Mixed Convection Flow of Viscous Fluid in Vertical Channel Filled with Porous Stratum in Presence of First Order Chemical Reaction with an Effect of Thermal Conductivity and Variable Viscosity

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Abstract - *A present study reveals the steady characteristic flow of viscous fluid along viscosity dependent temperature and thermal conductivity in vertical channel embedded with porous medium. A viscous dissipation included in an energy equation while first order chemical reaction between diffusing species and fluid is considered in diffusion mass equation. The channel walls preserved by two different constant temperatures. A resulting non-linear, coupled ordinary differential equation were solved by using bvp4c Matlab code. It is found that presence of porous stratum reduces flow field. An effect of velocity, temperature and concentration distributions are discussed numerically and explained graphically. Skin friction and Nusselt number values near walls of channel are discussed and numerical values were presented through tables for various values of physical parameters.*

Key Words: Thermal conductivity, Viscous fluid, Viscous dissipation, porous medium, chemical reaction.

1.INTRODUCTION

The mixed convective flow over parallel vertical plate channel with porous stratum has been studied widely. Mixed convection flow along porous media have variety of applications such as drying of porous solid processing of food, cooling of electronic materials, packed-bed reactors, aerodynamics heating, ground water flows, industrial and agricultural assignment. In mixed convection flows, forced convective flow effect and free convection flow effects are of related magnitude.

Many works have been conducted over convective motion in porous stratum. A large number of technical applications are discussed in Nield and Bejan [1]. The impact of viscous dissipation past fully developed forced convection in two types of fluid in a parallel-plate channel fill up with porous medium investigated by Nield et al. [2]. Bhattacharyya [3] examined steady boundary-layer mass transfer and slip flow with chemical reaction over porous plate placed in porous stratum. An analysis of flow convective of two immiscible permeable fluids in vertical channel with heat source/sink in porous stratum by Umavathi [5, 6].

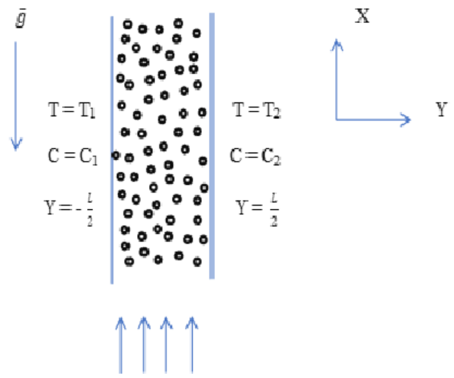
A viscosity variation with temperature on buoyancy transient induced free convective flow across flat vertical plate with saturated fluid porous medium examined by Mehta et al. [8]. Muthuraj et al. [9] investigated the mixed convection transfer heat and mass in vertical wavy channel over porous stratum by traveling thermal waves and thermal diffusion. Kou and Lu [10] examined combined effects and inertia effects on mixed convection fully developed vertical channel within porous media with non-Darcy flow model. Basant Jha et al. [12] analyzed effect of porosity over mixed convection flow through inclined channel fill up with porous material. Viscoelastic incompressible fluid flow on infinite porous vertical plate with first order chemical reaction is revealed by Damesh et al. [13]. Astanina et al. [14] investigated natural convection with differentially porous heated cavity of fluid along viscosity dependent temperature.

The analysis of porosity impact on natural convective problem fills with porous medium examined by Inna Aleshkova et al. [15]. Weng-Jeng Chang et al. [16] have analyzed fully developed laminar mixed convective flow on parallel plate channel partially with porous stratum. Hong et al. [17] considered natural convection against vertical plate in porous stratum with influence of non-Darcian and nonuniform permeability conditions. Pop et al. [18] examined mixed convective flow over narrow vertical duct fill up with porous medium with different temperature. Minkowycz et al. [19] examined the effect of buoyancy over stagnation and parallel flows within porous stratum about horizontal plate. Mastaneh Hajipour and Asghar Molaei Dehkordi [20] studied fully developed mixed convection heat transfer of Nano -fluid with partially filled porous stratum along vertical channel. Anwar Hossain and Mike Wilson [21] investigated the unsteady natural convection enclosed by partially filled porous medium over non-isothermal walls by internal heat generation. The present study is an extension of Umavathi et al. [22] analyzed mixed convective viscous fluid flow in vertical channel with viscosity and thermal conductivity in presence of chemical reaction of first order.

