THE ORDER REDUCTION OF HIGH ORDER CONTINUOS TIME MIMO USING MODIFIED POLE CLUSTERING AND SIMPLIFIED ROUTH APPROXIMATION METHOD

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Abstract - *In modeling physical systems, the order of the system gives an idea of the measure of accuracy of the modeling of the system. The higher the order, the more accurate the model can be in describing the physical system. But in several cases, the amount of information contained in a complex model may obfuscate simple, insightful behaviors, which can be better captured and explored by a model with a much lesser order. Pole clustering method is proposed for the Multiple Input Multiple Output linear time invariant system to obtain the stable reduced order system.*

The simplified Routh approximation is used at the tail end of the proposed scenarios to get error minimized reduced model. In this method, the common denominator polynomial of the reduced-order transfer function matrix is synthesized by using modified pole clustering while the co-efficients of the numerator elements are computed by simplified Routh approximation. The modified pole clustering generates more dominant cluster centres than cluster centres obtained by pole clustering technique already available. The proposed algorithm is computer-oriented and comparable in quality. This method guarantees stability of the reduced model if the original high-order system is stable. The algorithm of the proposed method is illustrated with the help of an example and the results are compared with the other well-known reduction techniques.

Key Words: MATLAB, DOMINANT POLE, ISE, ORDER REDUCTION, POLE CLUSTERING,STABILITY.

1. INTRODUCTION

Getting the reduced order model of a higher order model. The reduced order modeling of a large system is necessary to ease the analysis of the system. The approach is examined and compared to single-input single-output (SISO) and multi-input multi-output (MIMO) systems.

It is very easy to analyze both electrical and mechanical systems. Order reduction is a technique for reducing the computational complexity of mathematical models in numerical simulations.

High-fidelity models can involve large-scale, nonlinear dynamical system behavior whose simulations can take hours or even days. Some applications require the model to be simulated thousands of times. Running many high-fidelity simulations presents a significant computational challenge.

These disadvantages can be reduced by reducing the high order systems to low order systems without changing its physics. By this we can save time and we can get the output accurately.

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2. LITERATURE SURVEY

Every physical system can be translated into mathematical model. The mathematical procedure of system modeling often leads to comprehensive description of a process in the form of high-order differential equations which are difficult to use either for analysis or controller synthesis. It is, therefore, useful, and sometimes necessary, to find the possibility of finding some equations of the same type but of lower order that may be considered to adequately reflect the dominant characteristics of the system under consideration.

Some of the reasons for using reduced-order models of high-order linear systems could be as follows:

1. to have a better understanding of the system,
2. to reduce computational complexity,

3. to reduce hardware complexity,
4. to make feasible controller design.

Several reduction methods are available in literature for reducing the order of large-scale linear MIMO systems in frequency domain. Further, some mixed methods have been suggested by combining the algorithm of two different reduction methods .

In spite of having several reduction methods, none always gives the satisfactory results for all the systems. The optimization-based reduced-order modeling has already been suggested in the work in which the numerator coefficients are computed by minimizing the integral square error (ISE) between the step responses of the original and the reduced system while the denominator polynomial is obtained by using existing order-reduction technique.

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It is very easy to analyze both electrical and mechanical systems. Order reduction is a technique for reducing the computational complexity of mathematical models in numerical simulations. High-fidelity models can involve large-scale, nonlinear dynamical system behavior whose simulations can take hours or even days. Some applications require the model to be simulated thousands of times. Running many high-fidelity simulations presents a significant computational challenge.

These disadvantages can be reduced by reducing the high order systems to low order systems without changing its physics. By this we can save time and we can get the output accurately.

3. REDUCTION METHOD

Introduction

This proposes an algorithm for the reduction of high – order MIMO systems. The proposed method is based on formation of clusters of poles of original high order system and retention of time moments.

Proposed Reduction Procedure

Let the high-order linear dynamic stable MIMO continuous time system be defined as:

$X^* = AX + BU$ and

$Y^* = CX$......(3.1)

Where, $X \in R^n$,U,Y are scalar input and output variables respectively and A,B,C are matrices of compatible dimensions.

