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Verify Euler's Law of Buckling Failure Applying Various End Conditions for Mild Steel, Brass and Aluminium bar using ANSYS

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Abstract – *This paper presents verification of results for buckling failure of long column obtained by Euler's law using ANSYS 18.2 structural analysis*

In this paper we are considering buckling failure of mild steel ,brass and aluminium bar having varying end conditions for various cross sections and lengths.

Key Words: **Long column, Crippling stress, Crushing stress, Buckling failure, ANSYS 18.2**

1. INTRODUCTION

Columns or struts are members of structure which subjected to axial compressive load. When subjected to axial load the column can deform and can be buckled under variety of loading end conditions. There is necessity of investigation of buckling failure of various materials under different end conditions.

Columns which have lengths less than 8 times their respective diameters or slenderness ratio less than 32 are short columns. When short columns are subjected to compressive loads, their buckling is generally negligible and the buckling stress is very small as compared with direct compressive stress. Therefore, it is assumed that short columns are always subjected to direct compressive stress only .The columns having their lengths more than 30 times their respective diameters or their slenderness ratio more than 120 are Long columns. They are usually subjected to buckling stress only. Direct compressive stress is very small as compared with buckling stress and hence it is neglected.

Eigenvalue buckling analysis is used to find load required to just buckle the long column for various end conditions of axial loading. Here, in ANSYS structural analysis for buckling of column we are considering pre axial load as an initial loading condition.

1.1 Assumptions of Euler's Theory

- Column is initially perfectly straight and the load is applied axially.
- The cross section of column is uniform throughout the length.
- The column material is perfectly elastic, isotropic and homogeneous and obeys Hook's law.
- The length of column is considered to be very large as compared to lateral dimension.
- The direct stress considered to be very small as compared to bending stress.
- The column will fail only by buckling.
- The self-weight of column is considered to be negligible.

According to Euler's formula for crippling /buckling stress crippling stress is given by,

Crippling Stress $=\Pi^2 E\div (Le/k)^2$

Where, Le =Effective length, E =Young's Modulus of elasticity, K= Radius of Gyration.

Hence, crippling stress is directly proportional to Young's modulus of elasticity and square of radius of gyration of corresponding martial and inversely proportional to square of effective length of column. So that effect of each parameter on crippling stress is find out by fixing other parameters.

1.2 Finding Limiting Condition for Euler's Law Le/k Ratio

For a column with both ends fixed Le=l/2, hence crippling stress becomes as , $\Pi^2 E\div (Le/k)^2$.

where Le/k is slenderness ratio.

If the slenderness ratio is small, the crippling stress (or the stress at failure) will be high. But for the column material, the slenderness ratio is less than a certain limit, Euler's formula gives a value of crippling stress greater than the crushing stress. In the limiting case, we can find the value of slenderness ratio for which crippling stress is equal to crushing stress.

For example, for a mild steel column with both ends fixed Crushing stress=330 N/mm^2

Table -1: Properties of Mild Steel

Properties	Mild Steel
Young'Modulus (MPa)/(N/mm^2)	2.1*10^5
Poisson's Ratio	0.3
Density(kg/m^3)	7850
Tensile Yield Strength (MPa) Or (N/mm^2)	330

Equating the crippling stress to the crushing stress corresponding to the minimum value of slenderness ratio, we get
Crippling stress=Crushing stress

Let, p load applied axially on column of both ends fixed and have effective length Le, Young's Modulus of elasticity E, and moment of inertia I, radius of Gyration K, cross sectional area A

$P= \Pi^2 EI/ Le^2;$

Hence crippling stress σ,

$\sigma=\Pi^2 EI/ (Le^2 A);$

$\sigma= \Pi^2 Ek^2/ (Le^2);$

$\sigma=\Pi^2 E/ (Le /k)^2;$

$\Pi^2 E\div (Le/k)^2=330;$

Slenderness ratio (s) =Le/k;

Where Le is effective length, here for both ends fixed condition Le=l/2;

$(4\Pi^2*2.1*10^5)/ (l /k)^2=330;$

$(4\Pi^2*2.1*10^5)/ 330= (l /k)^2;$

$(4\Pi^2*2.1*10^5)/ 330=25097.1636;$

Let $I=Ak^2$;

$I= (\Pi/64) d^4;$
$A= (\Pi/4) d^2;$

Therefore, k=d/4;

$(l/k) = (25097.1635)^{1/2} = 158.42;$

s =Le/k =l/2k =158.42/2=79.21 say 80

Let k=d/4;

4 l/d=158.42

l/d=39.605, say 40.