In this present study we analyzed the flow, mass and heat transfer of viscous fluid in porous stratum by thermal conductivity and viscosity through vertical channel. The resulting coupled, non-linear ODE were solved by bvp4c Matlab code. An obtained numerical result for various physical parameters were presented via graphs and tables.

2. MODELLING OF THE PROBLEM

Consider flow of two-dimensional incompressible fluid between two parallel plates fill up by porous material. Let two parallel plates bounded the fluid separated by distance $2b$, origin of co-ordinate axis and gravitational acceleration $\vec{g}$ is located in left side and located in right side of channel, respectively drawn in Fig 1. The two parallel plates retained at two temperatures constant T_1 and T_2 for left and right plates, respectively. Let $-b \le Y \le b$ be region of space occupy by the channel.



An equation of state and Boussinesq approximation of the fluid which characterizing properties except viscosity, density and thermal conductivity are considered to be constant.

$$\rho = \left[1 - \beta_C (C - C_0) - \beta_T (T - T_0)\right] \rho_0 \quad (1)$$

The governing equations for an incompressible fluid are written by,

$$\frac{d}{dY}\left(\mu \frac{dU}{dY}\right) + \beta_T \rho_0 g (T - T_0) + \beta_C \rho_0 g (C - C_0) - \frac{\partial p}{\partial X} - \frac{\nu_0}{k} U = 0 \quad (2a)$$

$$\frac{d}{dY}\left(K \frac{dT}{dY}\right) + \mu \left(\frac{dU}{dY}\right)^2 - \frac{\nu_0}{k} U^2 = 0 \quad (2b)$$

$$D \frac{d^2 C}{dY^2} - \gamma (C - C_0) = 0 \quad (2c)$$

Where T_0 is the reference temperature.

The boundary conditions are given by

$$Y = -b; \quad U = 0, \quad T = T_1, \quad C = C_1 \quad (3a)$$

$$Y = b; \quad U = 0, \quad T = T_2, \quad C = C_2 \quad (3b)$$

Let μ be viscosity of fluid is presumed to vary by temperature

$$\mu = \mu_0 e^{-a(T - T_0)} = \left(1 - (T - T_0) a\right) \mu_0 \quad (4)$$

The fluid thermal conductivity is assumed to be

$$K = K_0 e^{-\tilde{b}(T - T_0)} = \left(1 - \tilde{b}(T - T_0)\right) \mu_0 \quad (5)$$

A fluid thermal conductivity is considered to linearly vary by temperature.

The dimensionless form of the equations (2a) to (2c) using the following dimensionless parameters to determine velocity, temperature and concentration distribution.

The non-dimensional variables are,

$$u = \frac{U}{\bar{u}}, \quad y = \frac{Y}{b}, \quad m = \frac{T_1 - T_2}{\Delta T}, \quad \theta = \frac{T - T_0}{\Delta T}, \quad \phi = \frac{C - C_0}{\Delta C},$$

$$GR_T = \frac{Gr_T}{\text{Re}}, \quad GR_C = \frac{Gr_C}{\text{Re}}, \quad Gr_T = \frac{g \beta_T b^3 \Delta T}{\nu^2},$$

$$\alpha = \frac{\gamma b^2}{D}, \quad Br = \frac{\mu_0 \bar{u}^2}{K_0 \Delta T}, \quad \text{Re} = \frac{\bar{u} b}{\nu}, \quad k = \frac{b^2}{\sigma^2},$$

$$Gr_C = \frac{g \beta_C b^3 \Delta C}{\nu^2}, \quad P = \frac{b^2}{\mu_0 \bar{u}} \frac{\partial p}{\partial x} \quad (6)$$

Using dimensionless parameters Eq. (6) in Eqs. (2a) – (2c) the following equations are derived:

$$\frac{d^2 u}{dy^2} - b_V \frac{d\theta}{dy} \frac{du}{dy} + (1 + b_V \theta)(GR_T \theta + GR_C \phi) - (1 + b_V \theta) P$$
$$-(1 + b_V \theta) \sigma^2 u = 0 \quad (7a)$$

$$\frac{d^2 \theta}{dy^2} - b_k \left(\frac{d\theta}{dy}\right)^2 + Br \left(\frac{du}{dy}\right)^2 + (b_k - b_V) \theta Br \left(\frac{du}{dy}\right)^2 - b_k b_V \theta^2 Br \left(\frac{du}{dy}\right)^2$$
$$-(1 + b_k) Br u^2 \sigma^2 = 0 \quad (7b)$$