The complex s-domain transfer function of the original high order system can be written as

G(s) = C$(sI - A)^{-1}B$

$=\sum_{i=0}^{n-1} B_i S^i / \sum_{i=0}^{n} A_i S^i$ ....(3.2) .

$=\sum_{i=0}^{n-1} B_i S^i / \prod_{i=1}^{n}(s + P_i)$ ......(3.3)

where 'pi' are the 'k' poles of R_k(s).

R_k(s) = N_k(s) / D_k(s)

The 'k' poles of R_k(s) (i.e., p1,p2,……,pk) are formed by the method of pole clustering described below.

Rules for cluster formation of poles:

The criterion for grouping the poles of G(s) into one particular cluster should be based on :

a) The relative distance between the poles and

b) The desired order of the reduced model,

Since each such cluster of poles of finally replaced by a single (pair of) real (complex) pole (poles).

The rules used for clustering the poles in the s-plane are:

a) Separate clusters should be made for real poles and complex poles and

b) Poles on the jω-axis (if any) are retained in the reduced order model.

Reduced Order Denominator $D_k(s)$

An obvious choice for obtaining the cluster center which will eventually be used to replace the poles lying in this cluster would be to use the "mass center". But the poles lying near the jω-axis have dominant effect on the system behavior and so these must be given proper weights in determining the cluster centre.

A simple method that will automatically provide larger weights to the dominant poles would be the use of the criteria of Inverse Distance Measure. Thus the poles are obtained by "Inverse Distance Measure" (IDM) method.

Let the 'r' real poles in one cluster be P1, P2, ...Pr,

then the IDM criterion identifies the cluster centre as :

$$\mathrm{P_{rc}} = \left[\left(\sum_{i=1}^{r} {}^{1}/_{\mathrm{P_i}}\right)/\mathrm{r}\right]^{-1} \quad \text{.........(3.4)}$$

Where 'P_{rc}' is the cluster centre for the real poles of the original system. 'P_{rc}' is then taken as a real pole of the reduced order model.

Let '2c' complex poles in a cluster be

$\alpha 1 \pm j\beta 1, \alpha 2 \pm j\beta 2, \ldots\ldots, \alpha c \pm j\beta c$

Then the "Inverse Distance Measure" (IDM) criterion identifies the complex cluster centre as (A + jB)

where $\mathrm{A}=\{[\sum_{i=1}^{c} 1/\alpha_i]/c\}^{-1}$ ......(3.7)

and $\mathrm{B}=\{[\sum_{i=1}^{c} 1/\beta_i]/c\}^{-1}$ ....(3.8)

Now, the reduced order model denominator is obtained by replacing the clusters of the poles of the original system by the respective cluster centers.

i.e., D_k(s) = (s+ Prc1)(s+ Prc2)…………(s+ P_{rc}(rg))(s+A1+ jB1)

(s+A2+ jB2)………… (s+ A_{cg}+ j B_{cg}) ……(3.9)

where 'r_g' is the number of real groups and

'cg' is the number of complex groups.

Reduced Order Numerator $N_k(s)$

Procedural steps:

1. Consider the form of original high order MIMO system obtained by its transfer function as :

$$G(s) = \frac{B_0 + B_1 s + B_2 s^2 + \cdots + B_{n-1} s^{n-1}}{A_0 + A_1 s + A_2 s^2 + \cdots + A_0 s^n}$$

.............(3.10)

2. Let the reduced order transfer function of lower order model to be required is :

$R_k(s) = \frac{b_0+b_1 s+b_2 s^2+\cdots+b_{k-1} s^{k-1}}{a_0+a_1 s+a_2 s^2+\cdots+a_k s^k}$ where $k < n$

.........(3.11)

3. Now, by using the Time-moment matching technique, for higher-order original system [i.e. for G(s)], the time moments can be calculated from the following relations :

First Time Moment

$T_0 = \frac{B_0}{A_0}$ (for $n = 0$)

...............(3.12)

and

N^{th} Time Moment $T_n = \left[\frac{B_n - \sum_{j=1}^{n} A_1 T_{n-j}}{A_0}\right]$ (for $n > 0$)

