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Finite Element Modelling of Cantilever Beam Bounded with Piezoelectric patch Subjected to Vibration for Energy Harvesting

Laxmi.B.Wali [1], Dr.Chandrashekara C V [2]

[1]Assistant Professor, Department of Mechanical Engineering RNSIT, VTU, Bengaluru, Karnataka, INDIA
[2]Professor, Design Lead, Department of Mechanical Engineering , PES University, Bengaluru, Karnataka, INDIA

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Abstract— *This paper presents the Finite element modeling of the cantilever beam bounded with piezoelectric patch to generate voltage in Matlab, The equations of motion is represented using Hamilton's principle. Frequency convergence study with varying number of elements is carried to known the number of elements required to analyze the resonance frequencies to generate voltage when the PZT is bounded continuously from fixed end to free end, PZT mounted on individual element and 2 PZT elements bounded on base beam. Energy harvesting is the process by which energy is derived from external sources; vibration is one such energy source which can be captured and stored. Piezoelectric materials are considered as a media to harvest vibration energy .Piezoelectric materials have received tremendous interest in energy harvesting technology due to its unique ability to capitalize the ambient vibrations to generate electric potential.*

Keywords: FEM, Vibration, Energy Harvesting, Piezoelectric

1. INTRODUCTION

In the recent years, research in the area of energy harvesting has attracted the attention of many researchers. The reduced power requirement of small electronic components has motivated the researchers for design of energy harvesting systems. Energy from vibration is free and meets the requirement at the local level without depending on the conventional sources. Recovering even a fraction of this energy would have a significant economic and environmental impact. [1]The blades of helicopters are heavily loaded and are critical components. Failure of any one blade will lead to loss of the aircraft .The measuring systems used in aircraft blade to calculate the actual loads, fatigue within the blades and end of life of blade need energy for sensing. Piezoelectric materials are considered as a media to harvest vibration energy of helicopter blades. Many Researchers are involved in Analytical, Numerical and Experimental study to understand the behavior of different energy harvesting systems.

2. FINITE ELEMENT MODEL

The Finite element method plays the major role in the complex problem analysis .In vibration based energy harvesting, piezoelectric unimorph beam forms the basic structure. The formulation of beam with piezoelectric (PZT) layer is considered as shown in Figure 1.

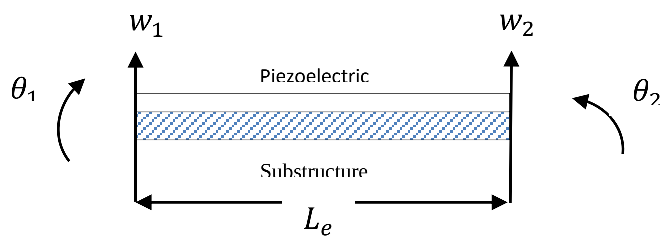

Figure 1. Beam Element with Substructure bounded with PZT

The beams with Piezoelectric have two degree of freedom at each node, transverse and rotational.
The displacement function of the beam element is represented by a cubic polynomial with four constant coefficients

$$w(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 \tag{1}$$

Eqn. (1) is the displacement equation with four unknown coefficients. Applying boundary conditions at node 1 ($x = 0$) and node 2 ($x = L$) of element, the four coefficients in polynomial equation are solved.

The shape functions are given in equation (2-5)

$$[N] = [N_1 \quad N_2 \quad N_3 \quad N_4]$$

where

$$[N_1] = \left(1 - 3\left(\frac{x}{L_e}\right)^2 + 2\left(\frac{x}{L_e}\right)^3\right) \tag{2}$$

$$[N_2] = \left(L_e\left(x - 2\left(\frac{x}{L_e}\right)^2 + \left(\frac{x}{L_e}\right)^3\right)\right) \tag{3}$$

$$[N_3] = \left(3\left(\frac{x}{L_e}\right)^2 - 2\left(\frac{x}{L_e}\right)^3\right) \tag{4}$$

$$[N_4] = \left(L_e\left(-\left(\frac{x}{L_e}\right)^2 + \left(\frac{x}{L_e}\right)^3\right)\right) \tag{5}$$

Strain is represented by

$$S(x) = -z\frac{d^2 w(x)}{dx^2} = -zB \quad (6)$$

$Where\ B = [B_1, B_2, B_3, B_4]$

$$\frac{d^2N_1}{dx^2} = \frac{-6}{{L_e}^2} + \frac{12x}{{L_e}^3} = B_1 \quad (7)$$

$$\frac{d^2N_2}{dx^2} = \frac{-4}{L_e} + \frac{6x}{{L_e}^2} = B_2 \quad (8)$$

$$\frac{d^2N_3}{dx^2} = \frac{6}{{L_e}^2} - \frac{12x}{{L_e}^3} = B_3 \quad (9)$$

$$\frac{d^2 4}{dx^2} = \frac{-2}{L_e} + \frac{6x}{{L_e}^2} = B_4 \quad (10)$$

Table 1 Geometry and Material Properties [2]