$$\frac{d^2 \phi}{dy^2} - \alpha \phi = 0 \quad (7c)$$

boundary conditions,

$$y = -1; \quad u = 0, \quad \theta = m + 1, \quad \phi = 1 + n \quad (8a)$$

$$y = 1; \quad u = 0, \quad \theta = 1, \quad \phi = 1 \quad (8b)$$

2.1 Solutions:

The solutions of equations (7a) and (7b) are solved by bvp4c Matlab code. The solution of equation (7c) is obtained directly by Integration is given by

$$\phi = C_1 e^{\sqrt{\alpha y}} + C_2 e^{-\sqrt{\alpha y}} \qquad (9)$$

Where C_1 and C_2 are constants of integrating and are given by

$$C_1 = \frac{(1+n)e^{-\sqrt{\alpha}} - e^{\sqrt{\alpha}}}{e^{-2\sqrt{\alpha}} - e^{2\sqrt{\alpha}}} \quad \text{and} \quad C_2 = \frac{1 - C_1 e^{\sqrt{\alpha}}}{e^{-\sqrt{\alpha}}}$$

The most interest physical quantities of the problem were skin friction and Nusselt number.

The shear stress and Nusselt number given as,

$$C_f = \left(\mu \frac{dU}{dY}\right)_{Y=\pm b} \quad \text{and} \quad Nu = -\frac{b\left(K\dfrac{dT}{dy}\right)_{Y=\pm b}}{K_0 \Delta T} \qquad (10)$$

By introducing dimensionless parameters Eq. (6) we get,

$$C_{f1} = e^{-b_V(1+m)}\left(\frac{du}{dy}\right)_{y=-1} \quad \text{and} \quad C_{f2} = e^{-b_V}\left(\frac{du}{dy}\right)_{y=1} \qquad (11)$$

$$Nu_1 = -(1 + b_k(m+1))\left(\frac{d\theta}{dy}\right)_{y=-1} \quad \text{and}$$

$$Nu_2 = -(1 + b_k)\left(\frac{d\theta}{dy}\right)_{y=1} \qquad (12)$$

3. RESULTS AND DISCUSSION

A mixed convection flow of fluid inside vertical channel embedded in porous stratum by an influence of thermal conductivity and viscosity along chemical reaction of first order is analyzed. An equation (7a) and (7b) are coupled and non-linear are solved by bvp4c Matlab code. A solution of equation (7c) is obtained directly by integration using the boundary conditions (8a) and (8b). The result were presented graphically for b_v; viscosity variation variable, b_k;Conductivity variation variable, thermal Grashof number, GR_C; σ; porosity variable, Br ; Brinkman number, m; ratio of wall temperature, GR_T; mass Grashof number, and α; chemical reaction of first order.

Figure 2 represents velocity profile u for varying of viscosity variation variable b_v. It shows the velocity enhances for increasing of b_v. The velocity profile is move towards left wall when values of b_v are negative and velocity profile is move towards right wall when values of b_v positive. The figure 3 elaborates temperature θ when varying viscosity variation variable b_v. Figure shows for increasing b_v increases temperature.

Figure 4 which illustrate values of conductivity variation variable b_k on velocity profile. It is evident that the velocity suppresses for increasing b_k. Figure 5 is plotted for several values of conductivity variation parameter b_k. We can observe that for increasing conductivity variation variable b_k the temperature decays gradually at cold and hot walls.

The effects of thermal Grashof number GR_T on velocity and temperature profiles as drawn in figures 6 and 7. We infer from figure, velocity and temperature were increases. Figure 8 is plotted for velocity profile for varying mass Grashof number GR_C. We observe that for increasing GR_C velocity profile rises up. Figure 9 demonstrate impact of mass Grashof number GR_C along temperature profile. From this plot it is observed that for increasing GR_C temperature profile enhances.

Figures 10 and 11 which is a graphical representation of velocity and temperature for varying Brinkman number Br. Figures presented for increasing Br both velocity and temperature profiles enhances. An influence of ratio of wall temperature m along velocity and temperature were plotted by figures 12 and 13 respectively. It can be observed for increasing ratio of m velocity decays and growth has been observed in temperature profile.

Figures 14 and 15 presents the impact of α about velocity and temperature. In figure 14, velocity reduces for increasing α, while temperature profile enhances with increasing α in figure 15. Figure 16 depicts velocity for porosity parameter σ. The figure elaborates increasing porosity parameter σ velocity enhances. Figure 17 is plotted for various values of porosity parameter σ. It shows that for increasing porosity parameter σ the temperature decreases. Figure 18 depicts the concentration profile for varying first order chemical reaction α. It shows concentration profile reduces by increasing α.