....(3.13)

Similarly, for the lower order system [i.e. for R_k(s)], the time moments can be calculated from the following relations.

first time moment

$t_0 = \frac{b_0}{a_0}$ (fork $= 0$)

...............(3.14)

K^{th} time moment $t_k = \left[\frac{b_k - \sum_{j=1}^{k} a_1 t_{k=j}}{a_0}\right]$ (for $k > 0$)

.....(3.15)

Now, from the equations (3.12 & 3.13) and (3.14 & 3.15), by matching the time moments of the original high-order system, with that of the reduced lower order system, we can obtain the coefficients of the numerator polynomial of the reduced order system and thereby we can obtain the zeros of the reduced order model.

To obtain the reduced order model:

Finally, we can obtain the required reduced order model of the given high-order system using the equation (3.11) i.e.

$R_k(s) = \frac{N_k(s)}{D_k(s)}$

$= \frac{b_0+b_1 s+b_2 s^2+\cdots+b_{k-1} s^{k-1}}{a_0+a_1+a_2 s^2+\cdots+a_k s^k}$ where $k < n$

3.1 FLOWCHART

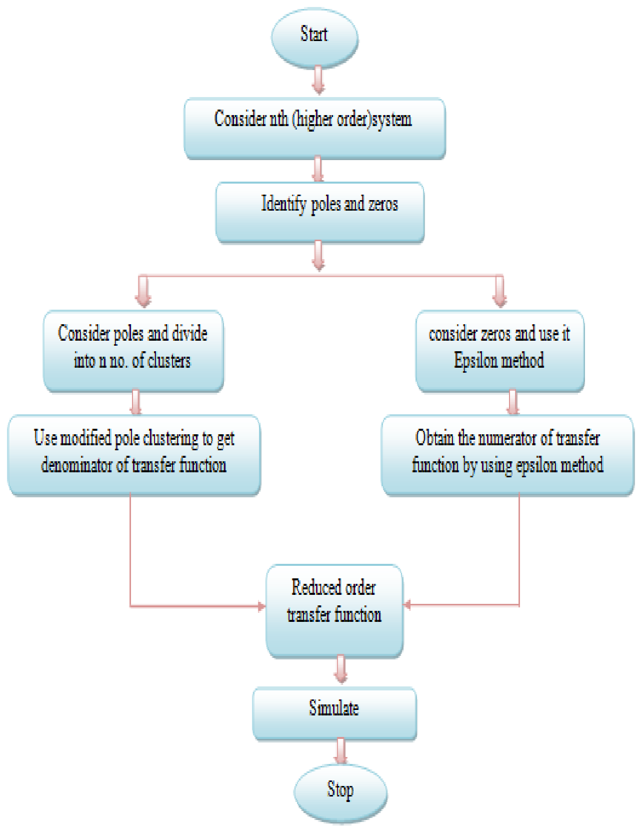

figure 1: Flow chart for Proposed method

4. NUMERICAL EXAMPLE

Example 1: 4th order system

Consider a MIMO system, with transfer function given by G(s)=

$$\begin{pmatrix} \frac{1}{(s+2)} & \frac{1}{(s+3)} \\ \frac{1}{(s+4)} & \frac{s+5}{(s+1)(s+2)} \end{pmatrix}$$

The poles are -1,-2,-3,-4

Let the first pole cluster be (-1,-2) and

second pole cluster be (-3,-4)

Consider first cluster as -1,-2

$P_1 = -1$, $P_2 = -2$

$Cj = [\sum i = 1r \ \ (-1/IPiI) \ \div r] - 1$

$C_1 = [(-1/i\text{-}1i+-1/i\text{-}2i)\div 2]^{-1}$

C_1=-1.33

C_2= $[((-1/\text{ i } P_1 \text{ i})+(-1/ \text{ I}C_{j-1}\text{I}))\div 2]^{-1}$