Dimensional	Parameter
Length of the beam, *l*	100(mm)
Width of the beam, *b*	20(mm)
Thickness of the substructure, *hs*	0.5(mm)
Thickness of PZT	0.4(mm)
Young's Modulus of the substructure, *Es*	100(GPa)
Young's Modulus of the PZT, *Ep*	66(GPa)
Density of the substructure layer, $\rho^{(1)}$	7165(kg/m3)
Density of the PZT layer, $\rho^{(2)}$	7800(kg/m3)
Piezoelectric constant, *d*31	-190(pm/V)
Permittivity, ε^s	15.93 (n F/m)

3. ELECTROMECHANICAL COUPLED FIELD PIEZOELECTRIC EQUATIONS

The electromechanical coupling of a piezoelectric can be demonstrated in two ways. In a harvesting mode, an applied mechanical pressure produces a proportional voltage response, which is known as the direct piezoelectric effect. Conversely, an applied voltage produces a deformation of the material, which is known as the indirect piezoelectric effect.

The direct effect and the converse effect may be modeled by the following matrix equations

Direct Piezoelectric Effect:

$$\{D\} = [e]^T\{S\} + [\varepsilon^S]\{E\} \quad (11)$$

Converse Piezoelectric Effect:

$$\{T\} = [c^E]\{S\} - [e]\{E\} \quad (12)$$

Where $\{D\}$ is the electric displacement vector, $\{T\}$ is the stress vector, $[e]$ is the dielectric permittivity matrix, $[c^E]$ is the matrix of elastic coefficients at constant electric field strength, $\{S\}$ is the strain vector, $[\varepsilon^S]$ is the dielectric matrix at constant mechanical strain, and $\{E\}$ is the electric field vector.

The constitutive equation for 1-dimensional form with constant electric field and strain

$$\begin{Bmatrix} T_1 \\ D_3 \end{Bmatrix} = \begin{bmatrix} c_{11}^E & -e_{31} \\ e_{31} & \varepsilon_{33}^S \end{bmatrix} \begin{Bmatrix} S_1 \\ E_3 \end{Bmatrix} \quad (13)$$

The substructure plane stress field equation is given by

$$T_1{}^{(1)} = c_{11}^{(1)} S_1^{(1)} \quad (14)$$

The piezoelectric plane stress field equation is given by

$$T_1{}^{(2)} = c_{11}^{(2)} S_1^{(2)} - e_{31}E_3 \quad (15)$$

$\vartheta(z) = \frac{z - z_n + h_p}{h_p}$ is the shape function over the interval $z_n - h_p \le z \le z_n$

$z_n = \frac{c_{11}^{(1)}{h_s}^2 + c_{11}^{(2)}{h_p}^2 + 2c_{11}^{(1)}h_s h_p}{2(c_{11}^{(1)}h_s + c_{11}^{(2)}h_p)}$ = distance from the neutral axis to the top layer of the beam with piezoelectric

$\varphi(z) = \vartheta(z)v_p$ is electrical potential

$\Omega(z)$ = first derivative of shape function $= \frac{d\vartheta(z)}{dz} = 1/h_p$,

The electric field equation

$$E_3 = -\,\Omega(z)v_p \quad (16)$$

$where\ v_p = voltage$

Substituting Equation (19) into Equation (23)

$$T_1{}^{(1)} = -zc_{11}^{(1)}\mathrm{B} \quad (17)$$

$$T_1{}^{(2)} = -zc_{11}^{(2)}\mathrm{B} + e_{31}\Omega(z)v_p(t) \quad (18)$$

$$D_3 = -ze_{31}\mathrm{B} - \varepsilon^S{}_{33}\,\Omega(z)v_p \quad (19)$$

Element Stiffness matrix is given by

$$[K] = EI\int [B^T]\,[B]dx \quad (20)$$

Where
$E = Youngs\ modulus,$
I = moment of inertia,
Element Mass matrix is given by

$$[M] = \int \rho[N^T]\,[N]dv \quad (21)$$

Damping matrix is given by

$$C = \alpha M + \beta K \quad (22)$$

Mechanical Force is given by

$$Q = \int \rho^{(1)}N^T dV^{(1)} + \int \rho^{(2)}N^T dV^{(2)} \quad (23)$$

$$F = -Q\ddot{w}_{base} \quad (24)$$

$Where\ Q = charge,$
$\ddot{w}_{base} \quad = \quad base\ excitation\ acceleration$

