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RELIABILITY BASED STABILITY ANALYSIS OF A CONCRETE DAM BY PROBABILISTIC METHODS

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Abstract - *Concrete gravity dams are important infrastructure assets in many countries, the failure of which may lead to catastrophic consequences with major social and economic impacts. The need to evaluate the safety of existing concrete gravity dams in seismically active regions is very important.*

In this paper, the reliability analysis of a hypothetical dam section is done for various IS Load Combinations, different water levels and drainage conditions using First order second moment method, Rosenbleuth point estimate method and Advanced first order second moment method and the results are compared with the results obtained by Monte Carlo Simulation method.

Key Words: **Gravity dam, Reliability Index, Factor of safety**

1. INTRODUCTION

Recent publication of dam safety guidelines and research around the world reflect a growing concern on the seismic safety of hydroelectric and flood control gravity dams and other similar water retaining gravity structures built prior to 1950's. Although probable maximum flood (PMF) is a critical lateral load in the sliding stability of concrete gravity dams, by and large horizontal and vertical accelerations due to maximum credible earthquake (MCE) have become the main factor in the stability evaluation of concrete gravity dam structures. A typical medium to large size concrete gravity dam comprises various service or relief structures that include a number of expansion joints. By controlling temperature ingredients, expansion joints are mainly provided to minimize cracking of concrete mass due to heat of hydration of concrete during and immediately after construction of dams. Inclusion of expansion joints results in the creation of several rigid concrete monoliths or blocks along the length of the dam. During a strong seismic event in the upstream-downstream direction (i.e. along the centerline of river), concrete blocks tend to slide at the base.

2. Study Area

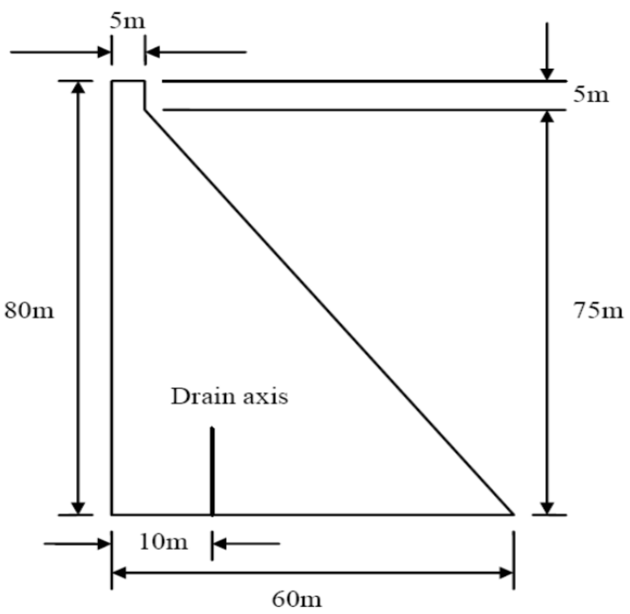

Figure 3.1: Hypothetical dam section

The Hypothetical dam to be analyzed in this thesis is taken from the theme C of the eleventh ICOLD benchmark workshop on numerical analysis of dams (3IWRDD ICOLD, 2011). The problem in the benchmark workshop aims at analyzing the dam with a 2D model. Total height of the dam is 80 m with drain axis 10 m from the upstream end. The other dimensions are given in Figure 3.1. The objective of the ICOLD proposal was to benchmark numerical and analytical methods for the evaluation of the maximum sustainable reservoir level before dam collapsing and the evaluation of the uplift pressure distributions acting along the dambase.

Methodolgy:-

3. Random Variables

The variables are classified as deterministic or random. In the present thesis, the random variables considered are friction angle ‘φ’ and cohesion ‘c’ along the dam-foundation interface. Probability density functions are determined for friction and cohesion and the probability distribution is assumed normal. The friction angle is defined by a normal probability function with a mean value μ_f and standard deviation σ_f. Similarly, cohesion is normally distributed with a mean μ_c and standard deviation σ_c.

