Mathematical Equipment of Lagrange's Equation Describes Real Mechanics.

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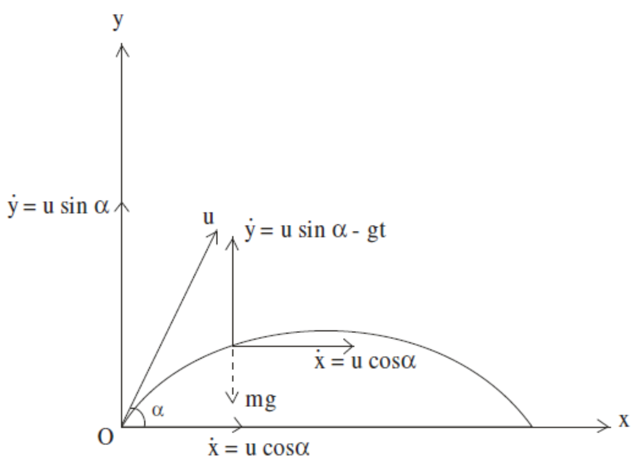
Abstract - *Mathematics is a branch of science that deals with all concepts in the world according to its definition of Mathematics. And technology is a very important scientific building study of movement, but it is based on a specific mathematical formulation. In mechanics there is a study of moving particles under the action of force. Movement equilibrium describes how particles move under the action of force. We must know that all particle movement is not free movement, but rather restricted by imposing certain conditions on the movement of a particle or particle system. Thus basic concepts such as movement equilibrium, issues and type of barriers to particle movement, integrated links, conservation forces, conservation scholars, D'Alembert's policy, etc. . This research work emphasizes the movement of particles that describe machines and is actually part of the Partial Differential Equation of mathematics and is largely based on the Lagrange equation on ancient machines.*

Keywords: D'Alembert system, standard power, normal speed, Virtual function, virtual velocity, direct migration.

1. INTRODUCTION

Suppose a bullet is fired at a point determined by the initial velocity, not directly upwards but at an angle horizontally.[1]The mechanical branch of the mathematical instrument used is related to the movement of a particle or particle system and energy.[2] Of course, to answer such questions, mathematicians do not need a meter to measure the distance covered by a bullet immediately, they do not need a speedometer, and they do not need a clock to determine the time required to cover a particular distance.[3] In fact, they are not required to perform any such tests. What they need to explain the movement of the bullet is simply a link.[4] So one of the most important concepts in technology is the concept of links. Co-coordinates, however, play only the role of markers or codes and will not affect or influence the movement of the character.[5]These are just mathematical tools in the hands of a mathematician. So the instruments in the hands of a mathematician are the links. With the help of these compounds, the movement of a particle or a particle system can be fully explained.[6]



For example, to discuss the movement of a character, take P (x, y) for any point in the alphabetical order. The only force acting on this character is the force of gravity on the downward side. To solve this force horizontally and vertically, we write about Newton's second law of motion.

$$\ddot{x} = 0, \quad \ddot{y} = -g. \qquad \ldots (1)$$

Integrating the above two equations and using the initial conditions we readily obtain

$$\dot{x} = u\cos\alpha, \quad \dot{y} = u\sin\alpha - gt, \qquad \ldots (2)$$

where u is the initial velocity of the bullet when t = 0. Integrating equations (2) once again and using the initial conditions we obtain

$$x = u\cos\alpha \cdot t, \qquad \ldots (3)$$

$$y = u\sin\alpha t - \frac{1}{2}gt^2. \qquad \ldots (4)$$

Equations (2) determine the velocity of the bullet at any time t, while equations (3) and (4) determine the position of the bullet at that instant. Further, eliminating t from equations (3) and (4), we get

$$y = x\tan\alpha - \frac{1}{2}\frac{gx^2}{u^2\cos^2\alpha}. \qquad \ldots(5)$$

This figure provides a bullet method and the method is parabola. However, the links used to describe the movement of a particle or a particle system must be equally independent. Otherwise the number of calculations describing the movement of the system will be less than the value of the variable and in this case the solution cannot be determined separately. For example if a particle moves freely in space, then three independent links are used to describe its movement. This can be a Cartesian combination (x, y, z) or circular polar connectors (r, θ,φ). In that case, the particle travels to one of the connecting axes in space, then all three links are independent, so these three links cannot be used for its meaning. In conjunction with a single coefficient only one co-editing varies and the other two are times and only different compounds can be used to define particle movements.

