# ON THE INJECTIVE DOMINATION OF JUMP GRAPHS

**N. Pratap Bau Rao**

*Department of Mathematics S.G. College, Koopal(Karnataka) INDIA*

---------------------------------------------------------------------***---------------------------------------------------------------------

**ABSTRACT:** Let J(G)=(V,E)be a jump graph. A subset D of J(V) is called injective dominating set (inj-dominating set) if every vertex v∈ J(V) – D there exists a vertex u∈ D such that |Γ(u,v)|≥1, where |Γ(u,v)| 9s the number of common neighbors between the vertices u and v. The minimum cardinality of such dominating set denoted by $\gamma_{inj}$(J(G)) and is called injective dominating number (Inj-dominating number) of J(G). In this paper, we introduce the injective domination of a jump graph J(G)and analogous to that , we define the injective independence number (Inj-independence number)$\beta_{inj}$(J(G). and injective domatic number (Inj-domatic number) $d_{inj}$(J(G)). Bounds and some interesting results are established.

Mathematics subject classification 95C69

**Key words** : Injective domination number ,Injective independence number, Injective domatic number

**1 .Introduction:**

By a graph means a finite, undirected with no loops and multiple edges. In general we use

< X > to denote the sub graph induced by the set of vertices X and N(v), N[v] denote the open and closed neighborhood of a vertex v, respectively. The distance between two vertices u and v in J(G) is the number of edges in a shortest path connecting them this is also known as the geodesic distance .The eccentricity of a vertex v is the greatest geodesic distance between v and any other vertex an denoted by e(v)

A set D of vertices in a graph J(G) is a dominating set if every vertex in J(V) – D is adjacent to some vertex in D. The dominating number $\gamma$(J(G)) is the minimum cardinality of a dominating set of J(G).We denote to the smallest integer greater than or equal to x by $\lceil x \rceil$ and the greatest integer less than or equal to x by $\lfloor x \rfloor$. A strongly regular jump graph with parameter ( n, k, $\lambda$ $\mu$ ) is a graph withn vertices such that the number of common neighbors of two vertices u ans v is k, $\lambda$ or $\mu$ according to whether u and v are equal, adjacent , respectively. When $\lambda$ = 0the strongly regular graph J(G) is called primitive if J(G) and J($\bar{G}$ ) are connected.

For terminology and notations not specifically defined here we rfer the rader to [5] For more details about domination number and neighborhood number and their related parameters. We refer to [3] ,[4]

The common neighborhood domination in graph has introduced in [2]. A subset D of J(V) is called common neighborhood dominating set (CN-dominating set)if every vertex v ∈ J(V) – D there exists a vertex u ∈ D such that uv ∈ E(J(G)) and | Γ(u,v)|≥ 1, where | Γ(u,v)| is the number of common neighborhood between the vertices u and v. The minimum cardinality of such dominating set denoted by $\gamma_{cn}$(J(G)) and is called common neighborhood domination number (CN-domination number) of J(G). The common neighborhood(CN-neighborhood) of a vertex

u∈ V(J(G)) denoted by $N_{cn}$(U) is defined as $N_{cn}$(u) = {v ∈ V(J(G)): uv ∈ E(J(G)) and | Γ(u,v)|≥ 1 }.

The common neighborhood graph (congraph) of J(G), denoted by con(J(G)), is the graph with the vertex set $v_1$, $v_2$,……$v_p$, in which two vertices are adjacent if and only if they have at least one common neighbor in the graph J(G) [1].

In this paper, we introduce the concept of injective domination in jump graph. In ordinary domination between vertices is enough for a vertex to dominate another in practice. If the persons

Have common friend then it may result in friendship. Human being have a tendency to move with others when they have common friends.

**2. Injective Dominating Sets:**

If defense and domination problem in some situations there should not be direct contact between two individuals but can be linked by a third person this motivated us to introduced the concept of injective domination.

