ANALYSIS OF WAVELET, RIDGELET AND CURVELET TRANSFORMS ON IMAGE DENOISING

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Abstract - *Image processing refers to digital image processing to extract more information than is shown in the original image. It is an integral part of the applications used in publishing, satellite imagery analysis, medical fields, earthquakes, and many different fields. Images are often problematic with high-level components of noises. Image denoising is the technique of removing various types of noises or distortions from the corrupted image while retaining the edges and other features as detailed as possible. To achieve this goal, it makes use of a mathematical function known as the wavelet transform. Wavelet transform has unique advantages when it comes to working with a single point, but has some disadvantages when it comes to working with the borders and the line features of an image. There was also a Ridgelet transform that could only represent images of line singularities in two-dimensional space and not the curve singularity. For better representations of images with curve singularities in high dimensions, a Curvelet transform was introduced. An image's edges can be represented in a very useful way with the Curvelet transform since it is anisotropic with strong directional characteristics. The objective of this research is to analyze the wavelet, ridgelet, and curvelet transform for removing multiple types of noise from an image and determining the most appropriate transform among them. The wavelet, ridgelet, and curvelet transform for image denoising are implemented using Matlab. We analyze the accuracy, scalability, and graphical representation of image denoising based on wavelets, Ridgelets, and Curvelets.*

Key Words: Wavelet transform, Ridgelet transform, Curvelet Transform, Denoising, MATLAB

1. INTRODUCTION

An image is a multidimensional array of numbers between 0 and 255, known as pixels. Each number is identified by a combination of horizontal and vertical coordinates. Three different types of Images –

- Binary image - Image where pixels have either a value of 0 (dark) or 255 (bright).
- Grayscale image - The grayscale image has a pixel value ranging from 0 to 255.
- Color image - The color image is composed of 3 channels red (R), green (G), and blue (B). Each pixel value is between 0 and 255.

Color Image Grayscale Image Binary Image

Fig -1: Different types of Images

Image Processing is a type of signal processing in which input is an image and output may be either an image or the characteristics and features associated with it. It enables some operations to be performed on an image, to enhance it, or to extract some useful information from it. Image Processing techniques are required in the field of research and technology. Image processing is a very important and base technology for many real-time applications like image search engines, image clustering, image segmentation, Entropy detection, etc. Image clarity is an important property of image and the maximum percentage is expected for image processing applications. The accuracy of image processing results is determined by this factor. The increased level of clarity will increase the result accuracy dramatically and also makes the results reliable. All-natural phenomena and transmission errors are degrading the image quality thereby noise is introduced in the image.

Image blurriness is a common problem in the area of image processing, which is also called a noisy image noise, often referred to as "noise in pictures," is a variation in brightness or color information incorporated into images, obscuring the desired information. During transmission, acquisition, coding, and processing steps, noise is introduced into the image.

Noise models involved are as follows:

- **Additive Noise -** The process of adding noise to your original image results in a noisy image that is corrupted.

C(x, y) = O(x, y) + N(x, y)

- **Multiplicative Noise -** In this case, the image noise is multiplied by the original image, resulting in a corrupted noisy image.

C(x, y) = O(x, y) * N(x, y)

C(x, y) = Corrupted Noisy Image; O(x, y) = Original Image;

N(x, y) = Image Noise;

There is a need for image denoising to reduce the noise level present in the image to produce the denoised image that is nearer to the original image. Image denoising is a quality enhancement method in image processing, where the noise is removed from the noisy image and recovers the original image by retaining its quality, which gets corrupted during its acquisition or transmission. There are many fields of image processing that depend on denoising to recover anatomical details that may have been masked in the data due to different types of noise. Edge preservation needs to be balanced with denoising since edges are the most important aspect of biomedical images and other field images. As a result, the high-frequency component of the noisy image is filtered while the low-frequency information is left intact. In this paper, Image denoising is done based on Wavelet, Ridgelet, and Curvelet transforms to obtain the best results by removing the different types of noise from noisy images.

2. PROPOSED METHODOLOGY

In the proposed paper, the wavelet, ridgelet, and curvelet transforms are compared to determine the best one for removing noise from images to retain the features and quality of the original image.