2. Another Basic Definition.

2.1 Velocity: Allow the particles to move in any direction in relation to the fixed point O. If r $\vec{}$is is its stop vector, then the particle velocity is defined as the measurement time of the local vector change. i.e.,$v = \dot{\vec{r}}$,
where dot denotes the derivative with respect to time.
If further,

$\vec{r}= xi + yj + zk$ is the position vector,

then velocity of the particle is,

$v = \dot{\vec{r}} = \dot{x}i + \dot{y}j + \dot{z}k$, where ẋ,ẏ, ż are called the components of the velocity along the coordinates axes.

2.2 Direct pressure: Equal particle pressure is defined as the product of particle mass and its velocity. The vector size is also defined by p $\vec{}$. So we have p $\vec{}$ = mv $\vec{}$. The direction of force corresponds to the same velocity track. Depending on the direct particle pressure the movement equation is given by F $\vec{}$ = p $\vec{}$

2.3 Angular Pressure: The angular pressure of an aparticle by any fixed point O as Origin is defined as
r $\vec{}$ × p $\vec{}$. Vector size and defined by LL. So L $\vec{}$ = r $\vec{}$ × p $\vec{}$
Angular pressure is eye-catching in both the vector of position and the same particle strength.

2.4 Torque

4. **Torque (Moment of a Force):** The time rate of change of angular momentum $\bar{L}$ is defined as torque, It is denoted by $\bar{N}$. Thus

$$\bar{N} = \frac{d\bar{L}}{dt} = \frac{d}{dt}(\bar{r}\times\bar{p})$$

$$\bar{N} = \frac{d}{dt}(\bar{r}\times m\dot{\bar{r}}),$$

$$= \dot{\bar{r}}\times m\dot{\bar{r}} + \bar{r}\times\bar{F},$$

$$\bar{N} = \frac{d\bar{L}}{dt} = \bar{r}\times\bar{F}.$$

2.5 Virtual Work :

If the particle system is equal to its zero value then the work performed is zero. So in the event that it is a particle into equilibrium there is no movement, so there is no question of migration. In this case we assume that the particle undergoes a small visual migration (system migration that does not cause real movement is called visual or imagine migration)

Virtual displacement δr_i is assumed to take place only in the co-ordinates and at fixed instant t, hence δ change in time t is zero.

$$\delta r_i = (dr_i)_{dt=0}.$$

Thus the work done by the system of forces in causing imaginary displacement is called virtual work. It is the amount of work that would have been done if the actual displacement had been caused. Hence the expression for the virtual work done by the forces is given by

$$\text{Virtual work } \delta W = \sum_i F_i\delta r_i. \qquad \ldots(1)$$

2.6 Actual Virtual Work:

If the forces are in balance then the result is zero. So the algebraic amount of visual activity is zero. On the other hand, if the algebraic amount of visible activity is zero then the forces are in balance.

Note that this policy applies to statics. However, the same law of flexibility was put forward by D'Alembert.

2.7 D'Alembert's System :

D'Alembert started with the equation of motion of a particle $F_i = \dot{p}_i$, where p_i is the linear momentum of the i^{th} particle. This can be written as $F_i - \dot{p}_i = 0$.