Table 3.1 : Data for friction and cohesion at the interface (3IWRDD ICOLD, 2011)

Sample	Friction angle ’φ’ in degrees	Cohesion ’c’ (MPa)
1	45	0.5
2	37	0.3
3	46	0.3
4	45	0.7
5	49	0.8
6	53	0.2
7	54	0.6
8	45	0.0
9	49	0.1
10	60	0.2
11	63	0.2
12	62	0.4
13	60	0.7
14	56	0.1
15	62	0.4

Table 3.2: Mean and standard deviation values used for friction and cohesion

Normal Probability Distribution	Friction Angle (φ) Degrees	Cohesion (c) Mpa
Mean	52.4^0	0.3367
Standard Deviation	7.989	0.2468

3.2.1 Deterministic variables

The considered deterministic variables are

1. Concrete density (kN/m^3)

2. Water density (kN/m^3)

3. Water Pressure 'P' (kN/m^2)

4. Self-weight of the dam 'W' (kN/m)

5. Horizontal load due to water pressure acting on the upstream face of the dam 'H' (kN/m)

6. Uplift load acting on the base of the dam 'U' (kN/m)

7. Tensile stress 'σ' (kN/m^2)

The Stability Analysis is carried out by deterministic method for the following load cases:

1. Load combination A: (Empty reservoir condition) : When the reservoir is empty, the force acting on the dam profile will be due to the self weight only, which acts at inner middle third. Other forces such as water pressure and uplift will be zero.

2. Load combination B (normal operating conditions): Full reservoir elevation, normal dry weather tail water, normal uplift, ice and silt (if applicable). Here for Indian conditions the silt load is neglected

3. Load combination C: (Flood discharge condition) - Reservoir at maximum flood pool elevation all gates open, tailwater at flood elevation, normal uplift, and silt (if applicable)

4. Load combination D: Combination of A and earthquake

5. Load combination E: Combination B, with earthquake but no ice

6. Load combination F: Combination C, but with extreme uplift, assuming the drainage holes to be Inoperative

7. Load combination G: Combination E but with extreme uplift (drains inoperative)

3.4 TAYLOR'S SERIES METHOD OF RELIABILITY ANALYSIS

Performance function M for a particular water level is defined as:

$$M = \frac{R}{H} - 1 \qquad \text{------ 3.1}$$

Where R is the sum of stabilizing forces and H is the sum of horizontal forces. The methodology used for calculating the probability of failure for the two given cases is presented below. The stepwise procedure below is in detail the same as summarized before in section 2.5.2 according to USACE (Duncan MJ, 1999).

R and H vary with the change in water level 'h'. With different drain conditions, the parameter R and H have varying values depending on the height and the uplift pressure but the methodology remains the same for both drainage conditions and different water levels. The values of R and H can be given as described in eq. 3.2 and eq. 3.3

$R = (W-U)\cdot Tan(\varphi) + A.c$ ------ 3.2

$H=0.5\cdot\rho\cdot g\cdot h2$ ------ 3.3

Mean and standard deviations for both 'φ' and 'c' are

Friction angle 'φ': mean = μf standard deviation = σf

Cohesion 'c': mean = μc standard deviation = σf

The performance function M can be evaluated by varying the mean value of each variable according to its standard deviation. It is done by computing the performance function M with each parameter increased by one standard deviation and then decreased by one standard deviation from its most likely values as described previously in chapter 2. It will generate different values of M, i.e. M1, M2, M3, and M4 as shown in eq. 3.4 to eq. 3.7 where M1 and M2 belongs to the friction 'φ' and M3, and M4 belongs to the cohesion 'c'.

M1 (μf + σf, μc) = (W-U)·Tan·(μf + σf) + A·μc ------ 3.4

M2 (μf - σf, μc) = (W-U)·Tan (μf - σf) + A·μc ------- 3.5

M3 (μf, μc + σf) = (W-U)·Tan (μf) + A·(μc + σf) ------- 3.6

M4(μf,μc-σf)=(W-U)·Tan(μf)+A·(μc-σf) -------- 3.7

3.5 ROSENBLEUTH POINT ESTIMATE METHOD:

In engineering, for conventional slope structure stability analysis, we can build the following state function according to its structure, the failure mechanism and the stress condition:

$$z = F(x_1, x_2 \ldots, x_n) = \frac{R(x_1, x_2 \ldots, x_n)}{S(x_1, x_2 \ldots, x_n)}$$

------- 3.8

Where R(x1,x2....xn) means Sliding resistance force or sliding resistance torque S(x1,x2....xn) means Down force or Downturn torque

When the distribution function of the state variables is unknown, the variation does not need to be considered, we can choose 2 data point symmetrically in the interval (xmin,xmax) .

$$\left.\begin{aligned} x_{i1} &= \mu_{xi} + \sigma_{xi} \\ x_{i2} &= \mu_{xi} - \sigma_{xi} \end{aligned}\right\}$$

------- 3.9

If there are n state variables, there are 2n values, all possible combinations of values maybe 2n. For this combination, according to the equation of state, we can obtain 2n state functions, so there are 2n stability coefficient (safety factor).

