 1.4\ \times 21\ V\ (\ C\ e\ b + Pt)}{0.75\ b\ Q\ Dp\ (0.27\ \times 9.81\ \sin(\alpha)\cos(\alpha)\left(\frac{1}{Ep}+\frac{1}{Eg}\right)(21V + \sqrt[2]{C\ e\ b + Pt})}}$$

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Input Data		
Parameters	Value	Unit
Track width	1600	mm
Min turning radius	4900	mm
Central Turning Radius (Rt)	**4900**	**mm**
Inner wheel turning radius	4100	mm
Outer wheel turning radius	5700	mm
Wheel od	540	mm

Fig 5.1- Input data for speed calculations

Data	Value	Unit
Car speed	35	km/h
Tire diameter	0.56	m
Ramp speed	34.7222	rad/sec
	331.573	rpm
side (inner)	29.0533	rad/sec
side (outer)	40.3912	rad/sec
spider	5.66893	
V (Pitch line velocity of side g)	323.129	m/s

Fig 5.2- Speed Calculations

Data		Value	Unit
k (tooth form constant) for 20 degree involute		0.111	-
C (deformation facto	k/(1/Ep + 1/Eg)	11655	Mpa
e (max expected error)			
Class 1		0.05	mm
Class 2		0.025	mm
Class 3		0.0125	mm
Calculations	Formula	Value	Unit
Q (ratio factor)	2Zg/(Zg+Zp*tan(γp))	1.36541	-
σc (Surface enduran	0.27*9.81*(BHN)	1441.37	
Hardness (BHN)	sqrt((cos(γ)*21*V*C*(C*e*b + Pt)*Fs)/(3.	544.181	BHN
Hardness (HRC)		53.7	HRC

Fig 5.3- Hardness calculations

6. ANALYSIS OF BEVEL GEAR

Static analysis has been done in Hypermesh software to validate the design. A force of 16000 N was applied on the tooth of a bevel gear towards the pitch cone and tangential to the cone. The axis of the gear is constrained. Max stress of magnitude 685MPa was generated in at the base of the tooth for a 20MnCr5 with a yield strength of 970-1250 MPa.

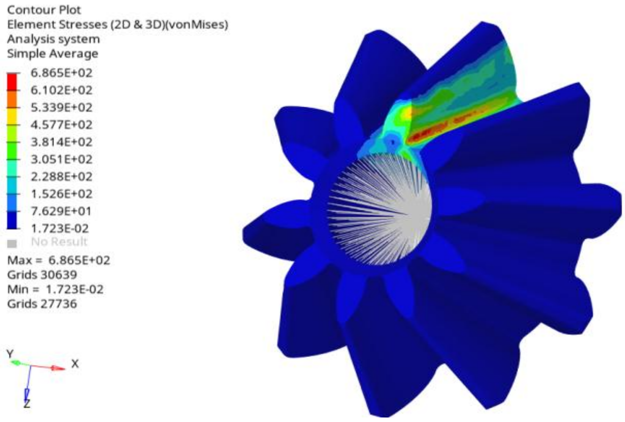

Fig 6.1- Stress analysis

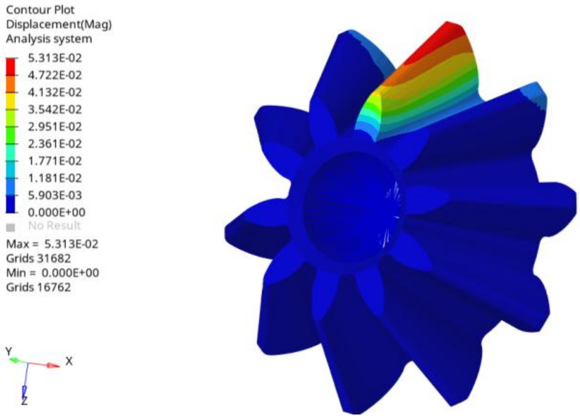

Fig 6.2- Displacement analysis

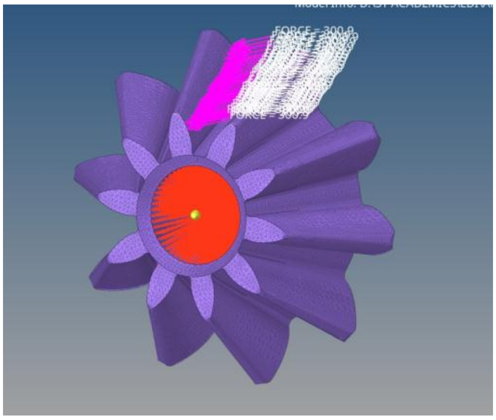

Fig 6.3- Force applied

7. CONCLUSION

The bevel gears are an integral part of an automotive differential that allows speed differentiation. The design finalized by using the derived equations for module and surface strength yields similar levels of safety through FEA analysis compared to the factor of safety assumed while calculating. The factor of safety assumed while calculating was 1.35, and the factor of safety obtained through FEA is

1.41. Thus, we can conclude that the design done using the direct equations for module and surface strength is safe and can be used for the preliminary stage of gear design to estimate module, dimension values, and surface hardness.

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