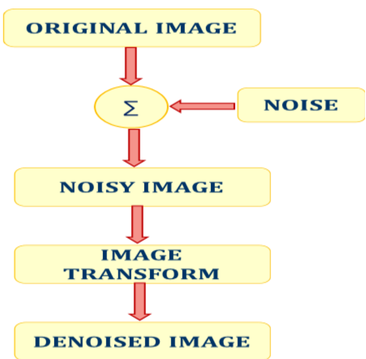


Fig -2: Flow Chart of Image Denoising using Transform

2.1 Different Types of Noises

A) Gaussian Noise

In mathematical terms, it is statistical noise with a probability density function (PDF) equal to the Normal Distribution.

Sources of Gaussian Noise –

- During Image Acquisition

  Example - Due to poor illumination and high temperatures, there is sensor noise.

- During Transmission.

  Example - Electronic circuit noise.

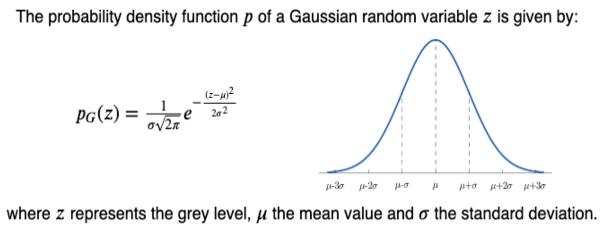


Fig -3: Gaussian function

Fig -4: Gaussian noise

B) Salt and Pepper Noise

- It is also called Data drop-out.
- It is a fixed valued Impulse Noise. The results for an 8-bit image are 255 (bright) for salt noise and 0 (dark) for pepper noise.
- Sources of Salt and Pepper Noise –
  - Sharp and sudden disturbances in the image signal.
  - Malfunctioning of camera's sensor cell.

The probability density function 'S' is given by:

$$S(u) = \begin{cases} s_p \; for \; u = 0 \; (pepper) \\ s_s \; for \; u = 2^n - 1 (salt) \\ 1 - (s_p + s_s) \; for \; u = k (0 < k < 2^n - 1) \end{cases}$$

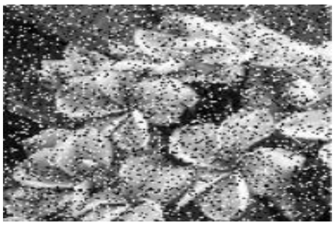


Fig -5: Salt and Pepper noise

C) Speckle Noise

An image can be corrupted by speckle noise, which is a rough noise that naturally exists. Random pixel values can be multiplied with different pixels in an image to model speckle noise.

Sources of Speckle Noise –

- Effect of environmental conditions during imaging sensor in the process of image transmission.

Speckle noise follows gamma distribution as follows:

$$F(u) = \frac{u^{\alpha-1}}{(\alpha-1)!a^{\alpha}} e^{-\frac{u}{a}}$$

Where:
u: Grey level
a^{α}: Variance

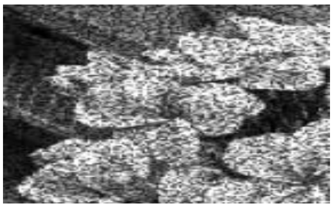


Fig -6: Speckle noise

D) Poisson Noise

It can also be referred to as Quantum (Photon) Noise or Shot Noise. The noise of this type has a Poisson distribution probability density function.

Sources of Salt and Pepper Noise –

- Random fluctuation of photons.

Fig -7: Poisson noise

2.2 Image Denoising

The technique of image denoising is the removal of noise from a noisy image so that a clear image can be obtained. It is hard to distinguish noise, edges, and textures in the process of denoising, and the denoised images are likely to lose some details. Today, a significant challenge in obtaining high-quality images through the noise removal process to recover meaningful information from noisy images. An image is preserved in its details, while the different types of noise are removed as much as possible. This process involves reconstructing an image from a noisy one.

A key objective of image denoising is to remove noise and retain useful information about the image. Generally, a noisy image is applied with the image transform to obtain a denoised image by removing the different types of noises in an image.