C_2= $[((-1/\text{ I-1I})+(-1/\text{ I-1.33I}))\div 2]^{-1} = (-0.875)^{-1}$

P_{e1}= C_2=-1.1428

C_1= $[[(-1/\text{ I-3I}) +(-1/\text{ I-4I})]\div 2]^{-1}$

C_1= $((-0.3-0.25)\div 2)^{-1}$

C_1=-3.636

C_2= $[[(-1/\text{ I-3I}) +(-1/\text{ I-3.636I})]\div 2]^{-1}$

C_2= $[(-0.635) \div 2]^{-1}$

P_{e2}= C_2= -3.2893

P_{e1}= -1.1428

P_{e2}=-3.2893

By doing the pole clustering

The modified pole clusters are

P_{e1} = -1.1428
P_{e2} = -3.2893
Therefore modified Denominator D1(s)= S^2+4.4321S+3.759
common denominator
D(s)=(s+1)(s+2)(s+3)(s+4)
=s^4+10s^3+35s^2+50s+24

TRANSFER FUNCTIONS:

% Original system 1%

$$G1 = \frac{2s^3+15s^2+33s+20}{s^4+10s^3+35s^2+50s+24}$$

% Original system 2%

$$G2 = \frac{2s^3+18s^2+58s+66}{s^4+10s^3+35s^2+50s+24}$$

Now by Proposed Epsilon method
Considering N1(s)/D(s)
a_0 value from denominator by pole clustering = 3.759
A_0, B_0 values from original system are 24, 20 respectively
Then
$T_1 = (a_0/A_0)*B_0$
= (3.759/24)*20
= 3.1375
$M_1 = (B_5 / A_5)$
= 2/1
= 2
R11= $(T_1+sM_1)/D_1(s)$
% Proposed Epsilon System 1%

$$R11 = \frac{2 s + 3.1375}{s^2 + 4.432 s + 3.759}$$

Considering N2(s)/D(s)
a_0 value from denominator by pole clustering = 3.759
A_0, B_0 values from original system are 24, 66 respectively
Then

$T_2 = (a_0/A_0)*B_0$
= (3.759/24)*66
= 10.337
$M_2 = (B_5 / A_5)$
= 2/1
= 2
R12= $(T_2+sM_2)/D_1(s)$
% Proposed Epsilon System2%

$$R12 = \frac{2 s + 10.337}{s^2 + 4.432 s + 3.759}$$

Example 2: 6th Order system
Let us consider an example of 6th order system

$$G(s)= \begin{pmatrix} \dfrac{2(s+5)}{(s+1)(s+10)} & \dfrac{s+4}{(s+2)(s+5)} \\ \dfrac{s+10}{(s+1)(s+20)} & \dfrac{s+6}{(s+2)(s+3)} \end{pmatrix}$$

The poles are -1,-2,-3,-5,-10,-20
Let the first pole cluster be (-1,-2) and second pole cluster be (-3,-5,-10,-20)
Consider first cluster as -1,-2
P_1 = -1, P_2 = -2
$Cj = [\sum i = 1r \ (-1/IPiI) \ \div r] - 1$
$C_1 = [(-1/I-1I + -1/I-2I)\div 2]^{-1}$
C_1=-1.33
$C_2 = [((-1/ IP_1I)+(-1/ IC_{j-1}I))\div 2]^{-1}$
$C_2 = [((-1/ I-1I)+(-1/ I-1.33I))\div 2]^{-1} = (-0.875)^{-1}$
$P_{e1} = C_2$=-1.1428
$C_1 = [[(-1/ I-3I) +(-1/ I-5I)+ (-1/ I-10I)+ (-1/ I-20I)]\div 4]^{-1}$
$C_1 = ((-0.333-0.2-0.1-0.05)\div 4)^{-1}$
$C_1 = (-0.683\div 4)^{-1}$
C_1=-5.8565
$C_2 = [[(-1/ I-3I) +(-1/ I-5.8565I)]\div 2]^{-1}$
$C_2 = [(-5.504) \div 2]^{-1}$
$P_{e2} = C_2$= -3.19678
P_{e1}= -1.1428
P_{e2}=-3.1967
By doing the pole clustering
The modified pole clusters are
P_{e1} = -1.1428
P_{e2} = -3.1946
Therefore modified Denominator $D_1(s)$= S^2+4.3374S+3.65
Now, for the Numerator
By doing Genetic Algorithm
N11(s)=(s+2)(s+5)(s+3)(s+20)(s+5)
=s^5+35s^4+459s^3+1805s^2+3850s+3000
N12(s)=(s+1)(s+4)(s+3)(s+20)(s+4)
=s^5+38s^4+459s^3+2182s^2+4160s+2400
N21(s)=(s+2)(s+5)(s+10)((s+3)(s+10)
=s^5+42s^4+601s^3+3660s^2+9100s+6000
N22(s)=(s+1)(s+5)(s+10)(s+20)(s+6)
=s^5+30s^4+331s^3+1650s^2+3700s+3000
N1(s)=N11(s)+N12(s)
=2s^5+73s^4+840s^3+3987s^2+8010s+5400
N2(s)=N21(s)+N22(s)
=2s^5+72s^4+932s^3+5310s^2+2800s+9000
common denominator
D(s)=(s+1)(s+2)(s+3)(s+5)((s+10)(s+20)
=s^6+41s^5+571s^4+3430s^3+10060s^2+13100s+6000