Hence $$\sum_i (F_i - \dot{p}_i) = 0,$$

implying a system of particles is in equilibrium. This equation states that the dynamical system appears to be in equilibrium under the action of applied forces F_i and an equal and opposite 'effective forces' $\dot{p}_i$. In this way dynamics reduces to static. Thus

$$\sum_i (F_i - \dot{p}_i) = 0 \Leftrightarrow \text{ the system is in equilibrium (the resultant is zero).}$$

Hence the virtual work done by the forces is zero. This implies that

$$\sum_i (F_i - \dot{p}_i)\delta r_i = 0.$$

This is known as the mathematical formula of the D'Alembert system. This means that "the particle system moves in such a way that the total work done by the applied forces and converting the active force is not zero".

Remark :

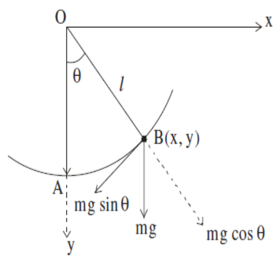
1. The D'Alembert Principle describes the movement of a system by considering its proportions.

2. All machine rules can be removed from this one system. D'Alembert's system has therefore been called the basic mechanical engineering. We will resolve some examples using this principle.

Example:

Use the D'Alembert principle to determine the balance of simple pendulum movements.

Solution : Consider a particle of mass m attached to one end of the string and other end is fastened to a fixed point 0. Let l be the length of the pendulum and θ the angular displacement of the pendulum shown in the fig.



According to the D'Alembert's principle we have

$$\sum_i (F_i - \dot{p}_i)\delta r_i = 0$$

where i is the number of particles in the system.

$$\Rightarrow \quad (F - m\ddot{r})\delta r = 0,$$

where r is the particle distance from where you start the curve. Resolving active energy is a particle according to the movement and direction of movement we have,

$$(-mg\sin\theta - m\ddot{r})\delta r = 0,$$

where the negative sign indicates the force is opposite to the direction of motion. Since $\delta r \neq 0$ we have

$$\ddot{r} = -g\sin\theta. \quad \ldots (1)$$

From the figure we have r = arcAB $\Rightarrow \quad r = l\theta \quad \Rightarrow \quad \ddot{r} = l\ddot{\theta}$.

Equation (1) becomes

$$\ddot{\theta} = -\frac{g}{l}\sin\theta. \quad \ldots (2)$$

For small angle, we have $\sin\theta = \theta \quad \Rightarrow \quad \ddot{\theta} = -\frac{g}{l}\theta$.

2.8 Actual Generalized Velocities :

From transformation equations we have

$$r_i = r_i(q_1, q_2, q_3, \ldots q_n, t), \quad \ldots (1)$$

Differentiating this with respect to t we get

$$\dot{r} = \sum_j \frac{\partial r_i}{\partial q_j}\dot{q}_j + \frac{\partial r_i}{\partial t} \quad \ldots (2)$$

where $\dot{q}_j, \quad j = 1,2,3,\ldots,n$ are called generalized velocities.

2.9 Actual Virtual displacement :

We find δ variation (change) in the transformation equation (1) to get

$$\delta r_i = \sum \frac{\partial r_i}{\partial q_j}\delta q_j$$

Note here that δt term is absent because virtual displacement is assumed to take place at fixed instant t, hence $\delta t = 0$.

2.10 Actual Generalized force :

If F_i are forces acting on a dynamical system with position vectors r_i then virtual work done by these forces is given by

$$\begin{aligned}\delta W &= \sum_i F_i \delta r_i, \\ &= \sum_i \sum_j F_i \frac{\partial r_i}{\partial q_j}\delta q_j, \\ &= \sum_j \left(\sum_i F_i \frac{\partial r_i}{\partial q_j} \right)\delta q_j, \\ &= \sum_j Q_j \delta q_j,\end{aligned}$$

where

$$Q_j = \sum_i F_i \frac{\partial r_i}{\partial q_j} \quad \ldots (1)$$

are called the components of generalized forces.

Note :

1. If forces are conservative then they are derived from potential V and are given by

$$F_i = -\nabla_i V = -\frac{\partial V}{\partial r_i}.$$

Consequently, the generalized forces are given by $Q_j = -\frac{\partial V}{\partial q_j}$.