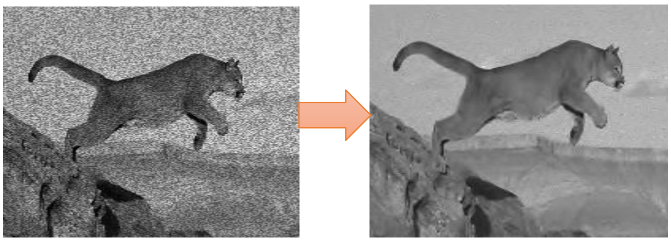


Image with noise Denoised Image

Fig -8: Image Denoising

2.3 Image Transforms

To convert an image from one domain to another, an image transform can be applied. Processing an image in a nonspatial domain such as frequency makes it possible to identify features that may otherwise be difficult to identify in the spatial domain.

Image transforms include:

- Wavelet Transform
- Ridgelet Transform
- Curvelet Transform

A) Wavelet Transform

One of the most famous image analysis tools is the wavelet transform. A feature of this is the ability to localize "point singularities" in the image as a result of its advantageous properties. An image is represented using the Discrete Wavelet Transform as a series of approximate coefficients and detailed coefficients at different resolutions. When an image is filtered from a higher resolution to a lower resolution, high pass filtration gives approximation coefficients, while low pass filtration gives detailed coefficients. Wavelet representations contain a set of detail coefficients at all resolutions and an approximation coefficient at the lowest resolution.

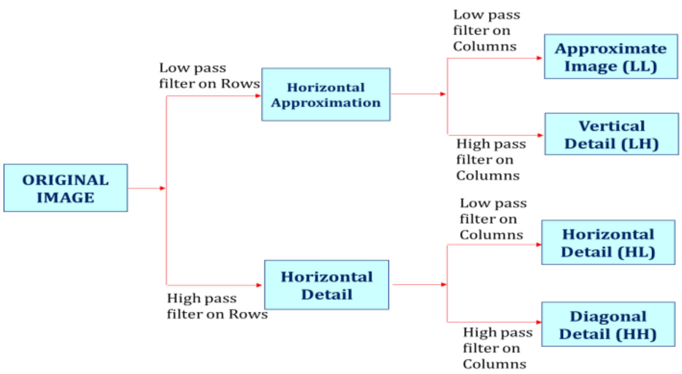


Fig -9: Block diagram for the wavelet transform

Wavelets oscillate like waves but are quickly attenuated. An admissibility condition for a wavelet is that it is a function ψ of the square of L(R):

$$C_\psi = \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(\xi)|^2}{|\xi|} d\xi < +\infty$$

Wavelet coefficients are obtained by decomposing a noisy image with the discrete wavelet transform. By using wavelet thresholding, the wavelet coefficients are denoised. To denoise the images, the modified coefficients are subjected to the inverse discrete wavelet transform.

The main theme of wavelet transforms:

- The time domain does not provide frequency information.
- The Fourier transform does not include time information.

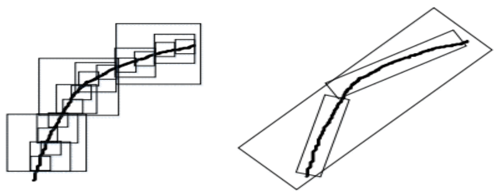


Fig -10: The difference between wavelet and curvelet in sparse approximation

B) Ridgelet Transform

Ridgelet transform is a dense representation of functions with discontinuities along lines that arises from a need to find a sparse representation. Ridgelet transform is used because the wavelet transforms only work for point discontinuities in 1D while failing when applied to edges in 2D. Edges in 2-D are not better represented by the Ridgelet transform than by the wavelet transform.

Ridgelet transform is a powerful tool for extracting lines from edge-dominated images. Radon transform facilitates the implementation of Ridgelet, which achieves a very compact representation of linear singularities. In this way, they are particularly important from a visual point of view, since they can significantly contribute to detecting and representing.

In the Radon domain, it can be conceptualized as a wavelet analysis applied to the Ridgelet domain. The Radon transform is a shape detection tool. The Ridgelet transform belongs to a family of discrete transforms that employ basis functions. In essence, the Ridgelet transform can be used as a tool to detect the ridges or shapes of objects in an image, as well as to make the detection of ridges or shapes easier. Recently, it has become an alternative method to overcome problems with wavelet transforms. A 2D wavelet transform produces large wavelet coefficients at every scale of decomposition. Since the Ridgelet transform has so many large coefficients, de-noising of noisy images is difficult, so de-noising of noisy images with an efficient legacy wavelet mechanism becomes problematic. Ridgelet transform was successfully used to process digital images with different orientations and locations. The ridgelet transform calculates integrals based on the data's orientation and location before processing it. A ridgelet is constant along the lines x1 cos β + x2 sin β = constant. In the direction orthogonal to these ridges it is a wavelet. De-noising images using ridgelets has recently been successful.