Definition 2.1 ([1]). Let J(G) be a jump graph with vertex set $V(J(G))= \{ v_1, v_2,v_3,\ldots\ldots v_p\}$, For $I \neq j$ the common neighborhood of the verticed $v_i$ and $v_j$, denoted by $\Gamma(v_{i,} v_j)$ is the set of vertices different from $v_i$ and $v_j$, which are adjacent to both $v_i$ and $v_j$.

Definition 2.2. Let J(G)= ( V,E) be a graph . A subset D of V is called injective dominating set

(Inj-dominating set) if for every vertex $v\in V$ either $v \in D$ or tere exists a vertex $u\in D$ such that

$|\Gamma(u, v)| \geq 1$. The minimum cardinalioty of Inj-dominating set of J(G) denoted by

$\gamma_{inj}(J(G))$ and called injective domination number (Inj-domination number) of J(G).

**Proposition 2.3:** Let J(G)= (V,E)be a graph and $u\in V$ be such that $|\Gamma(u, v)|=0$ for all $v\in V(J(G))$. Then u is every injective dominating se, such vertices are called injective isolated vertices.

**Proposition 2.4 :** Let J(G)= (V,E) be strongly regular graph with parameters ( n, k, $\lambda$ $\mu$ ). Then

$\gamma_{inj}(J(G)) = 1$ or 2.

**Proposition 2.5.:** For any graph J(G), $\gamma_{inj}(J(G)) \leq \gamma_{cn}(J(G))$.

**Proof:** From the definition of the CN-dominating set of a graph J(G), For any graph J(G). For any graph J(G) any CN-dominating set D is also Inj-dominating set. Hence $\gamma_{inj}(J(G) \leq \gamma_{cn}(J(G))$.

We note that Inj-domination number of a graph J(G) may be greater than, smaller than or equal to the domination number of J(G).

**Example 2.6.**

i) $\gamma_{9jn(J}(J(P2))=2$ $\gamma(J(P_2)) = 1$
ii) $\gamma_{9jn(J}(J(C_5))=2$ $\gamma(J(C_5)) = 1$
iii) If J(G) is the Petersen graph, then $\gamma_{inj}(J(G)) =2$ $\gamma_{cn}(J(G))$. = 3

**Proposition 2.7:**

**i)** For any complete graph $J(K_p)$ where $p \neq 2$ $\gamma_{inj}(J(K_p)) = 1$
**ii)** For any wheel graph $J(G) \cong J(W_p))$ $\gamma_{inj}(J(G)) =1$
**iii)** For any complete bipartite graph $J (K_{r,m})$ $\gamma_{inj}(J( K_{r,m})) =2$
**iv)** For any graph J(G), $\gamma_{inj}(Kp + J(G)) =1$ where $p \geq 2$

**Proposition2.8.** For any graph J(G) with vertices $1 \leq \gamma_{inj}(J(G)) \leq p$

**Proposition 2.9:** Let J(G) be a graph with p vertices .Then $\gamma_{inj}(J(G)) =p$ if and only if J(G) is a forest with

$\Delta (J(G)) \leq 1$.

**Proof: Let** J(*G) be a forest with $\Delta(J(G)) \leq 1$ . Then we have two cses.

**Case 1.** If J(G) is connected. Then either $J(G) \cong J(K_2)$ or $J(G) \cong J(K_1)$. Hence $\gamma_{inj}(J(G)) =p$

**Case 2.** If J(G) is is disconnected, then $J(G) \cong J(n_1K2 \cup n_2K_2 )$, thus $\gamma_{inj}(J(G)) =p$

Conversely, If $\gamma_{inj}$(J(G)) =p then all the vertices of J(G) are Inj-isolated that means J(G) is isomorphic to $J(K_1)$ or $J(K_2)$ or to the disjoint union of $J(K_1)$ and $J(K_2)$, that is $J(G) \cong J(n_1K2 \cup n_2K_2)$, for some $n_1,n_2 \in \{0,1,2,\ldots\}$ Hence J(G) is a forest with $\Delta(J(G)) \leq 1$.