2. If the forces are non-conservative, the scalar potential U may be function of position, velocity and time. i.e., $U = U(q_j, \dot{q}_j, t)$. This is called velocity dependent potential or generalized potential. Such a potential exits in the case of a motion of a particle of charge q moving in an electromagnetic field. We will see latter in example (8) that how the generalized potential can be determined in the case of a particle moving in an electromagnetic field. In this case generalized forces are given by

$$Q_j = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{q}_j}\right).$$

3. If however, the system is acted upon by conservative forces F_i and non-conservative forces $F_i^{(d)}$, in this case generalized forces Q_j are given by

$$Q_j = \sum \left(F_i + F_i^{(d)}\right)\frac{\partial r_i}{\partial q_j} \Rightarrow Q_j = -\frac{\partial V}{\partial q_j} + Q_j^{(d)},$$

where

$$Q_j^{(d)} = \sum_i F_i^{(d)} \frac{\partial r_i}{\partial q_j}$$

They are non-conservation forces that are not available in either V. Such a situation often arises when combat forces or dividing forces are present in the system. It is obtained by examining whether the distribution or competition strength is usually equal to the particle speed.

$$\Rightarrow \quad F_i^{(d)} = -\lambda_i \dot{r}_i,$$

where λ_i are constants. In such cases the generalized forces are obtained as

$$Q_j^{(d)} = \sum_i F_i^{(d)} \frac{\partial r_i}{\partial q_j},$$

$$= -\sum_i \lambda_i \dot{r}_i \frac{\partial r_i}{\partial q_j}$$

However, from transformation equation we obtain

$$\frac{\partial r_i}{\partial q_j} = \frac{\partial \dot{r}_i}{\partial \dot{q}_j}.$$

Thus we write

$$Q_j^{(d)} = \sum \frac{\partial}{\partial \dot{q}_j}\left(-\frac{1}{2}\lambda_i \dot{r}_i^2\right),$$

$$= -\frac{\partial R}{\partial \dot{q}_j},$$

where $$R = \frac{1}{2}\sum \lambda_i \dot{r}_i^2$$

is called the Rayleigh's dissipative function.

Now we will see Lagrange's Equation of Motion using above all fundamental definations,

2.11 Lagrange's Equations of motion:

The Newtonian method of particle definition incorporates a vector value. We now introduce another structure called the Lagrangian formulation of the definition of particle machinery or particle system based on standard, standard speeds with time t as a parameter. This design incorporates scalar masses as kinetic forces and potential forces and therefore proves to be as simple as that of Newtonian, because dealing with scales is easy than to deal with vectors.

Lagrange's Equations of motion from D'Alembert's System :

Theorem : we obtain the Lagrange's equations of motion from D'Alembert's principle.

Proof : Consider a system of n particles of masses m_i and position vectors r_i. We know the position vectors r_i are expressed as the functions of n generalized co-ordinates $q_1, q_2, q_3, ..., q_n$ and time t as

$$r_i = r_i\left(q_1, q_2, q_3, ... q_n, t\right), \qquad ...(1)$$

If F_i are the forces acting on the system, then by D'Alembert's principle we have

$$\sum_i \left(F_i - \dot{p}_i\right)\delta r_i = 0, \qquad ...(2)$$

where, $\dot{p}_i = m_i \ddot{r}_i$ is the linear momentum of the i^{th} particle of the system . From the transformation equations we obtain the expression for the virtual displacement

$$\delta r_i = \sum \frac{\partial r_i}{\partial q_j}\delta q_j,$$

where the term δt is absent because the virtual displacement is assumed to take place only in the co-ordinates and at the particular instant. Hence equation (2) becomes

$$\sum_i \sum_j F_i \frac{\partial r_i}{\partial q_j}\delta q_j = \sum_i \sum_j m_i \ddot{r}_i \frac{\partial r_i}{\partial q_j}\delta q_j .$$

$$\sum_j \left(\sum_i F_i \frac{\partial r_i}{\partial q_j}\right)\delta q_j = \sum_{i,j} m_i \ddot{r}_i \frac{\partial r_i}{\partial q_j}\delta q_j,$$

or

$$\sum_j Q_j \delta q_j = \sum_{i,j} m_i \ddot{r}_i \frac{\partial r_i}{\partial q_i}\delta q_j, \qquad ...(3)$$

where

$$Q_j = \sum F_i \frac{\partial r_i}{\partial q_j}. \qquad ...(4)$$

are called the components of generalized forces.