Hence, in case of Mild Steel, Euler's formula for both ends fixed will be valid for l/d ratio greater than 40 or slenderness ratio greater than 80.

For Brass bar with both ends are fixed,

Table -2: Properties of Brass.

Properties	Brass
Young'sModulus (MPa)/(N/mm^2)	1.17*10^5
Poisson's Ratio	0.331
Density(kg/m^3)	8450
Tensile Yield Strength (MPa) Or (N/mm^2)	140

Let,
Crushing stress=140 N/mm^2;
Young's modulus, E=1.17*10^5 N/mm^2;

$P=\Pi^2EI/ Le^2;$

$\sigma= \Pi^2EI/(Le^2A);$

$\sigma= \Pi^2Ek^2/ (Le^2);$

$\sigma= \Pi^2E/ (Le /k)^2;$

$\Pi^2E\div (Le/k)^2=140;$

Slenderness ratio (s) =Le/k;

Where Le is effective length, here for both end fixed condition Le=l/2;

$(4*\Pi^2*1.17*10^5)/ (l /k)^2=140;$

$(4*\Pi^2*1.17*10^5)/ 140= (l /k)^2;$

$(4*\Pi^2*1.17*10^5)/ 140=32959.2342;$

Let $I=Ak^2$;

$I=(\Pi/64)d^4;$
$A= (\Pi/4) d^2;$

Therefore, k=d/4;

$(l/k)= (32959.2342)^{1/2};$

s =Le/k =l/2k =181.5467/2=90.77335 say 91
l/d=45.38, say 46

Hence, in case of Brass, Euler's formula for both ends fixed will be valid for (l/d) greater than 46 or slenderness ratio greater than 91.

For Aluminum bar with both ends are fixed,

Table -3: Properties of Aluminium

Properties	Aluminium
Young'sModulus (MPa)/(N/mm²)	$0.689*10^5$
Poisson's Ratio	0.33
Density(kg/m³)	2770
Tensile Yield Strength (MPa) Or (N/mm²)	280

Let,
Crushing stress=280 N/mm^2;
Young's modulus, $E=0.689*10^5$ N/mm^2;

$P= \Pi^2 EI/ Le^2$;

$\sigma= \Pi^2 EI/ (Le^2 A)$;

$\sigma= \Pi^2 Ek^2/ (Le^2)$;

$\sigma= \Pi^2 E/ (Le /k)^2$;

$\Pi^2 E\div (Le/k)^2=280$;

Slenderness ratio (s) =Le/k;

Where Le is effective length, here for both end fixed condition Le=l/2;

$(4*\Pi^2*0.689*10^5)/ (l /k)^2=280$;

$(4*\Pi^2*0.689*10^5)/ 280= (l /k)^2$;

$(4*\Pi^2*0.689*10^5)/ 280=9704.6634$;

Let $I=Ak^2$;

$I=(\Pi/64)d^4$;
$A= (\Pi/4) d^2$;

Therefore=d/4;

$(l/k)= (9704.6634)^{1/2}$;

s =Le/k =l/2k =98.512/2= 49.256 say 50
Let k=d/4;

l/d=24.628, say 25

Hence, in case of Aluminium, Euler's formula for both ends fixed will be valid for l/d ratio greater than 25or slenderness ratio greater than 50.

2. Results and Discussion For mild steel bar with both ends are fixed,

Let,
Crushing stress=330 N/mm2;

Young's modulus, E=2.1*105 N/mm2;

Table -4: Mild steel results by theoretical and ANSYS for both ends fixed condition.