An impacts of viscosity variation variable b_v, Conductivity variation variable b_k, mass Grashof number GR_C, porosity parameter σ, Brinkman number Br, thermal Grashof number GR_T, ratio of wall temperature m and chemical reaction of first order α on C_f and Nu at wall were shown in Table 1. For increasing viscosity variation parameter b_v, the left wall C_f and Nu enhances and decays right wall C_f. The skin friction increases and decreases respectively for increasing Conductivity variation parameter b_k at both walls and same result can be observed for Nusselt number.

For increasing GR_C and GR_T the C_f and Nu enhances at left wall and suppresses at right wall. A C_f at left wall grows up decays due to increase wall temperature ratio m. The Nu at both walls decreases with increase m. The C_f and Nu at right wall increases and decays at left wall as Brinkman number Br enhances. For increasing chemical reaction of first order α the left wall C_f and Nu suppresses and enhances at right wall. The Nu at both walls enhances as porosity variable σ enhancing whereas C_f at left and right walls were increases and decreases respectively.

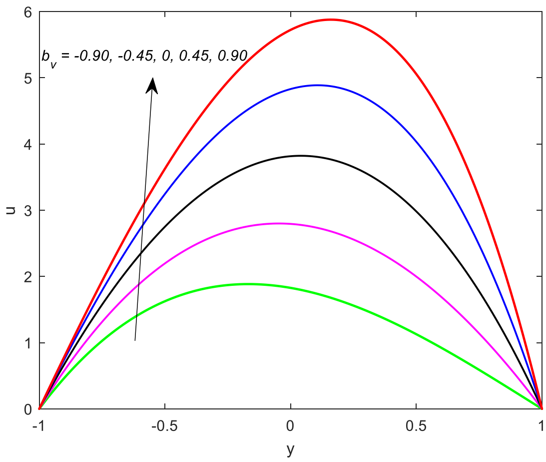


Fig. 2. Varying b_v against velocity

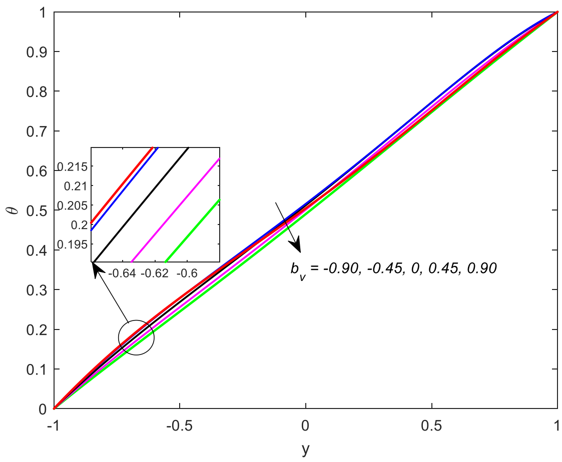


Fig. 3. Varying b_v against temperature