Consider

$$\frac{d}{dt}\left(\dot{r}_i \frac{\partial r_i}{\partial q_j}\right) = \ddot{r}_i \frac{\partial r_i}{\partial q_j} + \dot{r}_i \frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j}\right).$$

Substituting this in equation (3) we get

$$\sum_j Q_j \delta q_j = \sum_{i,j} m_i \left[\frac{d}{dt}\left(\dot{r}_i \frac{\partial r_i}{\partial q_j}\right) - \dot{r}_i \frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j}\right)\right]\delta q_j . \qquad ...(5)$$

Now from equation (1) we have

$$\dot{r}_i = \sum_k \frac{\partial r_i}{\partial q_k}\dot{q}_k + \frac{\partial r_i}{\partial t}. \qquad ...(6)$$

Differentiating this with respect to $\dot{q}_j$ we get

$$\frac{\partial \dot{r}_i}{\partial \dot{q}_j} = \frac{\partial r_i}{\partial q_j}. \qquad ...(7)$$

Further, differentiating equation (6) w. r. t. q_j, we get

$$\frac{\partial \dot{r}_i}{\partial q_j} = \sum_k \frac{\partial^2 r_i}{\partial q_k \partial q_j}\dot{q}_k + \frac{\partial^2 r_i}{\partial t \partial q_j}. \qquad ...(8)$$

Also we have

$$\frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j}\right) = \sum_k \frac{\partial^2 r_i}{\partial q_j \partial q_k}\dot{q}_k + \frac{\partial^2 r_i}{\partial q_j \partial t}. \qquad ...(9)$$

We notice from equations (8) and (9) that

$$\frac{\partial \dot{r}_i}{\partial q_j} = \frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j}\right)$$

In general we have

$$\frac{\partial}{\partial q_j}\left(\frac{d}{dt}\right) = \frac{d}{dt}\left(\frac{\partial}{\partial q_j}\right). \qquad ...(10)$$

On using equation (10) in equation (5) we get

$$\sum_j Q_j \delta q_j = \sum_{i,j}\left[\frac{d}{dt}\left(m_i v_i \frac{\partial v_i}{\partial \dot{q}_j}\right) - m_i v_i \frac{\partial v_i}{\partial q_j}\right]\delta q_j .$$

We write this as

We write this as

$$\sum_j Q_j \delta q_j = \sum_j \left[\frac{d}{dt}\left(\frac{\partial}{\partial \dot{q}_j}\left(\sum_i \frac{1}{2} m_i v_i^2 \right)\right) - \frac{\partial}{\partial q_j}\left(\sum_i \frac{1}{2} m_i v_i^2 \right)\right] \delta q_j ,$$

or

$$\sum_j Q_j \delta q_j = \sum_j \left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j}\right] \delta q_j ,$$

where $T = \frac{1}{2}\sum_i m_i v_i^2$

is the total kinetic energy of the system of particles.

$$\Rightarrow \quad \sum_j \left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} - Q_j \right] \delta q_j = 0. \qquad \ldots (11)$$

If the constraints on the motion of particles in the system are holonomic then δq_j are independent. In this case we infer from equation (11) that

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} - Q_j = 0,$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j, \quad j = 1,2,3,....,n. \qquad \ldots (12)$$

These are called the Lagrange's equations of motion. We see that, to derive the Lagrange's equations of motion the knowledge of forces acting on the system of particles will not be necessary.