Electromechanical coupling is given by

$$P_{sr} = -\int z\Omega\,(z)^T e_{31}\mathrm{B}dV^{(2)} \quad (25)$$

Capacitance matrix is given by

$$P_D = -\int \Omega\,(z)^T \varepsilon_{33}\Omega(\mathrm{z})dV^{(2)} \quad (26)$$

Representation of equations in matrix form utilizing extended Hamilton's principle [3]

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \ddot{w} \\ \ddot{v_p} \end{Bmatrix} + \begin{bmatrix} C & 0 \\ P_{sr} & P_D \end{bmatrix}\begin{Bmatrix} \dot{w} \\ \dot{v_p} \end{Bmatrix} + \begin{bmatrix} K & P_{rs} \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} w \\ v_p \end{Bmatrix} = \begin{Bmatrix} F \\ i_p \end{Bmatrix} \quad (27)$$

$where\ i_p = current$

4. CONVERGENCE STUDY

Frequency convergence study with varying number of elements is carried to known the number of elements required to analyze the resonance frequencies to generate voltage. The dimensions and Material properties are shown Table [1]

Table 2. shows the frequencies for various elements. The frequencies are shown for first ten modes with various elements

Number of Elements	Frequencies									
	Mode 1	**Mode 2**	**Mode 3**	**Mode 4**	**Mode 5**	**Mode 6**	**Mode 7**	**Mode 8**	**Mode 9**	**Mode 10**
1	48.04	473.29								
2	47.83	302.16	1021.95	2966.14						
3	47.81	300.6	849.38	1912.78	3599.86	7176.74				
4	47.81	299.96	845.43	1667.84	3102.11	4982	7898.13	12959.16		
5	47.81	299.76	841.94	1663.25	2760.58	4586.08	6707.18	9726.89	13817.78	20326.69
10	47.81	299.63	839.14	1645.54	2724.45	4081.54	5726.6	7672.06	9919.88	12331.84
15	47.81	299.62	838.97	1644.29	2719.03	4064.31	5682.58	7577.79	9755.77	12224.45
20	47.81	299.62	838.94	1644.07	2718.06	4061.15	5674.16	7558.45	9716.12	12150.15
25	47.81	299.62	838.94	1644.01	2717.79	4060.26	5671.76	7552.88	9704.52	12127.98
30	47.81	299.62	838.93	1643.99	2717.69	4059.93	5670.88	7550.84	9700.23	12119.72
35	47.81	299.62	838.93	1643.98	2717.65	4059.79	5670.51	7549.95	9698.36	12116.1
40	47.81	299.62	838.93	1643.98	2717.63	4059.72	5670.32	7549.51	9697.44	12114.33
45	47.81	299.62	838.93	1643.97	2717.62	4059.69	5670.22	7549.28	9696.95	12113.38
50	47.81	299.62	838.93	1643.97	2717.61	4059.67	5670.16	7549.15	9696.67	12112.83

The first mode of frequency reaches the converged result for increasing number of elements to 3. The second mode of frequency reaches the converged result for 15 elements while third mode of frequency converges to 30 elements.. If the analysis of the model only requires the first 2 natural frequencies, 15 discretized finite elements will be sufficient to show the correct resonance frequency value.

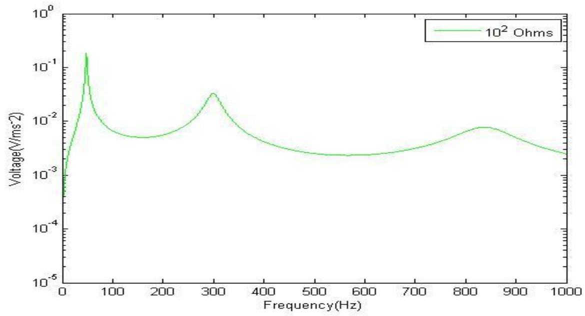

Figure 2. Voltage v/s Frequency for Resistance of 100 ohms

Figure 2. Shows the plot of voltage verses Frequency of a unimorph beam for resistance of 100 ohms. Here, At first natural frequency the voltage generated is 0.1798V and at Second natural frequency the voltage generated is 0.032 V.

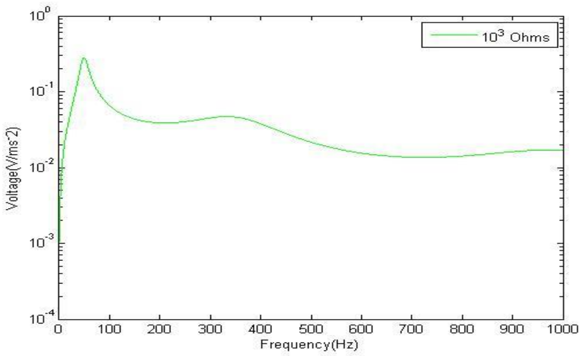

Figure 3. Voltage v/s Frequency for Resistance of 1000 ohms

Figure 3.Shows the plot of voltage verses Frequency of a unimorph beam for resistance of 1000 ohms. Here, at first natural frequency the voltage generated is 0.2817V and at Second natural frequency voltage generated is 0.047 V.