TRANSFER FUNCTIONS:
% Original system 1%

2 s^5 + 73 s^4 + 840 s^3 + 3987 s^2 + 8010 s + 5400
G1 = ---
s^6 + 41 s^5 + 571 s^4 + 3430 s^3 + 10060 s^2 + 13100 s + 6000

% Original system 2%

2 s^5 + 72 s^4 + 932 s^3 + 5310 s^2 + 12800 s + 9000
G2= ---
s^6 + 41 s^5 + 571 s^4 + 3430 s^3 + 10060 s^2 + 13100 s + 6000

By using Genetic Algorithm, Pole clustering

% Genetic Algorithm 1%

$$r11 = \frac{2.2\ s + 5.11}{s^2 + 4.337\ s + 3.651}$$

% Genetic Algorithm2%

$$r12 = \frac{2.18\ s + 5.44}{s^2 + 4.337\ s + 3.651}$$

Now by Proposed Epsilon method

Considering N1(s)/D(s)

a_0 value from denominator by pole clustering = 3.651

A_0, B_0 values from original system are 6000, 5400 respectively

Then

$T_1 = (a_0/A_0)*B_0$

= (3.651/6000)*5400

= 3.2858

$M_1 = (B_5 / A_5)$

= 2/1

= 2

$R11= (T_1+sM_1)/D_1(s)$

% Proposed Epsilon System 1%

$$R11 = \frac{2\ s + 3.285}{s^2 + 4.337\ s + 3.651}$$

Considering N2(s)/D(s)

a_0 value from denominator by pole clustering = 3.651

A_0, B_0 values from original system are 6000, 9000 respectively

Then

$T_2 = (a_0/A_0)*B_0$

= (3.651/6000)*9000

= 5.47

= (B_5 / A_5)

