

# Objectives of Probability Distribution and L-Moments Statics in Flood Frequency Analysis

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**Abstract** - Whenever, rainfall or river flow records are not available at or near the site of interest, it is difficult for hydrologists or engineers to derive reliable flood estimates directly. In this type studies, the regional flood frequency relationships or the flood formulae developed for the region are one of the alternative methods which may be adopted for estimation of design floods especially for small catchments. Most of the flood formulae developed for different regions of the country are empirical in nature and do not provide flood estimates for the desired return periods. Hence, there is a need for developing the regional flood formulae for estimation of floods of desired return periods for different regions of the country, using the recently developed, improved and efficient techniques of flood frequency analysis.

L-moments application with probability distribution for flood frequency analysis work as the robust method in calculating distribution parameters and useful in find out the most suitable distribution method. Application of Various frequency distributions based on the recently introduced goodness of fit criteria viz. L-moment ratio diagram and  $Z_i^{dist}$  statistic; useful for identifying the robust distribution for selected regional area. It is act as a robust method for estimation of floods of desired return periods for available catchments, the regional flood frequency relationship has been developed using the L-moment based on identified distribution method. Also, work as a whole for estimation of floods of different return periods for particular catchments of the study area, a regional flood formula has been developed by coupling the based-on L-moments, regional flood frequency relationship with the regional relationship between mean annual peak flood and the catchment area.

**Key Words:** Probability weighted moments, L-moments, L-moment ratio diagram, frequency distributions,  $Z_i^{dist}$  statistic as a goodness-of-fit measure

## 1. INTRODUCTION

L- Moments were found out by the Hosking, according to him the values are estimated as linear combination of Probability Weighted Moments. L-moments provides simple as well as proper estimation of distribution parameters and characteristics of the data which are used in hydrologic application. Some of the studies based on index flood

approach include Wallis and Wood (1985), Hosking et al. (1985), Hosking and Wallis (1986) etc. Based on some of the comparative flood frequency studies involving use of probability weighted moment (PWM) based on at-site. Farquharson et al. (1992) state that GEV distribution was selected for use in the Flood Studies Report (NERC, 1975) and has been found in other studies to be flexible and generally applicable. In 1995(Karim and Chowdhary) defined that both goodness-of-fit analysis and L-moment ratio diagram analysis indicated that the three-parameter GEV distribution is suitable for flood frequency analysis in Bangladesh while the two-parameter Gumbel distribution is not. Flood frequency analysis in India, using probability weighted moments (PWM) has been carried out (1992, NIH, Kumar *et al.*) for which GEV method found out to be the best. According to Karim and Chaudhary the three-parameter distribution i.e., GEV method is the best method for flood frequency analysis in case of Bangladesh flood using both the analysis L-moment ratio diagram and best-of-fit analysis.

### 1.1 Main Objectives

- i. L-moments, L-moments ratio are useful to find out the statistical properties of available maximum annual rainfall data for a particular regional area.
- ii. Work as the robust method in calculating distribution parameters and useful in find out the most suitable distribution method.
- iii. To estimates the statistical parameters such as weighted probability moments, L-moments, L-moments ratio for provided maximum discharge in a regional study.
- iv. To study the performance of standard methods of distributions namely generalized Pareto distribution, generalized extreme value distribution, general logistic distribution, Pearson type third, generalized normal distribution and wakeby distribution are used.
- v. To check the goodness of fit using  $Z_i^{dist}$ - statistic test applied for all distributions as used in a selected studied area.
- vi. More probability distributions can be used to calculate the quintile function value for different return periods for a study area.
- vii. Work as a part of hole in developing the flood frequency relationship in calculating floods for many more values of return periods for provided

catchment studied area based on the robust frequency distribution.

## 2. METHODOLOGY

The following aspects of methodology proved as useful tool for development of L-moment based flood frequency relationship for gauged catchments as well as development of regional flood formula for estimation of floods of various return periods for ungauged catchments are discussed as follows.