**Proposdition 2.10:** Let J(G)( b a nontrivial connect4d graph. Then $\gamma_{inj}$(J(G)) =1 if and only if there exists a vertex $v \in V(J(G))$ such that $N(v) = N_{cn}(v)$ and $e(v) \leq 2$

**Prof:** Let $v \in V(J(G))$ be any vertex in J(G) such that $N(v)= N_{cn}(v)$ and $e(v) \leq 2$. Then for any vertex

$u \in V(J(G))$-{v} if u is adjacent to v, Since $N(v)= N_{cn}(v)$ , then obvious $u \in N_{inj}(v)$. If u is not adjacent to v, then $|\Gamma(u,v)| \geq 1$. Thus for any vertex $u \in V(J(G))$-{v}, $|\Gamma(u,v)| \geq 1$.. Hence , $\gamma_{inj}$(J(G)) =1.

Conversely, 9f J(G) is a graph with p vertices and $\gamma_{inj}$(J(G)) =1., then there exist at least one vertex v

$\in V(G)$ such that $derg_{inj}(v) = p - 1$, then any vertex $u \in V(J(G))$-{v} either contained in trianglewith v or has distance two from v. Hence , $N(v)= N_{inj}(v)$ and e(v0 $\leq 2$.

**Theorem 2.11([6])** For any path $P_p$ and any cycle $C_p$ where $p \geq 3$, we hae

$$\gamma(P_p) = \gamma(C_p) = \lceil \frac{p}{3} \rceil$$

**Proposition 2.12 ☹[2]).For any path $P_p$ and any cycle $C_p$**

**i)** Con $(P_p) \cong P_{\lceil p/2 \rceil} \cup P_{\lfloor p/2 \rfloor}$.

**ii)**

$$Com(C_p) \cong \begin{cases} C_p & \text{if p is odd and } p \geq 3. \\ P_2 \cup P_2 & \text{if } p=4 \\ C_{p/2} \cup C_{p/2} & \text{if p is even} \end{cases}$$

From the definition of the common neighborhood graph and the Inj-domination in ajump graph the following propositions can easily verified.

**Proposition 2.13:** For any graph J(G), $\gamma_{inj}$(J(G)0 = $\gamma$(con(J(G))

The proof of the following proposition is straight forward crom Theorem 2.11 and proposition 2.12

**Proposition :2.14:** For any cycle $C_p$ with o9dd number of vertices $p \geq 3$.

$$\gamma_{inj(J(Cp))} = \gamma_{(J(Cp))} = \lceil \frac{p}{3} \rceil$$

**Theorem 2.15:** For any cycle Cp with even number of vertices $p \geq 3$.

$$\gamma_{inj(J(Cp))} = 2 \lceil \frac{p}{6} \rceil$$

**Proof:** From Proposition 2.13 ,Theorem 2.11 and proposition 2.12, if p is even then

$$\gamma_{inj(J(Cp))} = \gamma_{inj}( (J(C_{p/2}))\cup (C_{p/2})) = 2\gamma( J(C_{p/2})) = 2 \lceil \frac{p}{6} \rceil$$

**Proposition 2.16: For any odd number** $p \geq 3$

$$\gamma_{inj}(J(P_p)) = \lceil \frac{p+1}{6} \rceil + \lceil \frac{p-1}{6} \rceil$$

Proof: From proposition 2.13, Theorem 2.11 and proposition 2.12, if p is odd then,

$\gamma_{inj}(J(P_p)) = \gamma ( J( P_{\lceil p/2 \rceil}) \cup J( P_{\lceil p/2 \rceil}) ) = \gamma( (J(P_{p+1/2}) \cup J(P_{p-1/2})) =$

$\lceil \frac{p+1}{6} \rceil + \lceil \frac{p-1}{6} \rceil$

**Proposition 2.17** For any even number $p \geq 4$,

$$\gamma_{inj}(J(P_p)) = 2 \lceil \frac{p}{6} \rceil$$

**Proof:** From proposition 2.13, Theorem 2.11 and p[roposition 2.12 , if p is even then, $\lceil \frac{p}{2} \rceil = \lfloor \frac{p}{2} \rfloor = \frac{p}{2}$ Hence $\gamma_{inj}(J(P_p)) = 2 \lceil \frac{p}{6} \rceil$

**Theorem 2.18** Let J(G)= (v, E) be a jump graph without Inj-isolated vertices. If D is a minimal Inj-dominating set the V – D is an Inj-dominating set.