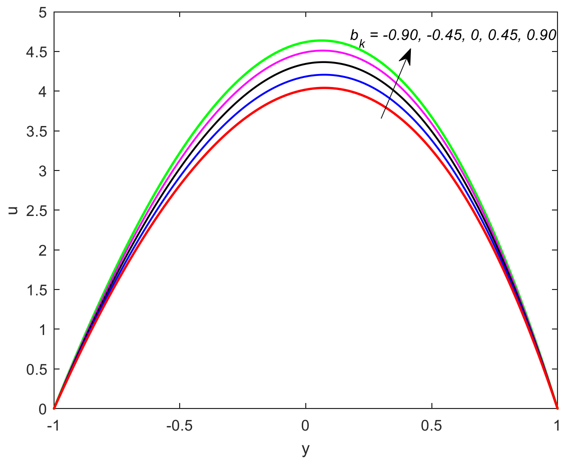


Fig. 4. Varying b_k against velocity

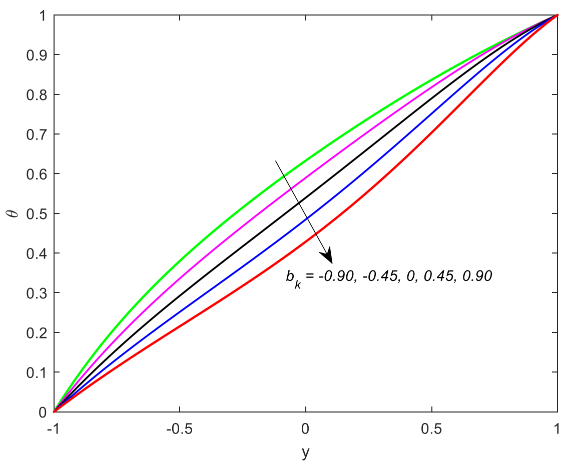


Fig. 5. Varying b_k against velocity

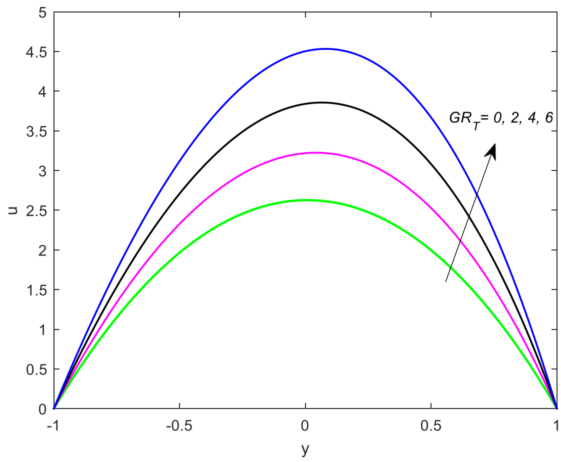


Fig. 6. Varying GR_T against velocity

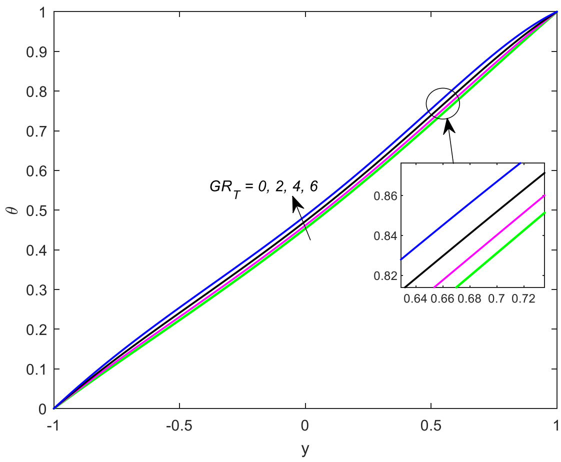


Fig. 7. Varying GR_T against temperature

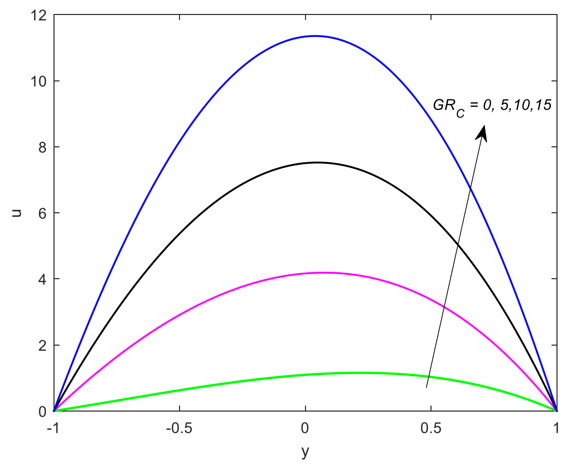


Fig. 8. Varying GR_C against velocity

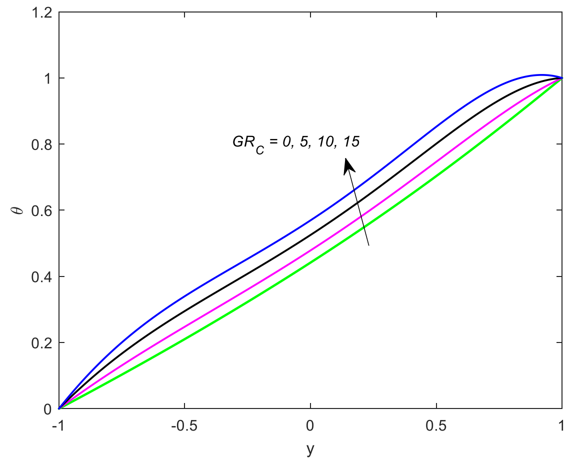


Fig. 9. Varying GR_C against temperature

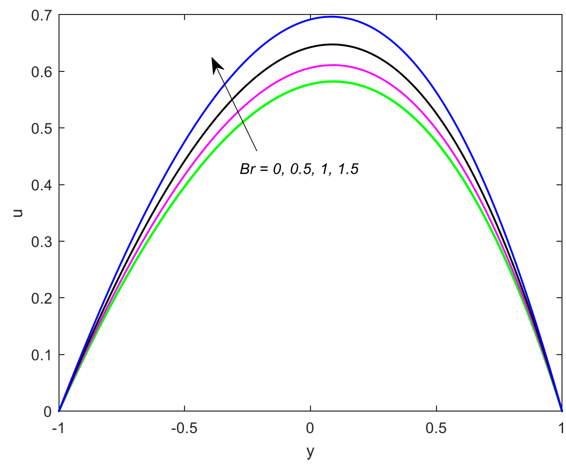


Fig. 10. Varying Br against velocity

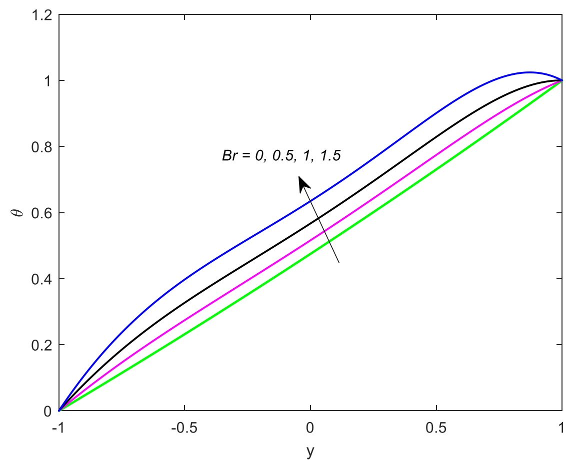


Fig. 11. Varying Br against temperature