1. L-moments
2. Frequency distributions used
3. Goodness of fit measures

### 2.1. L-moments Methods:

L-moments work as superior method to those that have been used previously, and are now being adopted by many organizations all over the world (Hosking and Wallis, 1997). L-Moments were found out by the Hosking, according to him the values are estimated as linear combination of Probability Weighted Moments. L-moments provides simple and proper estimation of distribution parameters and characteristics of the data which are used in hydrologic application.

Furthermore, authors mention that L-moments offer significant advantages over ordinary product moments, especially for environmental data sets, because of the following:

I. Useful and work as estimators for location, scale and shape of the probability distribution from which the observations arise (Hosking, 1990).

II. Useful for estimation as L-coefficient of variation, L-skewness, and L-kurtosis can exhibit lower bias than conventional product moment ratios, mostly for skewed samples.

III. Ratio of L-moments in calculation of L-coefficient of variation and L-skewness does not depend on sample size as do the ordinary product moment ratio estimators of coefficient of variation and skewness.

IV. L-moments are the form of linear combinations of the observations and thus are less sensitive to the largest observations in provided sample than product moment estimators.

V. L-moments work as robust method, particularly good in identification for the distributional properties of highly skewed data, whereas ordinary product moment diagrams are almost useless for this task (Vogel and Fennessey, 1993).

## 3. FREQUENCY DISTRIBUTIONS

Commonly adopted frequency distributions which can be used for flood frequency analysis as:

### A. Extreme value type-I distribution:

EV1 is a two-parameter distribution known as Gumbel distribution and the inverse form of the distribution is expressed as:

$$X(F) = u - a \ln(-\ln f) \quad (1)$$

Where,  $u$  and  $a$  represents the location and scale parameters.

### B. General extreme value distribution:

It is generalized three parameter extreme value distribution.

$$X(F) = u + a \{1 - (-\ln f)^k\} / k; \quad k \neq 0 \quad (2)$$

Where,  $u$ ,  $a$  and  $k$  represent location, scale and shape parameters. Extreme value type-I (EV1) distribution is the special case of the GEV distribution, when  $k = 0$ .

### C. Logistic distribution:

Expression for Logistic distribution is expressed as:

$$X(F) = u - a \ln\{(1-f) / f\} \quad (3)$$

Where,  $u$  and  $a$  represents location and scale parameters.

### D. Generalized logistic distribution:

GLO is expressed as:

$$X(F) = u + [a [1 - \{(1-f) / f\}^k] / k; \quad k \neq 0 \quad (4)$$

$$X(F) = u - a \ln\{(1-f) / f\}; \quad k = 0 \quad (5)$$

Where,  $u$ ,  $a$  and  $k$  represent location, scale and shape parameters.

LD (Logistic distribution) is the special case of the Generalized Logistic distribution, when  $k = 0$ .

### E. Generalized Pareto distribution (GPA)

GPA is expressed as:

$$X(F) = u + a \{1 - (1-f)^k\} / k; \quad k \neq 0 \quad (6)$$

$$X(F) = u - a \ln(1-f) \quad k = 0 \quad (7)$$

where,  $u$ ,  $a$  and  $k$  indicate location, scale and shape parameters respectively.

Exponential distribution is special case of Generalized Pareto distribution, when  $k = 0$ .

### F. Generalized normal distribution (GNO)

Generalized normal distribution (GNO) is given below.

$$F(x) = \varphi[-k^{-1} \log\{1 - k(x - \xi)/\alpha\}] \quad (8)$$

where, E, a and k represents its location, scale and shape parameters.

When  $k = 0$ , it becomes normal distribution with parameters  $\xi$  and  $\alpha$ . This distribution has no explicit analytical inverse form.

### G. Pearson Type-III distribution (PT-III)

Hosking and Wallis (1997) mention that the Pearson type-III distribution combines Gamma distributions.

### H. kappa distribution (KAP)

kappa distribution is a four-parameter distribution that includes as special cases the GLO (Generalized logistic), GEV (Generalized extreme value) and Generalized Pareto distribution (GPA).