If the n state variables are independent, the probability of each combination is equal, so the average of is:

$$\mu_Z = \frac{1}{2^n} \sum_{j=1}^{2^n} Z_i$$

------- 3.10

If the n state variables are Related, the probability of each combination is equal, so the average of is:

$$P_j = \frac{1}{2^n}(1 + e_1 e_2 \rho_{12} + e_2 e_3 \rho_{23} + \ldots + + e_{n-1} e_n \rho_{(n-1)n})$$

------- 3.11

$$\mu_Z = \sum_{j=1}^{2^n} P_j Z_j$$

------- 3.12

3.6 THE HASOFER-LIND RELIABILITY INDEX

Hasofer proposed the linearization about a point which lies on the failure surface (Hasofer-Lind, 1974). The point is known as the design point or most probable failure point. The Hasofer-Lind reliability index β HL computation is based on the transformation of the limit state surface into the space of standardized normal variates (Kisse, 2011) by defining the set of reduced variables Z 1 ,

Z 2 ,....Z n using

X' i = (X i - μ Xi) / σ Xi ..... (3.13)

Where μ Xi and σ Xi are the mean and standard deviation of variable X i . The limit state equation

g(X) = R(X) – S(X) is also transformed to standard space as shown in Figure 3.3.

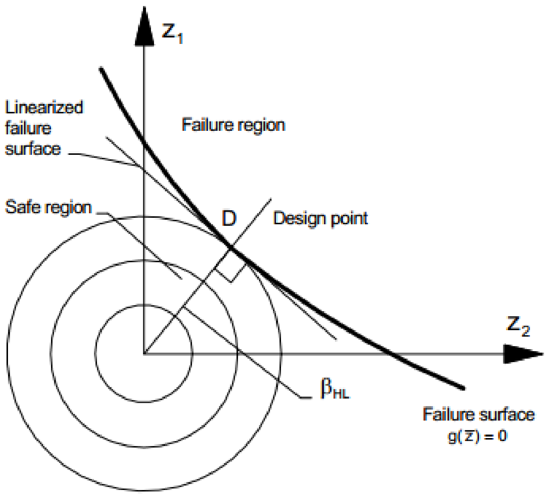

Figure 3.1 : Illustration of reliability index β in the plane (Burcharth, 1997 from Kisse 2011).

The reliability index is the distance from the origin of reduced variables to the nearest point D on the failure surface as shown in Figure 2.6. The point D is called the design point. Hasofer-Lind suggested linearizing the limit state function in this 'design point' in standard normal space (Hasofer-Lind, 1974). In standard normal space each variable has zero mean and unit standard deviation, thus the Hasofer-Lind safety index is defined as:

$$\beta_{HL} = \min_{X \in F} \left(\sum_{i=1}^{n} Z_i^2 \right)^2 = \min(Z^t \cdot Z)^{1/2} \quad ..... (3.14)$$

subjected to g(Z) = 0

, where Z i represents the coordinates of any point on the limit state surface. The point on the failure surface at which Z has minimum magnitude is the design point. Equation 3.2 can further be elaborated as

$$\beta_{HL} = \min_{X \in F} \sqrt{\left(\frac{x_i - \mu_i}{\sigma_i} \right)^T \cdot R^{-1} \cdot \left(\frac{x_i - \mu_i}{\sigma_i} \right)} \quad ..... (3.15)$$

Where X is a vector representing the set of random variables X i , μ i are the mean values, R is the correlation matrix, σ i is the standard deviation and F is the failure domain. According to Melchers (1999), the relationship between the design point D and β can be established as follows. From geometry of surfaces the outward normal vector to a hyperplane given by g(Z) = 0

has components given by

$$c_i = \frac{\sigma_g}{\sigma_Z} \quad .... (3.16)$$

the total length of the outward normal is

$$l = \left(\sum_i c_i^2 \right)^{1/2} \quad ..... (3.17)$$

direction cosines α i of the unit outward normal are

$$\alpha_i = \frac{c_i}{l} \quad .... (3.18)$$

where α i is also known as sensitivity factor or a relative measure of the sensitivity of the safety index β. Sensitivity factor indicates the importance on Hasofer-Lind reliability index of the value of parameters used to define the random vector X. In reliability analysis, a very small value of α i might end up with an assumption of considering its corresponding variable as a deterministic variable rather than a random variable, thus simplifying the probabilistic analysis (Melchers,1999). A higher value of α i implies more sensitivity of β to the standard normal variate. If α i is known, then the coordinates of the design point can be given in terms of the reliability index as

D = Z i = - α i β ..... (3.19)

The Hasofer-Lind index is commonly used in reliability analysis but somewhat advance computer software is needed for its computation. The greater the value of β the lower is the risk of failure but this will also increase the cost of the structure.