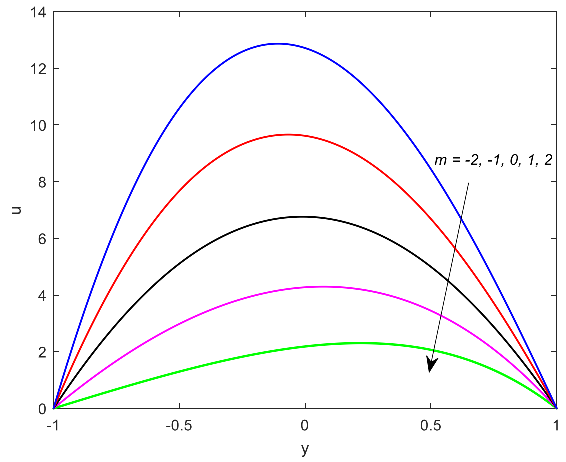


Fig. 12. Varying m against velocity

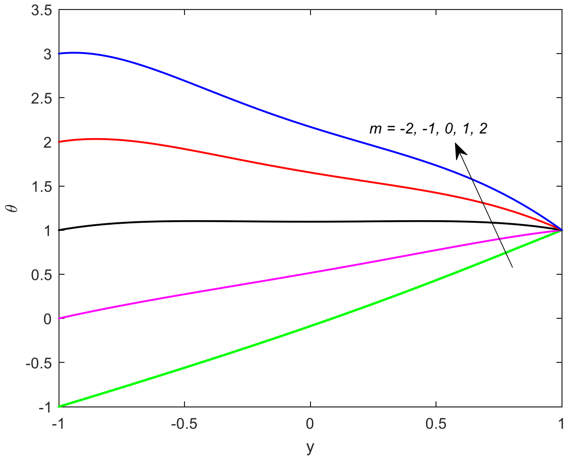


Fig. 13. Varying m against temperature

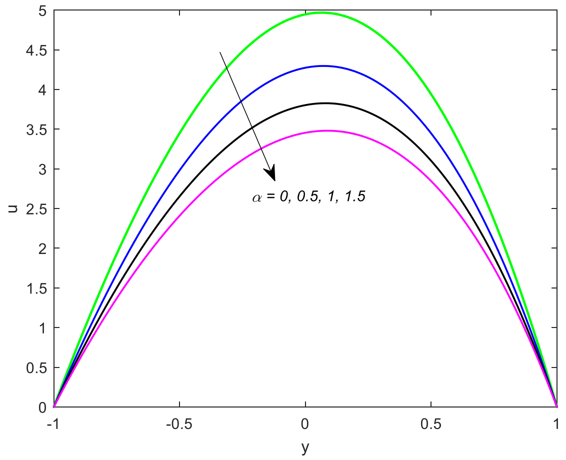


Fig. 14. Varying α against velocity

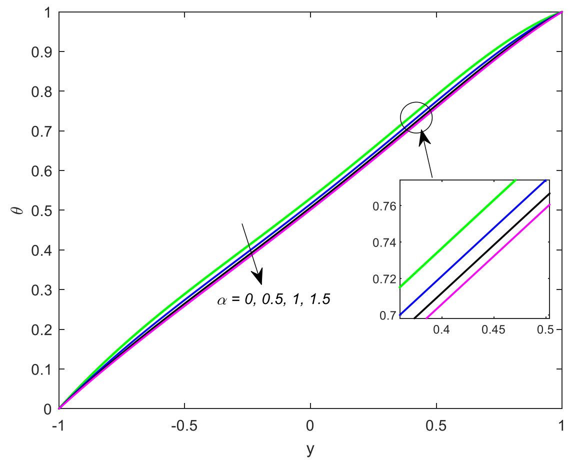


Fig. 15. Varying α against temperature

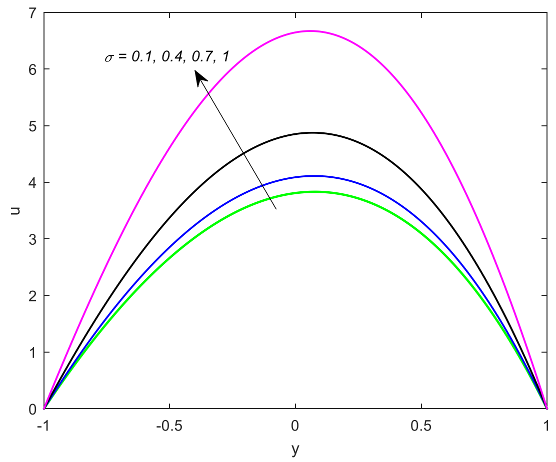


Fig. 16. Varying σ against velocity

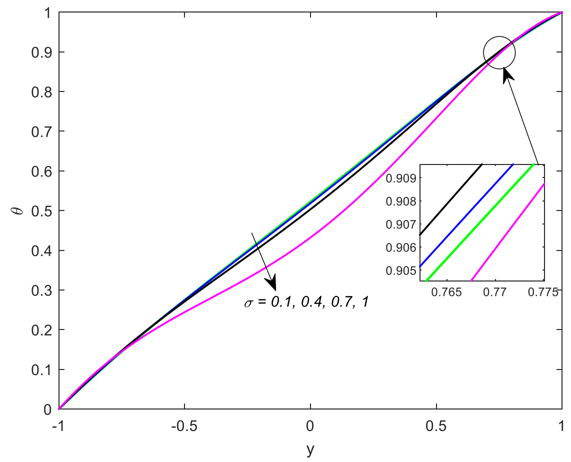


Fig. 17. Varying σ against temperature

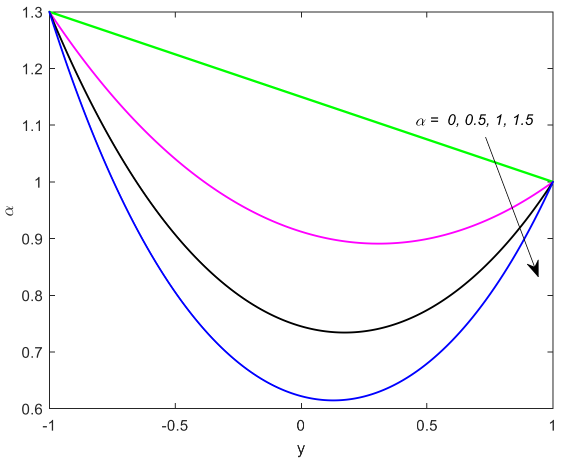


Fig. 18. Varying α against concentration

Table -1: Skin friction and Nusselt number values by varying b_V, b_k, GR_T, GR_C, m, α, P, σ, Br, n.