Dia. (mm)	C/S Area (mm)	Length (mm)	Slender-ness Ratio	Thr. Load (kN)	Ansys (KN)	%ERROR (Thr. Vs ANSYS)
8	50.24	770	192.5	2.807	2.9135	3.7940
8	50.24	720	180	3.21	3.341	4.0809
8	50.24	600	150	4.623	4.8514	4.9405
8	50.24	500	125	6.657	7.0514	5.9245
10	78.5	770	154	6.853	7.0861	3.4014
10	78.5	720	144	7.838	8.1238	3.6463
10	78.5	600	120	11.287	11.784	4.4032
10	78.5	500	100	16.253	17.107	5.2544
12	113.04	900	150	10.402	10.666	2.5379
12	113.04	770	128.33	14.211	14.63	2.9484
12	113.04	720	120	16.253	16.767	3.1624
12	113.04	600	100	23.405	24.284	3.7556
16	200.96	1000	125	26.629	26.806	0.6646
16	200.96	900	112.5	32.876	33.115	0.7269
16	200.96	770	96.25	44.914	45.292	0.8416
16	200.96	720	90	51.369	51.825	0.8876

P=Π2EI/ Le 2;
Slenderness ratio (s) =Le/k;
Where Le is effective length, here for both end fixed condition Le=l/2;
$P=4\Pi 2EI/ l^2$;
$P=4* (3.14*3.14) (2.1*10^5) I/ (l^2)$;

$P=4* (3.14*3.14) (2.1*10^5) ((\Pi/64) * d^4) / (l^2)$;

$P=4.06338*10^5 d^4/l^2$

For Brass bar with both ends are fixed,

Let,
Crushing stress=140 N/mm2;
Young's modulus, E=1.17*105 N/mm2;
$P=\Pi^2 EI/ Le^2$;
Slenderness ratio (s) =Le/k;
Where Le is effective length, here for both end fixed condition Le=l/2;

Table -5: Brass results by theoretical and ANSYS for both ends fixed condition.

Dia. (mm)	C/S Area (mm)	Length (mm)	Slender-ness Ratio	Thr. Load (kN)	Ansys (KN)	%ERROR (Thr. Vs ANSYS)
8	50.24	770	192.5	1.561	1.5843	1.470681
8	50.24	720	180	1.785	1.8138	1.587827
8	50.24	600	150	2.571	2.6205	1.888952
8	50.24	500	125	3.702	3.7848	2.187698
10	78.5	770	154	3.811	3.882138	1.832449
10	78.5	720	144	4.359	4.44282	1.886645
10	78.5	600	120	6.277	6.417431	2.18828
10	78.5	500	100	9.04	9.280222	2.588539
12	113.04	900	150	5.785	5.877526	1.574227
12	113.04	770	128.33	7.904	8.053392	1.855014
12	113.04	720	120	9.04	9.218135	1.932444
12	113.04	600	100	13.017	13.32317	2.298025
16	200.96	1000	125	14.8	14.54922	1.7237
16	200.96	900	112.5	18.285	17.89923	2.15521
16	200.96	770	96.25	24.98	24.43655	2.22394
16	200.96	740	92.5	27.047	26.325	2.74264

$P=4\Pi^2EI/ l^2;$
$P=4*(3.14*3.14)(1.17*10^5)I/(l^2);$
$P=4*(3.14*3.14)(1.17*10^5)((\Pi/64)*d^4)/(l^2);$

$$P=2.26*10^5\ d^4/l^2$$

For Aluminum bar with both ends are fixed,

Let,
Crushing stress=280 N/mm²;
Young's modulus, $E=0.689*10^5$ N/mm²;

Table -6: Brass results by theoretical and ANSYS for both ends fixed condition.

Dia. (mm)	C/S Area (mm)	Length (mm)	Slender-ness Ratio	Thr. Load (kN)	Ansys (KN)	%ERROR (Thr. Vs ANSYS)
8	50.24	770	192.5	0.921	0.932553	1.2388
8	50.24	720	180	1.053	1.0677	1.3767
8	50.24	600	150	1.516	1.542742	1.7334
8	50.24	500	125	2.184	2.228402	1.9925
10	78.5	770	154	2.248	2.28762	1.7319
10	78.5	720	144	2.571	2.615876	1.7155
10	78.5	600	120	3.703	3.778675	2.0026
10	78.5	500	100	5.332	5.464484	2.4244
12	113.04	900	150	3.412	3.46373	1.4934
12	113.04	770	128.33	4.662	4.742054	1.6881
12	113.04	720	120	5.332	5.433266	1.8638
12	113.04	600	100	7.679	7.845297	2.1197
16	200.96	950	125	9.67	9.479135	2.0164
16	200.96	900	112.5	10.786	10.53967	2.3371
16	200.96	770	96.25	14.736	14.35431	2.6590
16	200.96	740	92.5	15.955	15.54246	2.6542