The 2-D continuous Ridgelet transform in the square of R can be defined by the following basis function:

$$\psi_{a,b,\theta} = a^{-1/2} \psi\left(\frac{(x_1 \cos\theta + x_2 \sin\theta - b)}{a}\right)$$

Where,

a - Scale of the ridgelets; θ - Orientation; b – Location;

For each a > 0, b ∈ R and θ ∈ (0, 2π)

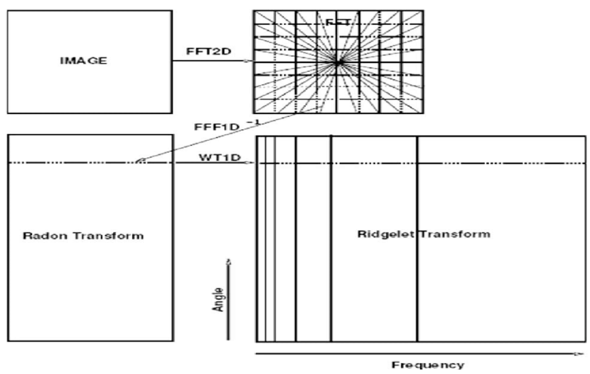


Fig -11: Flow chart of Discrete Ridgelet Transform

To calculate the continuous ridgelet transform, the Radon transform Rf (θ, t) is first computed by computing the inverse Fourier transform applied to the two-dimensional Fourier transform restricted to radial lines through the origin, and the one-dimensional wavelet transform is applied to these slices. The Radon transform is obtained by applying the inverse Fourier transform to the two-dimensional Fourier transform restricted to radial lines through the origin.

Thus, the Ridgelet coefficients are represented by:

$$R_f(a,b,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \psi_{a,b,\theta}(x_1,x_2) f(x_1,x_2)\,dx_1 dx_2$$

The reconstruction formula is given by:

$$f(x_1,x_2) = \int_0^{2\pi}\int_{-\infty}^{\infty}\int_0^{\infty} R_f(a,b,\theta)\psi_{a,b,\theta}(x_1,x_2)\frac{da}{a^3}db\frac{d\theta}{4\pi}$$

The Radon transform of an object is given by:

$$Rf(\theta,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1,x_2)\delta(x_1\cos\theta + x_2\sin\theta - t)\,dx_1 dx_2$$

The Ridgelet transform can be represented as follows:

$$R_f(a,b,\theta) = \int_{-\infty}^{\infty} Rf(\theta,t)a^{-1/2}\psi\left(\frac{(t-b)}{a}\right)dt$$

Ridgelet transforms are applied to slices of Radon transforms where angular value 'θ' remains constant while the angular value 't' varies. The approximate Radon transforms can be computed from discrete fast Fourier transforms as follows:

- Perform the two-dimensional Fast Fourier Transform (FFT) of the 'f' function.
- Convert the Fourier transform from cartesian to polar form using an interpolation scheme. That is, put the sampled values f on the polar lattice instead of the sampled values f on a square lattice.
- Compute the one-dimensional Inverse Fast Fourier Transform (IFFT) for each line that is for each angular parameter of each line.