= 2/1

= 2

$R12= (T_2+sM_2)/D_1(s)$

% Proposed Epsilon System2%

$$R12 = \frac{2\ s + 5.47}{s^2 + 4.337\ s + 3.651}$$

4.1 RESULTS

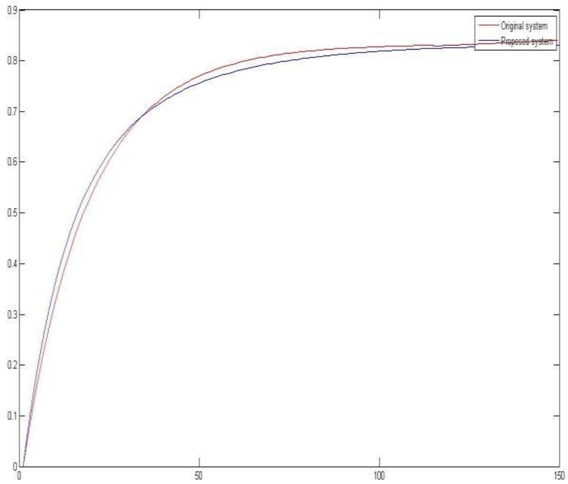

figure2: 4th Order system first output

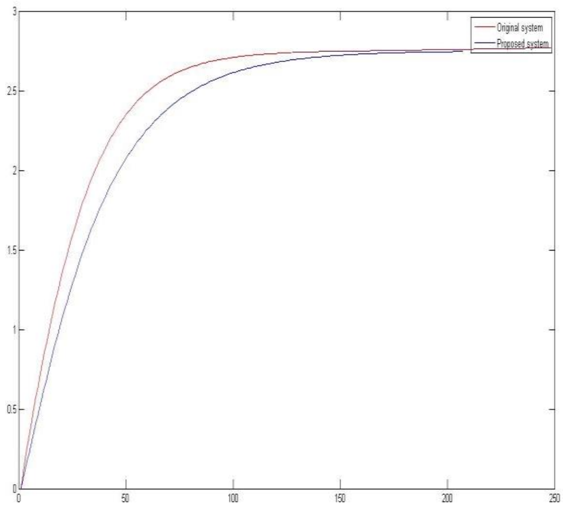

figure 3: 4th Order system second output

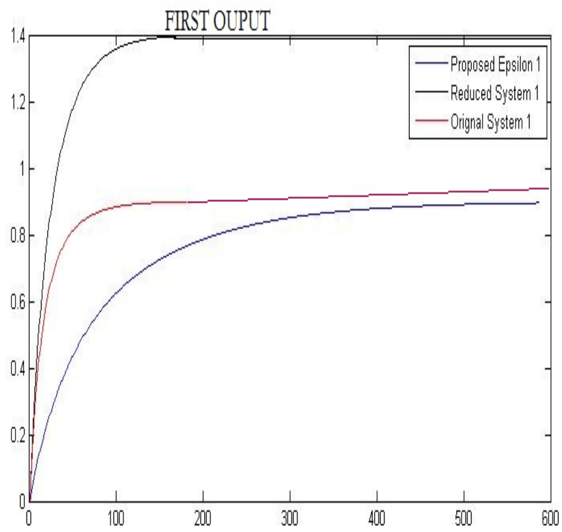

figure 4: 6th Order system first output

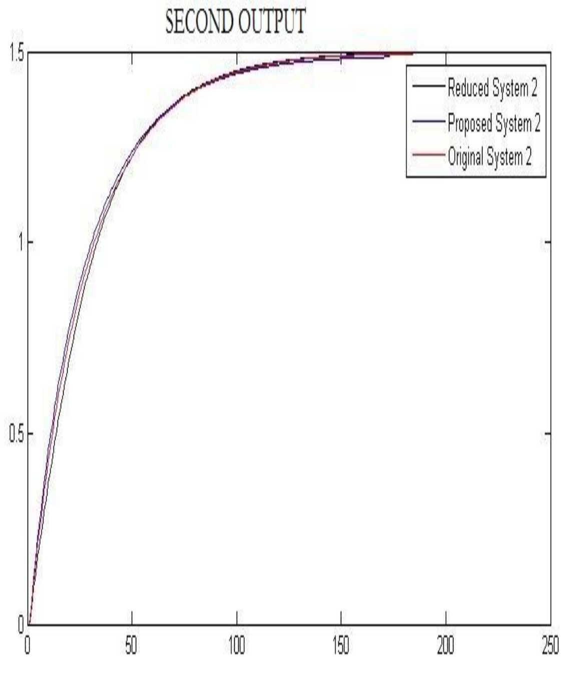

figure 5: 6th Order system second output

5. CONCLUSION

A new algorithm for the analysis and design of high order MIMO continuous time systems using Pole clustering model reduction technique is proposed. The proposed method is a mixed method, which is based on Pole clustering technique and time moment matching method to generate reduced order models and it guarantees the stability of the high order continuous time system in the reduced order models. The proposed method overcomes the limitations and drawbacks of some of the existing methods of continuous time MIMO systems discussed above. The effectiveness and computational simplicity of the proposed method is illustrated through typical numerical examples available in literature. The Proposed method is compared with the reduction method based on Genetic Algorithm.

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BIOGRAPHIES

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