$P=\Pi^2EI/ Le^2;$
Slenderness ratio (s) =Le/k;
Where Le is effective length, here for both end fixed condition Le=l/2;
$P=4\Pi^2EI/ l^2;$
$P=4*(3.14*3.14)(0.689*10^5)I/(l^2);$

$P=4*(3.14*3.14)(0.689*10^5)((\Pi/64)*d^4)/(l^2);$

$$P=1.3317*10^5\ d^4/l^2$$

For one end fixed and one end free,

Let, p load applied axially on column of one end fixed and other end free have effective length Le, Young's Modulus of elasticity E, and moment of inertia I, radius of Gyration K, cross sectional area A for mild steel column one end fixed one end free

Crippling stress=Crushing stress
$P=\Pi^2 EI/ Le^2;$

Hence crippling stress σ,

$\sigma=\Pi^2EI/(Le^2 A);$

$\sigma=\Pi^2 Ek^2/(Le^2);$
$\sigma=\Pi^2 E/(Le/k)^2;$

$\Pi^2 E\div(Le/k)^2=330;$

Slenderness ratio (s) =Le/k;

Where Le is effective length, here for both ends fixed condition Le=2l;

$(\Pi^2*2.1*105)/(2l/k)^2=330;$

$(\Pi^2*2.1*105)/330=(2l/k)^2;$

$(2l/k)^2=6274.29090;$

Let $I=Ak^2;$
$I=(\Pi/64)\ d^4;$
$A=(\Pi/4)\ d^2;$

$(2l/k)=(6274.29090)^{1/2};$

$(2l/k)=(6274.29090)^{1/2};$

$2l/k=79.21;$
$Le/k=79.21;$
s =Le/k =79.21 say 80
Let k=d/4;
l/d=9.9013 say 10

Table -7: Mild steel results by theoretical and ANSYS for one end fixed and one end free.

Dia. (mm)	C/S Area (mm)	Length (mm)	Slender-ness Ratio	Thr. Load (kN)	Ansys (KN)	%ERROR (Thr. Vs ANSYS)
8	50.24	770	192.5	0.175438	0.18209375	3.794086
8	50.24	720	180	0.200625	0.2088125	4.080997
8	50.24	600	150	0.288938	0.3032125	4.940515
8	50.24	500	125	0.416063	0.4407125	5.924591
10	78.5	770	154	0.428313	0.44288125	3.40143
10	78.5	720	144	0.489875	0.5077375	3.646338
10	78.5	600	120	0.705438	0.7365	4.403296
10	78.5	500	100	1.015813	1.0691875	5.254415
12	113.04	900	150	0.650125	0.666625	2.537973
12	113.04	770	128.33	0.888188	0.914375	2.94842
12	113.04	720	120	1.015813	1.0479375	3.162493
12	113.04	600	100	1.462813	1.51775	3.755608
16	200.96	1000	125	1.664313	1.675375	0.664689
16	200.96	900	112.5	2.05475	2.0696875	0.726974
16	200.96	770	96.25	2.807125	2.83075	0.841608
16	200.96	720	90	3.210563	3.2390625	0.887695

similarly, for Brass column one end fixed one end free

$(2l/k) = (8239.80107)^{1/2}$;

$(2l/k) = (8239.80107)^{1/2}$;

2l/ k =90.77335;

s =Le/k =90.77335 say 91

Let k=d/4;

l/d=11.34667 say 12

similarly, for Aluminium column one end fixed one end free

Therefore=d/4;

$(2l/k) = (2426.1658)^{1/2}$;

$(2l/k) = (2426.1658)^{1/2}$;

2l/ k =49.2561;

s =Le/k =49.2561 say 50

Let k=d/4;

l/d=6.1570 say 7

Table -8: Brass results by theoretical and ANSYS for one end fixed and one end free.