Note : If the constraints are non-holonomic then the generalized co-ordinates are not all independent of each other. Hence we can't conclude equation (12) from equation (11).

Note: In deriving Lagrange's equations of motion the requirement of holonomic constraints does not appear until the last step.

Case (1) : Conservative system :

If the system is Conservative so that the particles move under the influence of forces that depend on the connection only, then the forces are found in the V-provided by,

$$F_i = -\nabla_i V = \frac{\partial V}{\partial r_i}.$$

In this case the components of generalized forces becomes

$$Q_j = \sum_i \frac{\partial V}{\partial r_i}\frac{\partial r_i}{\partial q_j} = \frac{\partial V}{\partial q_j}, \text{ and } \quad V \neq V(\dot{q}_j).$$

Hence equation (12) becomes

$$\frac{d}{dt}\left(\frac{\partial (T-V)}{\partial \dot{q}_j}\right) - \frac{\partial (T-V)}{\partial q_j} = 0.$$

Define a new function $L = T - V$,

where L which is a function of $q_1, q_2, q_3, ..., q_n, \dot{q}_1, \dot{q}_2, \dot{q}_3, ..., \dot{q}_n$ and time t is called a Lagrangian function of the system of particles. Then the equations of motion become

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0. \; j = 1, 2, ..., n \qquad \ldots (13)$$

These are called the Lagrange's equations for motion for conservative holonomic system.

Note : The Lagrangian L satisfying equation (13) is not unique. Refer Example (13) bellow.

Case(2):Non-conservative system :

In the case of non-conservative system the scalar potential U may be function of both position and velocity. i.e., $U = U(q_j, \dot{q}_j, t)$. Such a potential is called as velocity dependent potential. In this case the associated generalized forces are given by

$$Q_j = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{q}_j}\right).$$

Substituting this in the equation (12) we get

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0, \quad j = 1,2,...,n$$

which are the Lagrange's equations of motion for non-conservative forces.

Case(3):Partially conservative and partially non-conservative system :

Consider the system is acted upon by conservative forces F_i and non-conservative forces $F_i^{(d)}$. Such a situation often occurs when frictional forces or dissipative forces are present in the system. In this case the components of generalized force are given by

$$Q_j = \sum \left(F_i + F_i^{(d)} \right) \frac{\partial r_i}{\partial q_j} \quad \Rightarrow \quad Q_j = -\frac{\partial V}{\partial q_j} + Q_j^{(d)},$$

where the non-conservative forces which are not derivable from potential function V are represented in $Q_j^{(d)}$. Substituting this in equation (12) we readily obtain

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j^{(d)}, \quad j = 1,2,...,n \qquad \ldots (14)$$

where the Lagrangian L contains the potential of the conservative forces, and $Q_j^{(d)}$ represents the forces not arising from the potential V. However, it is found by experiment that, in general the dissipative or frictional forces are proportional to the velocity of the particles.

$$F_i^{(d)} = -\lambda_i \dot{r}_i, \; \lambda_i \text{ are constants.}$$

Hence we have

$$Q_j^{(d)} = \sum_i F_i^{(d)} \frac{\partial r_i}{\partial q_j},$$

$$= -\sum_i \lambda_i \dot{r}_i \frac{\partial r_i}{\partial q_j}.$$

But we know that

$$\frac{\partial \dot{r}_i}{\partial \dot{q}_j} = \frac{\partial r_i}{\partial q_j}.$$

Hence

$$Q_j^{(d)} = \sum \frac{\partial}{\partial \dot{q}_j}\left(-\frac{1}{2}\lambda_i \dot{r}_i^2\right) = -\frac{\partial R}{\partial \dot{q}_j},$$

where $$R = \frac{1}{2}\sum_i \lambda_i \dot{r}_i^2$$

is called Rayleigh's dissipation function. Hence the Lagrange's equations of motion become

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} + \frac{\partial R}{\partial \dot{q}_j} = 0. \qquad \ldots(15)$$