**Proof:** Let d be the minimal Inj-dominating set of J(G) , Suppose V – D is not Inj-dominating set. Then there exists a vertex u in D such that u is not Inj-dominated by any vertex in V – D,that is $|\Gamma(u,v)| = 0$ for anyb vertex v in V –n D. Since J(G) has no Inj-isolated vertices, then there is at least one vertex in D – {u} has common neighborhood with u, Thus D – {u} is Inj-dominating set of J(G) which contradicts tht minimality ofb the Inj-dominating set D. Thus every vertex in D has common neighborhood with at least one vertex in V – D. Hence V – D is an Inj-dominating set.

**Theorem2.19;** A jump graph J(G) has a unique minimal Inj-dominating set , if and only if the set of all Inj-isolated vertices forms an Inj-dominating set.

**Proof:** Let J(G) has unique minimal Inj-dominating set D, and suppose S = { u∈ V : u is Inj-isolated vertex}. Thus $S \subseteq D$. Now suppose $D \neq S$

let $v \in D - S$ , Since v is no Inj-isolated vertex, then V – {v } is an Inj-dominating set. Hence there is a minimal Inj-dominating set $D_1 \subseteq V - \{v\}$ $D_1 \neq D$ a contradiction to the fact that J(G) has a unique minimal Inj-dominating set. Therefore S = D.

Conversely, if the set of all Inj-isolated vertices in J(G) forms an Inj-dominating set, then it is clear that J(G) has a uni9que Inj-dominating set.

**Theorem 2.20:** For any (p,q) graph J(G), $\gamma_{inj} J(G) \geq p - q$

**Proof:**Let D be a minimum Inj-isolated vertices in J(G), Since every vertex in V – D has common neighborhood with at least one vertex of D, then $q \geq |V - D|$, Hence $\gamma_{inj} J(G) \geq p - q$.

**Theorem 2.21:** Let J(G) be a graph on p vertices and $\delta_{inj}(J(G)) \geq 1$ Then $\gamma_{inj} J(G) \leq \frac{p}{2}$

**Proof:** Let D be any minimal Inj-dominating set in J(G). Then by Theorem 2.18, V – D is also an Inj-dominating set in J(G). Hence , $\gamma_{inj} J(G) \leq \min \{ | D_1| , | V - D |\} \leq \frac{p}{2}$.

**Theorem 2.22:** For any graph J(G) on p vertices $\gamma_{inj} J(G) \leq p - \Delta_{inj}(J(G))$.

**Proof:** Let v be a vertex of J(G) such that $deg_{inj}\{v\} = \Delta_{ionj}(J(G))$. Then v has common neighborhood with $|N_{inj}\{v\}| = \Delta_{inj}(J(G))$ vertices. Thus, V - $N_{inj}\{v\}$ is an Inj-dominating set. Therefore $\gamma_{inj} J(G) \leq | V - N_{inj}\{v\} |$, Hence

$\gamma_{inj} J(G) \leq p - \Delta_{inj}(J(G))$..