Dia. (mm)	C/S Area (mm)	Length (mm)	Slender-ness Ratio	Thr. Load (kN)	Ansys (KN)	%ERROR (Thr. Vs ANSYS)
8	50.24	770	192.5	0.097563	0.099019	1.49263
8	50.24	720	180	0.111563	0.113363	1.61345
8	50.24	600	150	0.160688	0.163781	1.92532
8	50.24	500	125	0.231375	0.23655	2.23663
10	78.5	770	154	0.238188	0.242634	1.86665
10	78.5	720	144	0.272438	0.277676	1.92292
10	78.5	600	120	0.392313	0.401089	2.23724
10	78.5	500	100	0.565	0.580014	2.65732
12	113.04	900	150	0.361563	0.367345	1.59941
12	113.04	770	128.33	0.494	0.503337	1.89008
12	113.04	720	120	0.565	0.576133	-1.97052
12	113.04	600	100	0.813563	0.832698	-2.35208
16	200.96	1000	125	0.925	0.909326	1.694489
16	200.96	900	112.5	1.142813	1.118702	2.109741
16	200.96	770	96.25	1.56125	1.527284	2.175553
16	200.96	740	92.5	1.690438	1.645313	2.669427

Table -9: Aluminium results by theoretical and ANSYS for one end fixed and one end free.

Dia. (mm)	C/S Area (mm)	Length (mm)	Slender-ness Ratio	Thr. Load (kN)	Ansys (KN)	%ERROR (Thr. Vs ANSYS)
8	50.24	770	192.5	0.057563	0.058285	1.25443
8	50.24	720	180	0.065813	0.066731	1.39598
8	50.24	600	150	0.09475	0.096421	1.76401
8	50.24	500	125	0.1365	0.139275	2.03305
10	78.5	770	154	0.1405	0.142976	1.76246
10	78.5	720	144	0.160688	0.163492	1.74549
10	78.5	600	120	0.231438	0.236167	2.04362
10	78.5	500	100	0.33325	0.34153	2.4847
12	113.04	900	150	0.21325	0.216483	1.51613
12	113.04	770	128.33	0.291375	0.296378	1.71717
12	113.04	720	120	0.33325	0.339579	1.89922
12	113.04	600	100	0.479938	0.490331	2.16561
16	200.96	950	125	0.604375	0.592446	1.973788
16	200.96	900	112.5	0.674125	0.658729	2.283801
16	200.96	770	96.25	0.921	0.897145	2.590162
16	200.96	740	92.5	0.997188	0.971404	2.585633

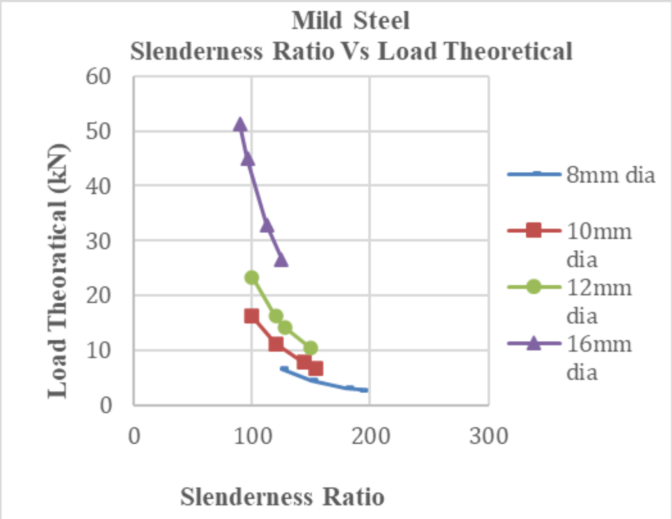

Chart -1: Mild Steel Slenderness Ratio Vs Load Theoretical for 8, 10, 12, 16mm dia. For both ends fixed.

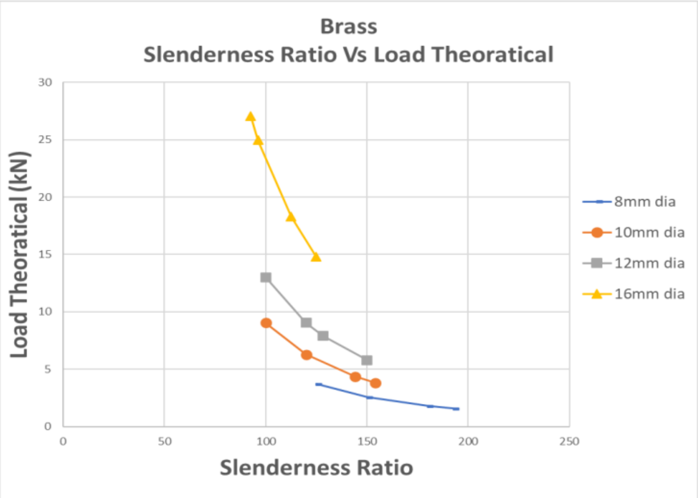

Chart -2: Brass Slenderness Ratio Vs Load Theoretical for 8, 10, 12, 16mm dia. For both ends fixed.