	C_{f1}	C_{f2}	Nu_1	Nu_2
b_V	$b_k = 0.2,\ GR_T = GR_C = 5,\ m = -1,$ $\alpha = P = \sigma = 0.5,\ Br = 0.01,\ n = 0.3$			
-0.90	5.0880	-2.3805	0.5177	0.4887
-0.45	6.1691	-5.0978	0.5636	0.4083
0	7.0630	-8.2748	0.6147	0.3288
0.45	7.6819	-11.8228	0.6566	0.3157
0.90	7.9753	-15.7028	0.6720	0.4473
b_k	$b_V = 0.2,\ GR_T = GR_C = 5,\ m = -1,$ $\alpha = P = 0.5,\ Br = 0.01,\ n = 0.3,\ \sigma = 0.5$			
-0.90	7.8409	-10.2692	1.0243	0.2854
-0.45	7.6627	-10.1060	0.8378	0.2736
0	7.4676	-9.9096	0.6906	0.2905
0.45	7.2624	-9.6835	0.5747	0.3465
0.90	7.0551	-9.4346	0.4836	0.4541
GR_T	$b_V = b_k = 0.2,\ GR_C = 5,\ m = -1,\ \alpha = P =$ $\sigma = 0.5,\ Br = 0.01,\ n = 0.3$			
1	5.6764	-6.2867	0.5476	0.4466
5	7.3772	-9.8124	0.6356	0.3099
10	9.9452	-14.7401	0.8224	0.0158
15	13.7682	-21.1342	1.2139	-0.5760
20	18.5741	-27.8643	1.8934	-1.5207
GR_C	$b_V = b_k = 0.2,\ GR_T = 5,\ m = -1,\ \alpha = P =$ $\sigma = 0.5,\ Br = 0.01,\ n = 0.3$			
1	2.5355	-4.7573	0.4819	0.5028
5	7.3772	-9.8124	0.6356	0.3099
10	13.6534	-16.3771	1.0445	-0.1662
15	20.2439	-23.2800	1.7494	-0.9599
20	27.2506	-30.6258	2.8480	-2.1663
m	$b_V = b_k = 0.2,\ GR_T = GR_C = 5,\ \alpha = P =$ $\sigma = 0.5,\ Br = 0.01,\ n = 0.3$			
-2	2.7619	-6.9079	0.8618	1.1354
-1	7.3772	-9.8124	0.6356	0.3099
0	13.4583	-12.8646	0.5085	-0.4893
1	21.0436	-15.9858	0.4669	-1.2832
2	30.0678	-19.0244	0.3243	-2.0593
Br	$b_V = b_k = 0.2,\ GR_T = GR_C = 1,\ m = -1,$ $\alpha = P = \sigma = 0.5,\ n = 0.3$			
0.1	0.9882	-1.4079	0.4865	0.5060
0.5	1.0215	-1.4466	0.6328	0.2997
1	1.0723	-1.5055	0.8541	-0.0070
1.5	1.1394	-1.5829	1.1431	-0.3982
2	1.2388	-1.6972	1.5643	-0.9526
α	$b_V = b_k = 0.2,\ GR_T = GR_C = 5,\ m = -1,$ $P = \sigma = 0.5,\ Br = 0.01,\ n = 0.3$			
0	8.6607	-13.4480	0.6529	0.2838
0.5	7.5987	-12.0577	0.5863	0.3702
1	6.8448	-11.0745	0.5454	0.4240
1.5	6.2807	-10.3417	0.5183	0.4600
σ	$b_V = b_k = 0.2,\ GR_T = GR_C = 5,\ m = -1,$ $\alpha = P = 0.5,\ Br = 0.01,\ n = 0.3$			
0.1	6.8254	-10.9364	0.5699	0.3837
0.4	7.2877	-11.6072	0.5800	0.3749
0.7	8.5770	-13.4731	0.6029	0.3630
1	11.5920	-17.8435	0.6042	0.4391

4. CONCLUSIONS

The study flow, mass and heat transfer characteristic of viscous fluid through vertical channel in porous stratum is analyzed with an impact of thermal conductivity and viscous at temperature. A resulting coupled nonlinear ODEs are solved by bvp4c Matlab code. A following result were observed from study are;

i). Increase in viscosity variation variable b_v, Conductivity variation variable b_k, thermal Grashof number GR_T, Brinkman number Br and mass Grashof number GR_C porosity parameter σ enhances velocity profile.

ii). The ratio of wall temperature m and first order chemical reaction α suppresses velocity.

iii). For increase in thermal Grashof number GR_T, ratio of wall temperature m, Brinkman number Br and mass Grashof number GR_C increases temperature profile.

iv). The viscosity variation variable b_v, Conductivity variation variable b_k, chemical reaction of first order α and porosity parameter σ decreases the temperature.

v). A concentration profile decays with increasing chemical reaction of first order α .

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