3.7 LEVEL 3 RELIABILITY METHOD (MONTE CARLO SIMULATION)

3.7.1 Introduction

Monte Carlo simulation is one of the techniques of the Level III methods to estimate the probability of failure. Level III methods are considered more accurate than level I and level II methods as they compute the exact probability of failure of the whole structural system (Bjerager, 1989). Numerical integration and Monte Carlo simulation are two examples of these methods.

3.7.2 Basis of simulation

The sampling selects the values of uncertain variables randomly according to their probability distribution functions (Hwang and Lee, 2008). What this simulation actually does is it allows a random number generator to select any value in a given range. If it's a normal distribution the values near the mean will be more frequently generated as compared to the values at the extreme as can be seen in Figure 3.4 (Hwang and Lee, 2008). For a simple approach in structural reliability analysis, the sampling includes each random variable randomly say X i to a given and the limit state function M(x) = 0 is then checked (Melchers, 1999). As sample value described by Melchers if M(x) 0, then the structure is considered as failed.

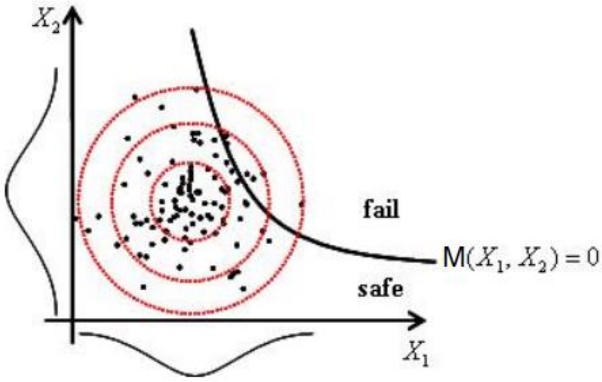

Figure 3.2 : Monte Carlo sampling (Lee and Hwang, 2008)

The basis of the simulation techniques can be well defined by rewriting the equation of probability of failure by means of an indicator function as shown in equation 3.8.

$$P_f = \int_{M(X)\le 0} f_x(x)dx = \int I[M(x) \le 0] f_x(x)dx$$ .... (3.20)

Where

I[M(x)] ≤ 0 is an indicator function. Its value is equal to 1 if M(x) ≤ 0 other wise it is 0

(Ang and Tang WH 2006, Melchers 1999). It is also known that if the value of M(x) is less than zero, it indicates failure. The experiment is repeated several times with randomly chosen vector x or xi . Now if there are N realizations of vector X, i.e. i , i=1,2...,N then the probability of failure as an unbiased estimator can be expressed as (Melchers 1999, Alfredo and Tang WH 2006)

$$P_f = \int \ldots \int I[M(x) \le 0] f_x(x)dx$$

$$P_f = \frac{1}{N}\sum_{i=1}^{N} I(M(x_i) \le 0$$ ..... (3.21)

Monte Carlo simulation technique mostly revolves around the application of the above equation. Now let n f be the number of cycles for which M(x) is less than 0 and N being the total number of simulation cycles then the probability of failure is estimated through (Melchers, 1999):

$$P_f = \frac{n_f}{N}$$.

.... (3.22)

The value estimated from the above equation may be considered as a sample of the expected value of the probability of failure. The equation actually takes part in the simulation, based on a concept that a large number of realizations of basic random variables X, i.e. i , i=1,2...,N are generated or simulated and for each generated value j it makes sure whether the limit state function taken in j i positive or not. If it is not positive, the simulations are considered under n f and after N simulations the probability of failure is estimated as shown in eq. (3.10).It is clear from the above explanation, and is also mentioned by Melchers, that the method used in Monte Carlo simulation actually creates a game of chance from the known probabilistic properties so that to solve the problem several times over and over again to give the required results. If N approaches infinity, then the failure probability becomes exact. For that case, the simulations are usually costly and of course the uncertainties that might take place, cannot be neglected. So a large number of simulations are required to achieve a better estimate.