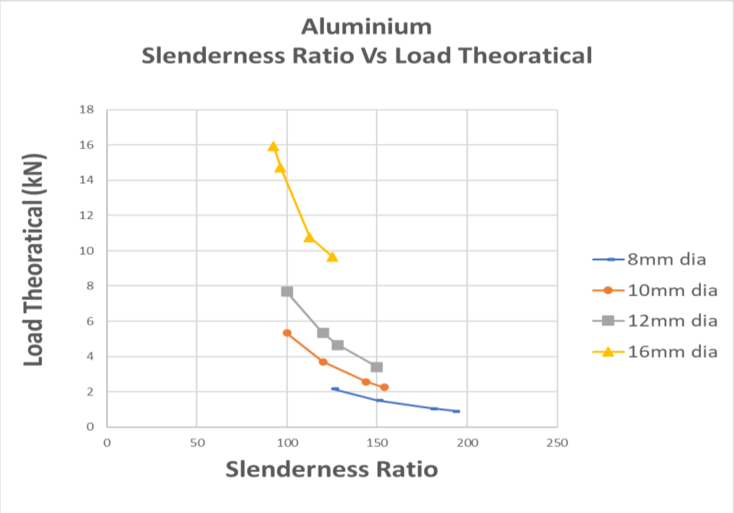

Chart -3: Aluminium Slenderness Ratio Vs Load Theoretical for 8, 10, 12, 16mm dia. For both ends fixed.

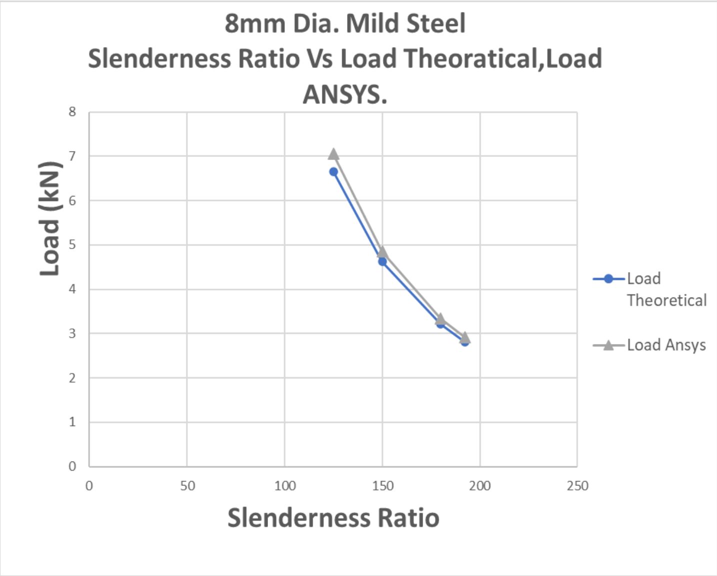

Chart -4: Mild Steel Slenderness Ratio Vs Load Theoretical, Load ANSYS For both ends fixed.

