THE AVERAGE LOWER DOMINATION NUMBER OF JUMPGRAPH

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ABSTRACT: The average lower domination number γ_{av} (J(G)) is defined as $\frac{1}{|V(J(G))}\sum_{v\in V(J(G))}\gamma_v(J(G))$ where γ_v(J(G)) is the minimum cardinality of a maximal dominating set that contains v. In this paper,the average lower domination number of complete k-ary tree and B_n tree are calculated .Moreover we obtain the $\gamma_{av}J(G^*)$ for thorn jump graph $J(G^*)$. Finally we compute the $\gamma_{av}(J(G_1) + J(G_2))$ of $J(G_1)$ and $J(G_2)$

2010 Mathematics subject Classificastion 05C35,05C40,05C69, 68M10,68R10.

Key words: Connectivity Network Degineand communication, Aerage lower Domination Number.

Introduction : A network is modeled with gaph in a situation which the centers are equal to the vertex of graphs and connection lines are equal to the edgesd of a graph. A graph J(G)(is denoed by J(G)= (V(J(G)). E(J(G)) where V(J(G)) and E(J(G)) are vertex and edge set of G, respectively. Let v be a vertex in V(J(G)).

In a jump graph J(G) = (V(J(G)), E(J(G)), a subset S⊆ V(J(G)) of vertices is a dominating setn if every vertex in V(J(G)) – S is adjacent to at least one vertex of S. The domination number of γ(J(G))is he minimum cardiality of a dominating set. A dominating set of cardinality γ(J(G)) is called a γ(J(G))-set.

Henning [12] introduce the concept of average domination . The lower domination number , denoted by γ_v(J(G)) is the minimum cardinality of a dominating set of (J(G)) that contains v.

The average lower domination number γ_{av} (J(G)) is defined as $\frac{1}{|V(J(G))}\sum_{v\in V(J(G))}\gamma_v(J(G))$ where γ (J(G)) is the minimum cardinality of a maximal dominating set that contains v.

Clear for the vertex v in a graph J(G), $\gamma(J(G)) \leq \gamma_{av}(J(G))$ with equality if and only if v belongs to a γ(J(G))-set. Consequently $\gamma_{av}J(K_n)$)= 1. While for a cycle C_n on $n\geq 3$ vertices

$\gamma_{av}(j(C_n)) = \gamma(J(C_n))= \lceil \frac{n}{3} \rceil$.

Poposition 1.1: [12] For any jump graph J(G) of order n with domination number γ,

$\gamma_{av}(J(G)) \leq \gamma + 1 - \frac{\gamma}{n}$, with equality if and o9nly if J(G) has unique γ (J(G))-set.

Theorem 1.1 : ([12]).: If T is a tree of order $n \geq 4$, then $\gamma_{av}(T) \leq \frac{n}{2}$ with equality if and only if T is the corona of tree.

In this paper, the average lower domination number of complete k-ary tree and B_n tree is calculated . Moreover we obtain the γ_{av} $J(G^*)$. For the thron graph $J(G^*)$. Finally we compute the $\gamma_{av}(J(G_1) + J(G_2))$ of $J(G_1)$ and $J(G_2)$

2. Average Lower Domination Number Of Some graphs.

Firstly we give the definition of a complete k-ary tree with depth n. The average lower domination number of complete k-ary tree are calculated. Moreover we obtain $\gamma_{av}(J(B_n))$ for binomial tree graph $J(G^*)$

Definition:2.1 ([3]) A complete k-ary tree with depth n is all leaves have the same depth and all internal vertices have degree k. A complete k-ary tree has $\frac{k^{n+1}-1}{k-1}$ vertices band $\frac{k^{n+1}-1}{k-1}$ - 1 edges

Theorem 2.1 : Let J(G) be a complete k-ary tree with depth n. Then

$$\gamma_{av}(J(G)) = \begin{cases} \gamma(J(G)) + 1 - \frac{\gamma(J(G))+k}{|V(J(G))|}, & n \equiv 0 \pmod 3 \\ \gamma(J(G)) + 1 - \frac{\gamma(J(G))}{|V(J(G))|}, & \text{otherwise} \end{cases}$$

Proof : Let G is a k-ary tree withg depth n then |V(J(G)| = $\frac{k^{n+1}-1}{k-1}$ we have two cases for n to find the average lower average number of J(G).