**Proposition 2.23;** For asny graph J(G) with diameter less than or equal three and maximum degree $\Delta$ (J(G)), $\gamma_{inj} J(G) \leq \Delta(J(G)) + 1$

**Proof:**Let $diam(J(G)) \leq 3$ and $v \in V(J(G))$ such that $deg(v) = \Delta(J(G))$, Clarlythat, if diam(J(G)) = 1 then J(G) is a complete graph and the result holds. Suppose diam(J(G)) = 2 or 3 Let $V_i(J(G)) \subseteq V(JG))$ be the set of vertices of J(G) which have distance I from v, where I = 1,2,3. Obv8ously, the set $S = V_1(J(G)) \cup \{v\}$ is an Inj-dominating set of J(G) of order $\Delta(J(G)) + 1$. Hence $\gamma_{inj}$ $J(G) \leq \Delta(J(G)) + 1$ .

**Definition:2. 24;** Let J(G)= (V, E) be a jump graph. $S \subseteq V(J(G))$ is called Inj-independent set if no two vertices in S have common neighbor. An Inj-independent set S is called maximal Inj-independent set if no superset of S is Inj-independent set. Ghe Inj-independent set with maximum size called the maximum Inj-independent set in J(G) and its size called the Inj-independence number of J(G) and is denoted by $\beta_{inj}(J(G))$

**Theorem 2.25:** Let S be a maximal Inj-independent set . Then S is a minimal Inj-dominating set.

**Proof:** Lt S be a maximal Inj-independent set and $u \in V - S$. If $u \notin N_{inj}(v)$ for every $v \in S$, then $S \cup \{ u \}$ is an Inj-independent set, a contradiction to the maximality of S. Therefore $u \in N_{inj}(v)$ for some $v \in S$. Hence , S is an Inj-dominating set. To prove that S is minimal Inj-dominating set. Suppose S is not minimal. Then for some $u \in S$ the set $S - \{u\}$ is an Inj-dominating set. Then there exist some vertex in S has a common neighborhood with u, a contradiction because S is an Inj-independent set. Therefore S is a minimal Inj-dominating set.

**Corollary: 2.26:** For any graph J (G), $\gamma_{inj}(J(G)) \leq \beta_{inj(J(G))}$.

**3. Injective domatic number in a jump graph.**

Let J(G)=(V, E) be a jump graph .A partition $\Delta$ of its vertex set VJ(G)) is called a domatic partition of J(G) if each class of $\Delta$ is dominating set in J(G). The maximum order of a partition of V(J(G)) into dominating sets is called domatic number of J(G) and is denoted by d(J(G)).

Analogously as to $\gamma(J(G))$ the domatic number d(J(G)) was introduced, we introduce the injr=ective domatic number $d_{inj}(J(G))$, and we obtain some bounds and establish some propoertieds of the injective domatic number of a jump graph J(G).

**Definition 3.1.:** Let J(G) = ( V, E ) be a jump graph . a partition $\Delta$ of its vertex set V(J(G)) is called an injective domatic (in short Inj-domatic) partitioned J(G)if each class of $\Delta$ is an Inj-dominating set in J(G). The maximum order of a partition of V(J(G)) into Inj-dominating sets called the Inj-domatic numbr of J(G) and is denoted by $d_{inj}(J(G))$.

For every jump graph J(G) there exists at last one Inj-domatic partition of V(J(G)) namely {V(J(G))}.Therefore $d_{inj}(J(G))$ is well-defined for any jump graph J(G).

**Theore4m 3.2;**

i) For anybcomplete4 graph $J(K_p)$ $d_{inj}(K_p) = d_{cn}(J(K_p)) = d(J(K_p))$ p
ii) $D_{inj}(J(G))$ = n1 if and only if J(G) has at least one Inj-isolatd vertex.
iii) For any wheel grph of p vertices, $d_{inj}(J(W_p)) = p$
iv) For any complete bipartite graph $J(K_{r,,m})$

$$D_{inj}(J(K_{r,m})) = \begin{cases} \text{Min}\{r, m\} & \text{if } r, m \geq 2, \\ 1, & \text{otherwise} \end{cases}$$

v) For any jump graph J(G), if $N_{inj}(v) = N(v)$ for any vertex v in V(J(G)), then $D_{inj}(J(G)) = d(J(G))$.