Example: Show that the Lagrange's equation

$$\frac{\partial \dot{T}}{\partial \dot{q}_j} - 2\frac{\partial T}{\partial q_j} = Q_j.$$

Solution: The kinetic energy T is in general a function of generalized co-ordinates, generalized velocities and time. Thus we have

$$T = T\left(q_j, \dot{q}_j, t\right). \qquad \ldots(1)$$

Differentiating this w. r. t. t we get

$$\frac{dT}{dt} = \dot{T} = \sum_k \frac{\partial T}{\partial q_k}\dot{q}_k + \sum_k \frac{\partial T}{\partial \dot{q}_k}\ddot{q}_k + \frac{\partial T}{\partial t}. \qquad \ldots(2)$$

Differentiating equation (2) partially w. r. t. $\dot{q}_j$ we get

$$\frac{\partial \dot{T}}{\partial \dot{q}_j} = \sum_k \left(\frac{\partial^2 T}{\partial \dot{q}_j \partial q_k}\dot{q}_k + \frac{\partial T}{\partial q_k}\delta_k^j\right) + \sum_k \frac{\partial^2 T}{\partial \dot{q}_j \partial \dot{q}_k}\ddot{q}_k + \frac{\partial^2 T}{\partial \dot{q}_j \partial t}$$

$$\frac{\partial \dot{T}}{\partial \dot{q}_j} = \sum_k \frac{\partial^2 T}{\partial \dot{q}_j \partial q_k}\dot{q}_k + \frac{\partial T}{\partial q_j} + \sum_k \frac{\partial^2 T}{\partial \dot{q}_j \partial \dot{q}_k}\ddot{q}_k + \frac{\partial^2 T}{\partial \dot{q}_j \partial t}. \qquad \ldots(3)$$

Also we find the expression

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) = \sum_k \frac{\partial^2 T}{\partial q_k \partial \dot{q}_j}\dot{q}_k + \sum_k \frac{\partial^2 T}{\partial \dot{q}_k \partial \dot{q}_j}\ddot{q}_k + \frac{\partial^2 T}{\partial t \partial \dot{q}_j}. \qquad \ldots(4)$$

From equations (3) and (4) we have

$$\frac{\partial \dot{T}}{\partial \dot{q}_j} - \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) = \frac{\partial T}{\partial q_j}. \qquad \ldots(5)$$

But it is given that

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) = \frac{\partial T}{\partial q_j} + Q_j.$$

Consequently equation (5) becomes

$$\frac{\partial \dot{T}}{\partial \dot{q}_j} - \left(\frac{\partial T}{\partial q_j} + Q_j\right) = \frac{\partial T}{\partial q_j}.$$

$$\Rightarrow \quad \frac{\partial \dot{T}}{\partial \dot{q}_j} - 2\frac{\partial T}{\partial q_j} = Q_j.$$

4. RESULT AND DISCUSSIONS

In obtaining Lagrange movement statistics the requirement for holonomic issues does not arise until the final step.

If the issues are not holonomic then the integrated systems are not all independent of each other.

If the system is Conservative so that the particles move under the influence of forces dependent on the contact only, then the forces are found in V.

In the case of a system that does not conserve the power of the scalar U can be the function of both position and velocity. Such forces are called velocity-dependent forces.

Lagrangian L contains the power of conservative forces, and represents the power that does not come from V.

5. CONCLUSION

With this paper we present applications for Formulation of Lagrange's Equation of Motion in various fields of engineering, such as Electronics, Mechanical, Physics ect. Apart from this, the Lagrange Equation of Motion is an effective tool to alleviate the most complex problems in Toatal Power and Various Problems. There is no denying that Lagrange's Equation of Motion is widely used in engineering. When energy is applied to the storage particle the full energy is retained. Impairment of absolute power is directly related to the presence of illegal power even if the conversion rate does not have time t. If the Lagrangian does not have time t obviously, the full power of the sequence system is saved. All sorts of conservative and supernatural forces are included in this movement.

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