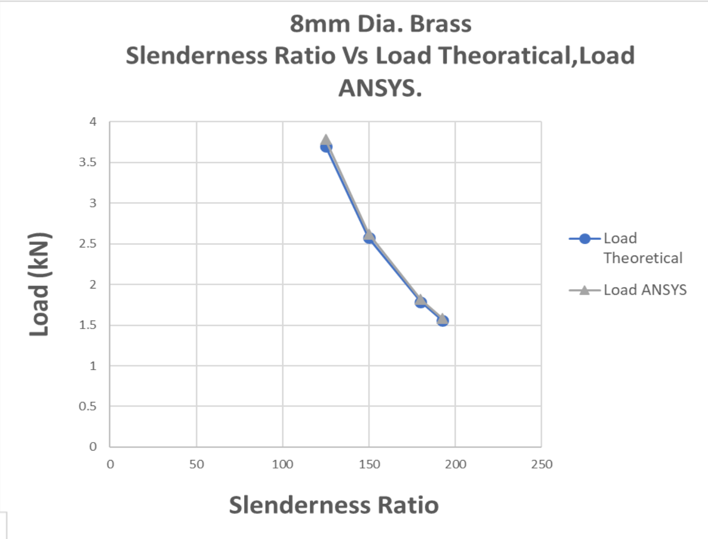

Chart -5: Brass Slenderness Ratio Vs Load Theoretical, Load ANSYS For both ends fixed.

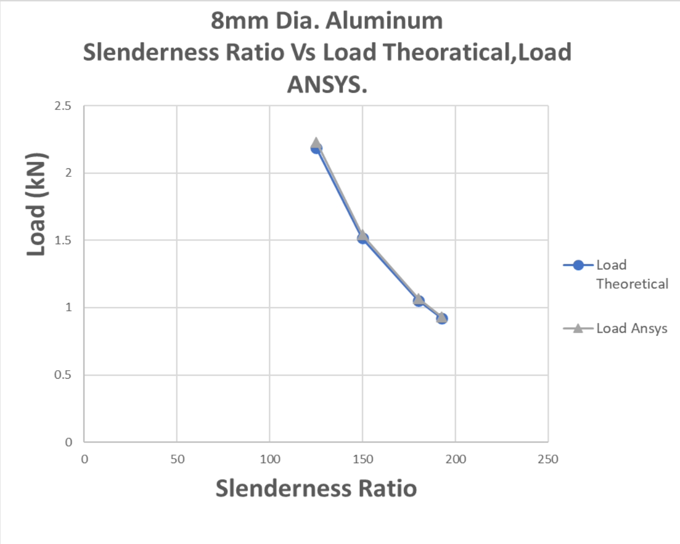

Chart -6: Aluminium Slenderness Ratio Vs Load Theoretical, Load ANSYS For both ends fixed.

3. Validation of Results obtained from Euler's formula with ANSYS 18.2

Results obtained by theoretical(Euler's formula) and practical(Experimental) are validated by ANSYS 18.2

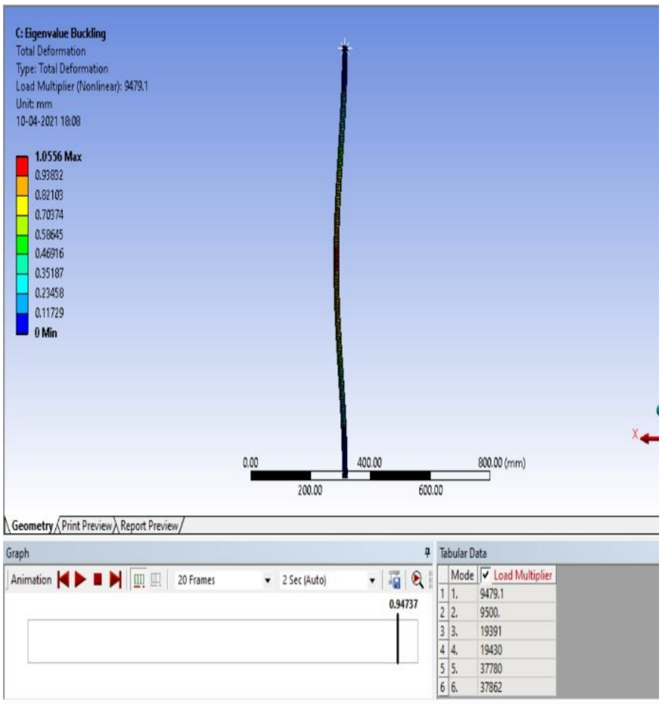

Fig -1: Structural Analysis of Long Column of Mild Steel Fixed at both ends on ANSYS 18.2.

Following steps are followed for analysis of columns

Step 1: Draw Geometry of component.

Step2: linking of geometry of component to static structural of→ Analysis system.

Step 3: Fill material properties in data library of Engineering Data.

Step 4: Static structural Analysis of column.

Step 5: Eigenvalue buckling analysis.