**Proof:**

**I)** If J(G) =( V, E ) is a complete graph $J(K_p)$, then for any vertex v the set {v} is a minimum CN-dominating set and also a minimum Inj-dominating set is p. Hence , $d_{inj}(J(K_p))= d_{cn}(J(K_p))$=p

**II)** Let J(G) be a graph which has an Inj-isolated vertex say v, then every Inj-dominting set of J(G) must contain the vertex v. Then $d_{inj}(J(G)) = 1$.

Conversely, if $d_{inj}(J(G)) = 1$. And suppose J(G) has no Inj-isolated vertex, then by Theorem 2.21 $\gamma_{inj}(J(G)) \leq \frac{p}{2}$, so if we suppose D is a minimal Inj-dominating set I J(G), then V – D is also a minimal Inj-dominating set. Thus $d_{inj}(J(G)) \geq 2$, a contradiction. Therefore J(G) has at last one Inj-isolated vertex.

iii) Since for every vertex v of the wheel graph the $deg_{ionj}\{v\} = p - 1$. Hence $d_{inj}(J(W_p)) = p$

(iv) and (v) the proof is obvious.

Evidently each CN-dominating set in J(G) is an Inj-dominating set in J(G) and any

CN- domatic partition is an Inj-domatic partition. We have the following proposition.

**Proposition 3.3.:** For any graph J(G) , $d_{inj}(J(G)) \geq d_{cn}(J(G))$.

**Theorem 3.4** : For any graph J(G) with p vertices , $d_{inj}(J(G)) \leq \; ^{p}/_{\gamma_{inj(J(G))}}$

**Proof:** Assume that $d_{inj}(J(G)) = d$ and $\{ D_1, D_2, D_3 \ldots\ldots D_d\}$ is a partition of V(J(G)) into

d numbers of Inj-dominating sets, clearly $| D_i | \geq \gamma_{inj}(J(G))$ for I = 1,2,……d. n we have $p= \sum_{i=1}^{d} | D_I | \geq d\, \gamma_{inj}(J(G))$, Hence $d_{inj}(J(G)) \leq \; ^{p}/_{\gamma_{inj}}J(G))$

**Theorem 3.5:** For any graph J(G) with p vertices, $d_{inj}(J(G)) \geq \llcorner\, ^{p}/_{p} - \delta_{inj}(J(G)) \,\lrcorner$

**Proof:** Let D be any subset of V(J(G)) such that $| D | \geq p - \delta_{inj}(J(G))$. For any vertex $v \in V - D$ we have $| N_{inj}[v\} | \geq 1 + \delta_{inj}(J(G))$. Therefore $N_{inj}(v) \cap D \neq \phi$. Thus D is an Inj-dominating set of J(G). So we can take any $|\; ^{p}/_{p} - \delta_{inj}(J(G))\; ]$ disjoint subset ach of cardinality $p - \delta_{inj}(J(G))$. Hence

$$d_{inj}(J(G)) \geq \llcorner\, ^{p}/_{p} - \delta_{inj}(J(G)) \,\lrcorner$$

**Theorem 3.6:** For any graph such that $d_{inj}(J(G)) \leq \delta_{inj}(J(G)) + 1$. Further the equality holds If J(G) is complete graph $J(K_p)$)

**Proof:** Let J(G) be a graph such that $d_{inj}(J(G)) > \delta_{inj}(J(G)) + 1$. Then there exists at least $\delta_{inj}(J(G)) + 2$ Inj-dominating sets which they are mutually6 disjoint. Let v be any vertex in V(J(G)) such that $deg_{inj}(J(G)) = \delta_{inj}(J(G))$. Then there is at least one of the Inj-dominating sets which has no intersection with $N_{IONJ}[v]$. Hence, that Inj-dominating set can not dominate v, a contradiction. Therefore $d_{inj}(J(G)) \leq \delta_{inj}(J(G)) + 1$. It is a obvious if J(G) is complete, then $d_{inj}(J(G)) > \delta_{inj}(J(G)) + 1$.