Case (i): if n ≡ 1(mod 3)or n ≡ 2 (mod 3) then J(G) has unique γ(H(G))-set. The minimal domination set of set J(G) contains the vertices on he level (n – 1 -3i) for 0≤ I ≤ $\lfloor \frac{n}{3} \rfloor$. Let vertices of J(G) be V(J(G) = V(J(G_1)) U V(J(G2)) where V(J(G_1)) The set contains the vertices on the levels (n – 1 – 3i) for 0≤ I ≤ $\lfloor \frac{n}{3} \rfloor$ V(J(G_2)) : The set contains the vertices of V(J(G)) –V(J(G_1)).

(i) If v ∈V(J(G_1)), then γ_v(J(G)) = γ (J(G)) since the vertex v is in the dominating set Sinc e this equality is satisfied for every vertex in V(J)(G_1)) we have

$$\sum_{v\in V(J(G1)} \gamma_v(J(G)) = \gamma(J(G)) \,.\, \gamma(J(G))$$

(ii) If v ∈V(J(G_2)), then γ_v(J(G)) = γ (J(G)) + 1 since he vertex not in the dominating set.Since the equality is satisfied for every vertex of V(G_2)). We have

$$\sum_{v\in V(J(G2)} \gamma_v(J(G)) = (|V(J(G)) - \gamma(J(G))\,)\,(\gamma(J(G)) +1\,)$$

Consequently,

$$\gamma_{av}(J(G)) = \frac{1}{|V9J(G))|} \sum_{v\in V(J(G1)} \gamma_v(J(G))$$

$$= \frac{1}{|V9J(G))|} \left(\sum_{v\in V(J(G1)} \gamma_v(J(G)) + \sum_{v\in V(J(G2)} \gamma_v(J(G)) \right)$$

$$= \frac{1}{|V9J(G))|} [\,(\gamma(J(G)) \,.\, \gamma(J(G))) + (|V(J(G)) - \gamma(J(G))\,)\,(\gamma(J(G)) +1\,)]$$

$$= \gamma(J(G)) + 1 - \frac{\gamma(J(G))}{|V(J(G))|} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Case (ii): If J(G) is a k-ary tree with depth n and n ≡ 0 (mod 3) , then J(G) has k+1 domination sets which give the domination number of J(G) The minimal domination setr of J(G). The minimal domination set of J(G) contains the vertices on the .levels (n - 1 - 3i) for 0≤ I ≤ $\lfloor \frac{n}{3} \rfloor$. But in this case the vertex on the 0^{th} level cannot be reached . Therefore the vertex on the 0^{th} level or one of the vertices on the 1^{st} level should be taken to the dominating set. Hence there are k + 1 dominating sets according to the choice of vertices.

(i) If v∈ γ(J(G))-set,then γ_v (J(G))= γ(UJ(G)) since the vertex is the dominating set. We have to respect this process for k + γ (J(G) vertices. Therefore

$$\sum_{v\in V(J(G)} \gamma_v(J(G)) = (\,\gamma(j(g)) + K\,)\,\gamma(j(g))$$

(II) If v∉ γ(J(G)) –set then γ_v(J(G)) = γ (J(G)) since the vertex v is the dominating set. We have to respect this process for |V(J(G))| - k – γ(J(G)) vertices. Hence

$$\sum_{v\in V(J(G)} \gamma_v(J(G) = (|V(J(G)| - (\,\gamma(J(G)) +k)\;\;(\,\gamma(J(G)) +1)\,]$$

As a result

$$\gamma_{av}(J(G)) = \frac{1}{|V(J(G))|} [\;(\,\gamma(J(G)) +k)\;\gamma(J(G)) \;+\; (|V(J(G)| - (\,\gamma(J(G)) +k)\;(\,\gamma(J(G)) +1)]$$

$$= \gamma(J(G) + 1 - \frac{\gamma(J(G))+k}{|V(J(G))|}, \quad \ldots\ldots\ldots\ldots{*}(2)$$

By (1) and (2) the proof is completed

Definition 2.2 : ([3]) The binomial tree of order n ≥ 0 with root R is the tree B_n defined as follows.

1) If n=0, B_n=B_0 = R i.e., the binomial tree of order zero consists of n single node R.