Step 6: Make outline of all required parameters.

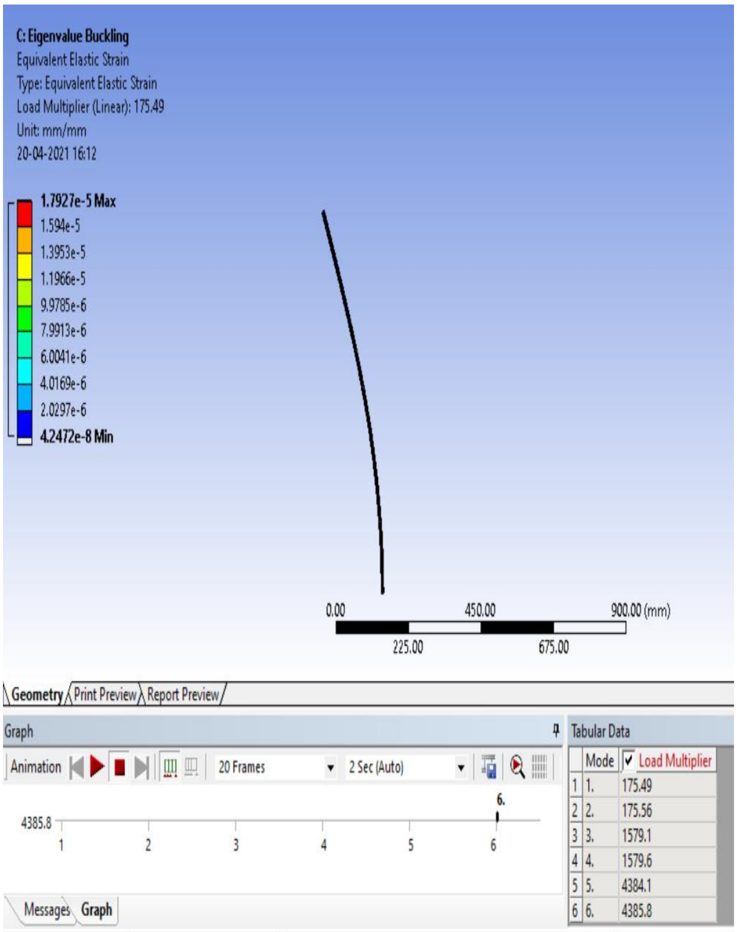

Fig -2: Structural Analysis of Long Column of Mild Steel one end Fixed and other end free on ANSYS 18.2

In case of mild steel critical buckling analysis Euler's formula gives the value of a crippling stress at which the buckling failure just occurs, when crippling load is equal to crushing load. Hence, in case of mild steel with both ends fixed condition Euler's formula is valid only for slenderness ratio greater than 80. In case of Brass , Euler's formula for both ends fixed will be valid for slenderness ratio greater than 91.

In case of Aluminium ,Euler's formula for both ends fixed will be valid for slenderness ratio greater than 50. Same will be valid for one end fixed and other end free conditions.

We have plotted the graph of slenderness ratio against load of the column we get the result as follows:

- As slenderness ratio decreases the load required to just buckle the long column increases.
- Similar to the mild steel, results are observed in case of Brass and Aluminum.
- For Particular slenderness ratio Steel column shows greater load bearing capacity of the brass and Aluminum columns.

In case of Brass and Aluminum for particular slenderness ratio brass shows greater buckling load bearing capacity than Aluminum

- By ANSYS static structural analysis method the values of load required to just buckle the column are 1 to 3% less than theoretical values obtained from Euler's formula method.
- In case of long columns as the diameter of column increases (or cross section area increases) load required to just buckle or crippling load increases.
- In other way, load bearing capacity is more for increasing cross section area for same length.

Conclusions

For particular slenderness ratio ,material having high values of crushing strength, tensile and compressive yield strength and Young's modulus of elasticity will show buckling at high load. As slenderness ratio decreases the load required to just buckle the long column increases.

In our case order of buckling strength as follows.

Mild steel >Brass>Aluminum.

By ANSYS static structural analysis method the values of load required to just buckle the column are 1 to 3% less than theoretical values obtained from Euler's formula method.
Results obtained from Euler's formula for buckling of log column validated by ANSYS software structural analysis.

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