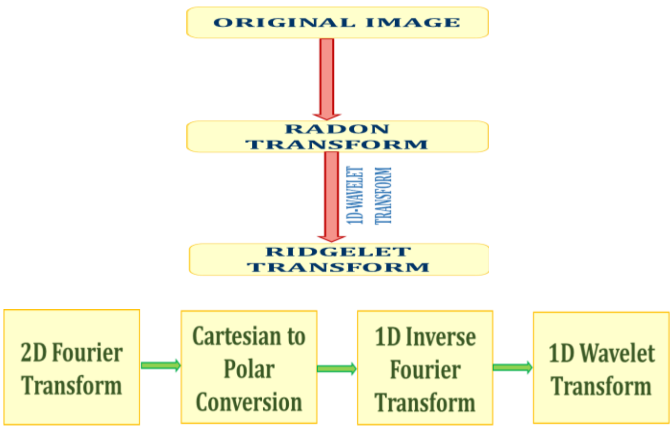


Fig -12: Flow charts of Ridgelet Transform

C) Curvelet Transform

It is one of the main disadvantages of wavelet transformations in image processing that the two-dimensional wavelet transform gives an enormous number of coefficients corresponding to the image edges. Moreover, the reconstruction of these edges is difficult, and many coefficients are required. Recent approaches like Ridgelets and Curvelets exploit this fact by exploiting the fact that wavelets are good only for point singularities and are not efficient to handle linear or curvilinear singularities in an image. Curvelet transforms are characterized by their sparsity and the ability to render edges or singularities.

Following are the steps of curvelet analysis:

- In the subband decomposition process, a bank of filters is applied to break down the image into dyadic scales.
- The subbands are divided into small squares through smooth partitioning.
- The discrete ridgelet transform is used to analyze each square.

Compared to traditional wavelets, curvelet transforms are much more efficient at representing edges and curve singularities. Wavelets have the disadvantage of poor directionality. DWTs are powerful tools in image processing, but they have three serious drawbacks: shift sensitivity, poor directionality, and no phase information. The Curvelet transform, which has a high degree of directional specificity, is proposed as a method that overcomes these disadvantages. It is difficult to design complex wavelets with ideal reconstruction properties and good filter characteristics, though the complex wavelet transform is one way to improve directional selectivity.

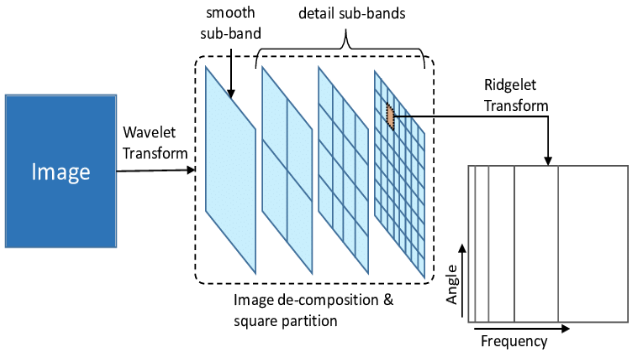


Fig -13: Curvelet Transform

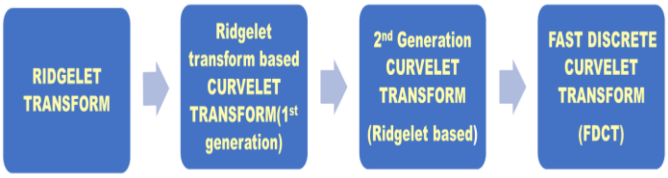


Fig -14: Stages of conversion from Ridgelet Transform to Curvelet Transform

An efficient method for solving partial differential equations (PDEs), image processing, fluid mechanics, and Partial differential equations(PGEs) is the Second Generation Curvelet Transform. There are many problems associated with it, such as high redundancy and poor sparse approximation of curve features beyond C^2 singularities.

To implement the newly implemented curvelet transform, also known as FDCT, there are two ways:

- Unequally Spaced Fast Fourier Transform(USFFT)
- Wrapping function.

Wrapping-based FDCT is faster than USFFT, therefore wrapping-based FDCT is widely used.

Generalized Curvelet Transform can be described as follows:

$$C(j,\theta,k_1,k_2) = \sum_{\substack{0\le x<M \\ 0\le y<N}} f[x,y]\cdot \varphi_{j,\theta,k_1,k_2}[x,y]$$

Where, j – Scale, θ – Orienation, k_1, k_2 – Spatial location of Curvelets
$\varphi[x,y]$ – Curvelet function
$f[x,y]$ – Input Image having dimension $M \times N$.