Kappa distribution useful as a general distribution with which to compare the fit of two and three parameter distributions and for use in simulating artificial data in order to assess the accuracy of statistical methods (Hosking and Wallis, 1997).

### I. Wakeby distribution (WAK)

Wakeby (WAK) distribution is expressed as:

$$x(F) = \xi + \frac{\alpha}{\beta} \{1 - (1 - F)^\beta\} - \frac{\gamma}{\delta} \{1 - (1 - F) - \delta\} \quad (12)$$

where,  $\xi$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  represents the parameters of the Wakeby distribution.

## 4. GOODNESS OF FIT MEASURES

The goodness of fit measures assesses the relative performance of various fitted distributions and help in identifying the robust viz. most appropriate distribution for the region. L-moment ratio diagram based on the approximations given by Hosking (1991) and the goodness of fit or behavior analysis measure for a frequency distribution given by statistic  $Z^{dist}$ , also used to identify the suitable frequency distribution.

### L-moment ratio diagram

The L-moment statistics of a sample reflect every information about the data and provide a satisfactory approximation to the distribution of sample values. The L-moment ratio diagram can therefore be used to identify the underlying frequency distribution. The average L-moment statistics of the region is plotted on the L-moment ratio diagram and the distribution nearest to the plotted point is identified as the underlying frequency distribution. The best advantage of L-moment ratio diagram is that one can compare fit of several

distributions using a single graphical instrument (Vogel and Fennessey, 1993).

### $Z_i^{dist}$ statistic as a goodness-of-fit measure

The main objective is to identify a distribution which fits the observed data acceptably closely it is described how much values of L-Skewness and L-Kurtosis fitted distribution match the regional average L-Skewness and L-Kurtosis of the observed data.

According to Hosking (1993), distribution is considered to give good fit if  $|Z^{dist}|$  is sufficiently close to zero, a reasonable criterion being  $|Z^{dist}| \leq 1.64$ .

## 5. RESULT AND DISCUSSION

Goodness of fit has been tested using the L-moment ratio diagram as well as  $Z^{dist}$  statistic for identifying the robust distribution. The aim of goodness-of-fit measure or the behavior analysis is to identify a distribution that fits the observed data acceptably closely. Used for describing Regional Flood Frequency Relationship for provided Catchment area. For estimation of T-year return period flood at a site, the estimate for mean annual peak flood is required. For ungauged catchments at-site mean cannot be computed in absence of the observed flow data. In such type of conditions, a relationship between the mean annual peak flood of gauged catchments in the region and their pertinent physiographic and climatic characteristics is needed for estimation of the mean annual peak flood.

## CONCLUSION

Accordingly, on the above objectives it is concluded that:

(i). Applying different types of distributions methods as Extreme value (EV1), General extreme value (GEV), Logistic (LOS), Generalized logistic (GLO), Generalized normal (GNO), Exponential (EXP), Generalized Pareto (GP), Kappa (KAP) and five parameter Wakeby (WAK) and more distributions methods could be used in for a study area. Studied L-moments approach useful for calculating parameters of these distributions.

(ii) Based on goodness of fit measures (the L-moment ratio diagram as well as  $Z^{dist}$  -statistic criteria) it is work as to identified the robust distribution for the provided study area.

(iii). Provided reliable estimation of the magnitude and frequency of occurrence of flood which are essential to the proper design of hydraulic structures across a river as well as to identify the flood risk area and in the design of hydraulic structures like drains, culverts and irrigation ditches.

(iv). For proper planning and design of hydraulic structures like dams, spillways, culverts, etc, reliable flood frequency estimation for large return periods at the site of interest is necessary. When the need to estimate how often a specific flood event will be for a particular probability of exceedance or recurrence interval arises, it could be realized through flood frequency analysis procedures.

(v). Useful for derived regional flood frequency relationship based on best fit distribution using L-moments approach for a study area in a particular zone.

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