**Theorem 3.7:** For any graph J(G) with p vertices $d_{inj}(J(G)) + d_{inj}(J(\bar{G}^{inj}) \leq p + 1$.

**Proof**: From Theorem 3.6, we have $d_{inj}(J(G)) \leq \delta_{inj}(J(G)) + 1$. and $d_{inj}(J(\bar{G}^{inj})) \leq \delta_{inj}(J(\bar{G}^{inj})) + 1$, and clearly $\delta_{inj}(J(\bar{G}^{inj})) = p - 1 - \Delta_{inj}(J(G))$. Hence

$$d_{inj}(J(G)) + d_{inj}(J(\bar{G}^{inj})) \leq \delta_{inj}(J(G)) + p - \Delta_{inj}(J(G)) + 1 \leq p + 1$$

**Theorem 3.8:** For any graph J(G) with p vertices and without Inj-isolated vertices, $d_{inj}(J(G)) + \gamma_{inj}(J(G)) \leq p + 1$.

**Proof:** Let J(G) be a graph with p vertices. Then by Theorem 2.22, we have

$$\boldsymbol{\gamma}_{inj}(J(G)) \leq p - \Delta_{inj}(J(G)) \leq p - \delta_{inj}(J(G)),$$

And also from Theorem 3.6, $d_{inj}(J(G)) \leq \delta_{inj}(J(G)) + 1$.Then

$$d_{inj}(J(G)) + \boldsymbol{\gamma}_{inj}(J(G)) \leq \delta_{inj}(J(G)) + 1. + p - \delta_{inj}(J(G))$$

Hence,

$$d_{inj}(J(G)) + \gamma_{inj}(J(G)) \leq p + 1$$

**References:**

[1] A.Alwardi, B.Arsit'c. I.Gutman and N.D.Sonar, The common neighborhood graph and its energy, Iran J.Math.Sci.Inf, 7(2) 1-8 (2012)

[2] Anwar Alwardi ,N.D.Sonar and Karn Ebadi, On the common neighborhood domination number, Journal of Computer and Mathemsatical Sciences 2(3),547=556 (2011)

[3] S.Armugum, C.Sivagnanam, Neighborhood connected and total domination in graphs,Proc.Int.Conf.on Disc.Math.,2334 B.Chaluvaraju, V.Lokesha and C.Nandesh kumar Mysore 45-51(2008).

[4] B.Chaluvarju, Some parameters on neighborhood number of a graph , Electronic Notes of Discrete Mathematics Elsevier,33 139-146 (2009)

[5] F.Harary, Graph theory, Addison-Wesley, Reading Mass (1969)

[6] T.W.Haynes,S.T. Hedetneimi and P.J.Slater,Fundamentals of domination in graph.Marcel Dekker.Inc., NewYork (1998).

[7] S.M. Hedetneimi, S.T. Hedetneimi, R.C. Laskar , L.Markus and p/J/Slater, Disjoint dominating sets in graphs,Proc.Int.Conf.on Disc.Math. IIM-IISc ,Banglore 88-101(2006).

[8] V.R.Kulli and S.C.Sigarknti,Further results on the neighborhood number of a graph Indian J.Pure and Appl.Math.23(8)575-577)1992).

[9] E.Sampathkumar and P.S.Neeralgi, The neighborhood number of a graph ,Indian Pure and Appl.Math.16(2)126-132 (1985).

[10] H.B.Walikar,B.D.Acharya and E.Sampathkumar, Recent developments in the theory of domination in graphs, Mehta Research institute,Allahabad,MRI Lecture Notes in Math.I (1979)

[11] N.Pratap Babu Rao , Sweta.N, Total efficient Domination in Jump graphs, International journal of Mathemtical Archieve 10(1) 2019 pp 21-25.

[12] Anwar Alwardi, R.Rangarajan and Akram Alqesmah On the injective domination of graphs,Palestine Journal of Mathematics Vol.7(1) (2008) 202-219.