Because curvelet transforms are typically applied in the frequency domain, the above equation can be expressed as:

Curvelet Transform = IFFT{FFT(Curvelet) x FFT(Image)}

Curvelets in the spatial domain are arranged in various orientations (θ) and Scale (j) (Coarser to finer) in such a way that the entire FFT plane is covered to avoid any signal loss. In the figure, 5-level curvelet digital tiling of an image is shown along with FFT of a curvelet (shaded wedge) at scale 4 and orientation 4. Approximation coefficients are shown in the center square (at scale 1), while Detailed Curvelet coefficients are shown in the other wedges at scales j=2,3,4,5, and so on. At Scale 2, there are 16 possible curvelet orientations. On scale 3, 32, and scale 4, there are 32 orientations and so on.

In the comparison of wavelet, ridgelet and curvelet transform for removing the different types of noises from an image, the best transform among the three transforms is obtained.

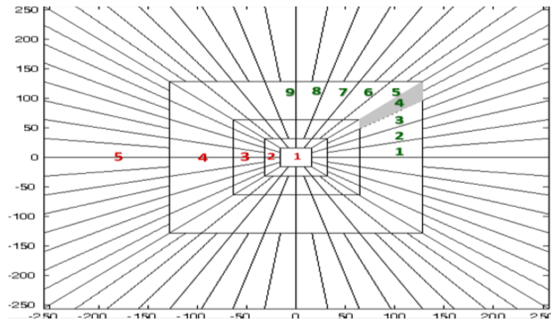


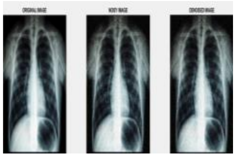
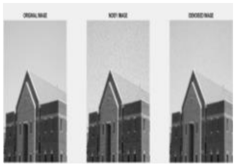
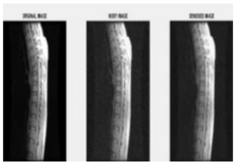
Fig -15: 5-level curvelet digital tiling of an image is shown along with FFT of a curvelet (shaded wedge) at scale 4 and orientation 4

3. EXPERIMENTAL RESULTS AND DISCUSSIONS

The different types of noise from images are removed using three transforms in this paper. MATLAB software was used to process the image and remove noises of various types. The Signal to Noise Ratio (SNR) values of the original input images with their noisy image and the denoised image is calculated to determine the best transform among wavelet, ridgelet, and curvelet transforms. Denoising the images has been achieved by using wavelet, ridgelet, and curvelet transforms. It shows that better SNR values determine the best transform among the three. The improvement in performance parameters values obtains the best transform that can effectively remove the different types of noises from the images. To compare the performance of each of these transformations on images, we take different types of images and add different types of noises to an image and note the SNR values to determine the best transform among them.

The results for removing different types of noises from the images using wavelet transform along with its SNR values are shown in the table.

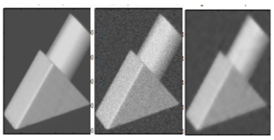
Table -1: Wavelet transform results

TYPE OF IMAGE	**TYPE OF NOISE**	**SNR** ORIGINAL VS NOISY IMAGE	**SNR** ORIGINAL VS DENOISED IMAGE
X-Ray 	GAUSSIAN	16.7167	24.5096
	POISSON	22.0291	27.3503
	RANDOM	15.2139	15.2380
	SALT & PEPPER	17.4552	17.5905
	SPECKLE	13.9162	15.7889
Cameraman 	GAUSSIAN	17.7122	22.9596
	POISSON	21.8056	23.9154
	RANDOM	16.4904	22.3245
	SALT & PEPPER	19.3795	19.5030
	SPECKLE	13.0491	19.1355
Building 	GAUSSIAN	19.3592	26.6154
	POISSON	22.6225	26.7079
	RANDOM	18.4168	26.2133
	SALT & PEPPER	21.0088	21.3765
	SPECKLE	13.7887	21.9778
MRI-Spine 	GAUSSIAN	14.3754	21.6094
	POISSON	21.2729	24.1396
	RANDOM	13.1690	21.1571
	SALT & PEPPER	15.4106	15.8091
	SPECKLE	13.9668	15.6001
Lena 	GAUSSIAN	20.8279	25.1971
	POISSON	20.8279	27.5101
	RANDOM	13.8260	13.9703
	SALT & PEPPER	16.4820	16.8714
	SPECKLE	13.1553	17.4107
Women and baby 	GAUSSIAN	19.1847	30.0657
	POISSON	22.5356	30.9337
	RANDOM	18.0668	18.0765
	SALT & PEPPER	20.7693	20.9552
	SPECKLE	13.7627	21.8921

Thus, we can now compare the wavelet transform results to the other transforms by comparing the SNR values.