2) If N>0, B_n= R, B_0, B_1…………B_{n-1} i.e., the binomial tree of order n>0 compress the root R, and n binomial sub trees, B_0, B_1…………B_{n-1}

Theorem 2.2: Let B_n be the binomial tree B_n consists of 2^n vertices, 2^{n-1} vertices with degree 1. While the domination set is found, all the vertices with degree 1 or the vertices adjacent to these vertices should be taken into the set. Therefore the domination number of B_n is $\gamma(J(B_n)) = 2^{n-1}$. Obviously the domination set satisfying the domination number can be obtained for every elemen of B_n. Since $\gamma_v(J(B_n)) = 2^{n-1}$ for every element v of B_n. Hence

$$\sum_{v\in V(J(Bn))} \gamma_v(J(B_n)) = 2^{n-1} \quad 2^n$$

From the definition of average lower domination number we have

$$\gamma_{av}(J(B_n)) = \frac{1}{2^n} 2^{n-1} \quad 2^n = 2^{n-1}$$

Definition 2.3 : ([13]) Let $p_1, p_2, \ldots\ldots p_n$ be non-negative integers and g be such a graph V(J(G)) = n. The rone graph of the graph, with parameters $p_1\ p_2, \ldots\ldots\ldots p_n$.is obtaind by attaching p, new vertices of degree 1 to the vertex v, of the graph J(G), i=1,2,.....n. The thorn graph of the graph J(G) will be denoted by $J(G^*)$ or by G^* ($p_1, p_2, \ldots\ldots\ldots p_n)_{...if}$ the respective parameters need to be specified.

Theorem 2.3 : Let G be a non complete connected graph with order n and G^* be a thorn graph of J(G) with every $P_k = 1$ Then $\gamma_{av}(J(G^*)) = n$

Proof: The number of vertices of $J(G^*)$ is 2n. While the dimention set is found every vertex of degree 1 or the vertex adjacent to it must be taken into the dominating set. Therefore the domination number of $J(G^*))$ is $\gamma(J(G^*))$=n. Thus the domination set satisfying the domination number can be obtained for every element of $J(G^*)$. Sin ce $\gamma_v(J(G^*) = n$ for every element of $J(G^*)$, therefore

$$\sum_{v\in J(G*)} \gamma_v(J(G^{*+}) = 2n\ .n$$

From the definition of average lower domination number we have

$$\gamma_{av}(J(G^*) = \frac{1}{2N} 2N\ .\ N = N$$

Theorem 2.4. Lert $J(G^*)$ be a thorn graph of J(G) with every $p_k > 1$ .Obviously $\gamma(J(G^*) = |V(J(G))|$,hence all of the vertices of J(G) should be taken into the dominating set .Let vertices set of $J(G^*)$ ve $V(J(G^*) = V(J(G_1) \cup V(J(G_2)$ where,

$V(J(G_1))$: The set contains the vertices of graph J(G).

$V(J(G_2))$: The set contains the vertices of $V(J(G)) - VJ(G_1))$

Then we have

$$\sum_{v\in V(J(G*)} \gamma(J(G^*)) = \sum_{v\in V(J(G1)} \gamma_v(J(G^*)) + \sum_{v\in V(J(G2)} \gamma_v(J(G^*))$$

i) If $v \in V(J(G_1))$, then $\gamma_v(J(G^*)) = |\ V(J(G))|$. We have torespect this processd for every vertices of $V(J(G_1))$ Hence

$$\sum_{v\in V(J(G1)} \gamma_v(J(G^*)) = |\ V(J(G))|\ |\ V(J(G))|$$

ii) If $v\in V(J(G_2))$, then $\gamma_v\ ((JG^*)) = |V(J(G))| + 1$. We have tp repeat this process for every vertices of $V(J(G_2))$. So,

$$\sum_{v\in V(G2)} \gamma_v(J(G^8)) = (|VJ(G^*)) - |V(J(G)|) (|V(J(G) | + 1)$$

From the definition of average lower domination number we have

$$\gamma_{av}(J(G^*)) = \frac{1}{|V(J(G*)|}(|V(J(G))||V(J(G))| + (|V(J(G^*)| - |V(J(G))|(|V(J(G))| + 1)$$

$$\underline{|V(J(G))|}$$

$$= |V(J(G))| + 1 \_ V(J(G^*))|$$

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