4.0 RESULTS AND DISCUSSIONS

4.1 Probability of failure using Taylor's series approximation

Table 4.1 : First order second moment method for sliding failure for different IS load combinations

Load Combination	β	Probability of failure pf
B	2.17	0.015
C	2.29	0.015
D	3.41	0.003
E	2.17	0.0192
F	1.935	0.0224
G	1.824	0.0314

Table 4.2: Rosenbleuth point estimate method for sliding failure for different IS load combinations

Load Combination	β	Probability of failure pf
B	2.10	0.0179
C	2.44	0.0073
D	3.57	0.0002
E	2.313	0.0103
F	2.00	0.0228
G	1.964	0.025

Table 4.3: Hasofer Lind method for sliding failure for different IS load combinations

Load Combination	β	Probability of failure pf
B	2.195	0.0143
C	2.56	0.0052
D	3.735	0.0002
E	2.475	0.0067
F	2.135	0.0163
G	1.975	0.0242

Table 4.4: Results of First order Second moment method for sliding failure Different Water levels – Drains Operative and No seismic Load

Water Level H in m	β	% contribution of φ	% contribution of c	Probability of failure pf
75	2.433	63.86	36.14	0.0075
76	2.342	63.87	36.13	0.0096

77	2.282	63.84	36.16	0.013
78	2.216	63.9	36.1	0.0136
79	2.115	63.7	36.3	0.0174
80	2.117	63.8	36.2	0.017

Table 4.5: Results of First order Second moment method for sliding failure for Different waterlevels with Seismic Loads – Drainage gallery Operative

Water Level H in m	β	% contribution of φ	% contribution of c	Probability of failure pf
75	2.066	78.09	21.91	0.0197
76	1.963	72.5	27.5	0.0281
77	1.89	70.08	29.92	0.0262
78	1.83	68.69	31.31	0.025
79	1.77	65.8	34.2	0.0307
80	1.727	63	37	0.0427

Table 4.6: Results of First order Second moment method for sliding failure for different water levels – Drainage gallery Inoperative

Water Level H in m	β	% contribution of φ	% contribution of c	Probability of failure pf
75	2.166	45.63	54.37	0.0154
76	2.098	45.6	54.4	0.0183
77	1.974	45.27	54.73	0.0244
78	1.875	45.65	54.35	0.0307
79	1.879	45.38	54.62	0.0301
80	1.801	45.59	54.41	0.0351

Table 4.7: Results of Rosenbleuth point estimate method for sliding failure for Different waterlevels Drains operative No Seismic load

Water Level H in m	β	Probability of failure pf
75	1.422	0.0778
76	1.375	0.0838
77	2.204	0.0125
78	1.305	0.0885
79	1.125	0.1297
80	1.206	0.1038

Table 4.8: Results of Rosenbleuth point estimate method for sliding failure for Different waterlevels with Seismic Loads – Drainage gallery Operative

Water Level H in m	β	Probability of failure pf
75	1.275	0.1151

76	1.24	0.1401
77	1.158	0.1379
78	1.13	0.1357
79	1.125	0.1539
80	1.095	0.1515

Table 4.9: Results of Rosenbleuth point estimate method for sliding failure for different water levels – Drainage gallery Inoperative

Water Level H in m	β	Probability of failure pf
75	1.105	0.1020
76	1.047	0.1492
77	1.035	0.1515
78	1.08	0.1401
79	0.975	0.1650
80	0.99	0.1685

Advanced First order Second Moment Method or Hasofer-Lind Method

The Hasofer-Lind method is applied for the different water levels for two random variables, friction angle and cohesion. The values of reliability index is summarised below.

Table 4.10: Results of Hasofer Lind method for sliding failure for Different Water levels – Drains Operative and No seismic Load

Water Level H in m	β	Probability of failure pf
75	2.85	0.0022
76	2.685	0.0037
77	2.508	0.0049
78	2.657	0.004
79	2.598	0.0047
80	2.605	0.004

Table 4.11: Results of Hasofer Lind method for sliding failure for Different waterlevels with Seismic Loads – Drainage gallery Operative

Water Level H in m	β	Probability of failure pf
75	2.865	0.0060
76	2.63	0.0075
77	2.52	0.0107
78	2.596	0.0096
79	2.435	0.0136
80	2.457	0.0122

Table 4.12: Results of Hasofer Lind method for sliding failure for different water levels – Drainage gallery Inoperative

Water Level H in m	β	Probability of failure pf
75	2.465	0.0021
76	1.987	0.0235
77	2.35	0.0094
78	2.297	0.0107
79	2.195	0.0141
80	2.205	0.0122

5. CONCLUSIONS

All the reliability methods produce fair results when compared to Monte carlo simulation method.

The Probability of failure is highest in case of Load Combination G which is the normal reservoir level with Extreme uplift and drains Inoperative condition in all methods.

It is also noted that the reliability index increases as the water levels in the dam increases.

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