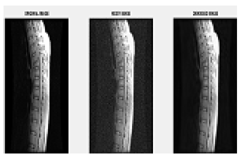
The results for removing different types of noises from the images using Ridgelet transform along with its SNR values are shown in the below table.

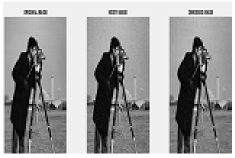
Table -2: Ridgelet transform result

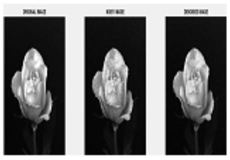
TYPE OF IMAGE	**TYPE OF NOISE**	**SNR** ORIGINAL VS NOISY IMAGE	**SNR** ORIGINAL VS DENOISED IMAGE
X-Ray 	GAUSSIAN	15.7167	22.5096
	POISSON	20.1466	25.6381
	RANDOM	13.1591	13.2619
	SALT & PEPPER	19.5251	19.6150
	SPECKLE	11.7534	17.8462
X-Ray 	GAUSSIAN	13.7284	25.9265
	POISSON	20.1466	25.6381
	RANDOM	13.1591	13.2619
	SALT & PEPPER	19.5251	19.6150
	SPECKLE	11.7534	17.8462

The results for removing different types of noises from the images using Curvelet transform along with its SNR values are shown in the below table.

Table -3: Curvelet transform results

TYPE OF IMAGE	**TYPE OF NOISE**	**SNR** ORIGINAL VS NOISY IMAGE	**SNR** ORIGINAL VS DENOISED IMAGE
MRI-Spine 	GAUSSIAN	14.3485	24.7859
	POISSON	21.2554	26.0992
	RANDOM	13.1970	24.9173
	SALT & PEPPER	15.5262	18.1427
	SPECKLE	13.9769	20.6194

Cameraman 	GAUSSIAN	17.7155	25.8694
	POISSON	21.8002	26.2786
	RANDOM	16.5063	25.7540
	SALT & PEPPER	19.3872	21.2435
	SPECKLE	13.0463	21.5012

Lena 	GAUSSIAN	17.3606	25.6830
	POISSON	21.5544	27.9469
	RANDOM	16.4290	25.5406
	SALT & PEPPER	13.1763	21.3743
	SPECKLE	13.1553	21.8809
Women and baby 	GAUSSIAN	14.3798	23.9250
	POISSON	21.7213	25.3337
	RANDOM	12.1398	25.0135
	SALT & PEPPER	13.7569	16.5982
	SPECKLE	13.7977	20.5677

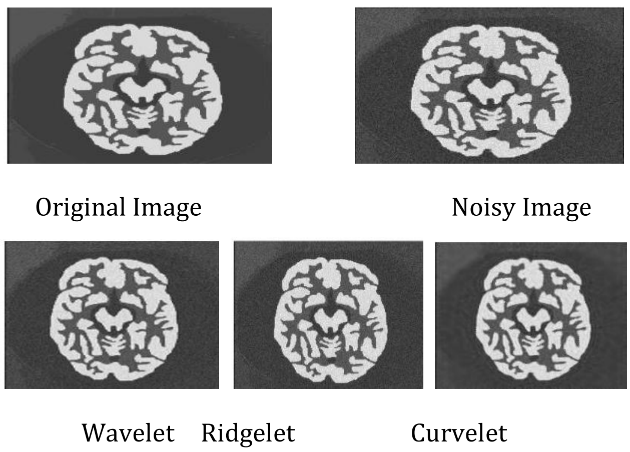


Fig -16: Original, Noisy, and Denoised images using wavelet, ridgelet, and curvelet transforms.

We can therefore conclude by looking into the output images and their SNR values that the curvelet transform is the best among the three transforms for removing the different types of noise.

4. CONCLUSIONS

This paper presents an analysis of wavelet, ridgelet, and curvelet transforms for image denoising. Using these transforms, different types of noises from an image can be removed from it, thereby improving its quality. In terms of performance as well as visual quality, the results obtained from the actual images along with the SNR values concluded that the curvelet transform was the best solution to remove the different types of noise in an image.

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