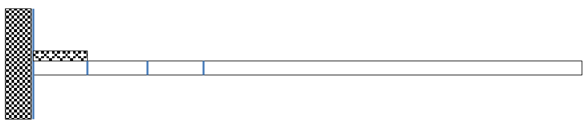

Figure 4 .Cantilever beam with Pzt on individual element

The base beam is divided into 15 elements and PZT patch is bounded on individual element from fixed end to free end.

Table 3. Gives the list of First, Second and Third natural frequencies, when the PZT is bounded on individual beam element. Plot is shown in Figure 5, 6 and 7 respectively

Table 3: Mode frequencies

PZT On Beam Element	1st Mode Frequency	2ndMode Frequency	3rd Mode Frequency
1	47.5	295.7	822
2	46.9	284.1	777.2
3	46.3	277.3	761.7
4	45.8	273.6	759.1
5	45.4	271.9	762.3
6	44.9	271.6	768.2
7	44.5	272.5	774.2
8	44.1	274.3	777.9
9	43.6	276.8	778.8
10	43.2	279.2	779.2
11	42.7	280.9	782.1
12	42.1	280.6	787.2
13	41.5	277.3	788.3
14	40.9	270.6	775.7
15	40.2	260.4	743.1

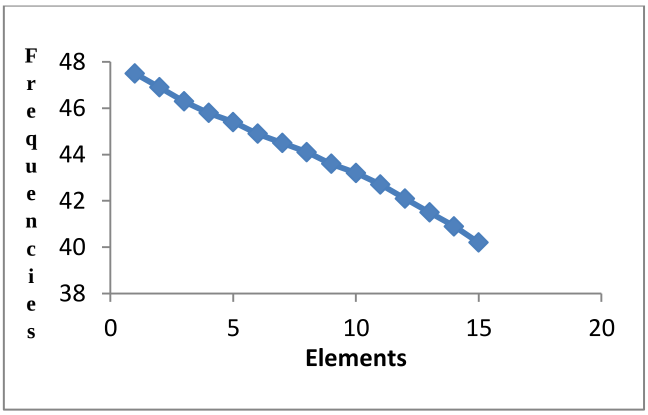

Figure 5. First Mode Frequencies

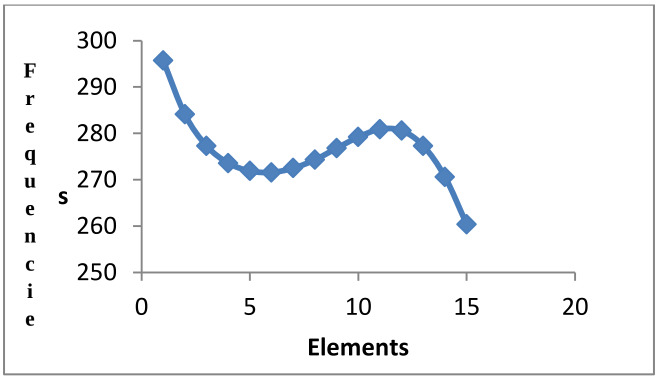

Figure 6. Second Mode Frequencies

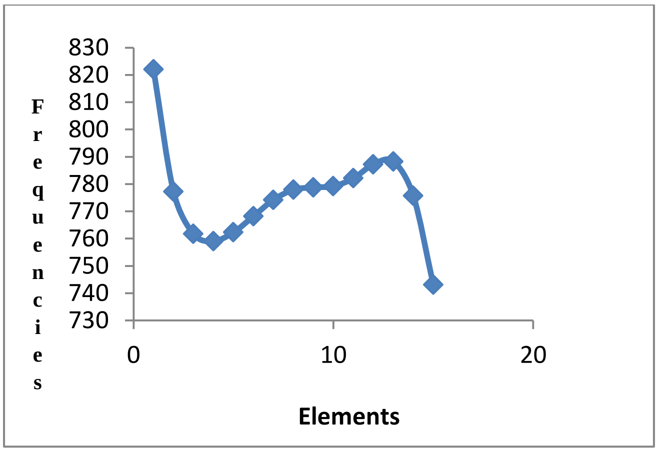

Figure 7. Third Mode Frequencies

Table 4 gives the voltage generated, when the PZT is bounded on individual beam element.

Table 4:Voltage generated	
PZT On Beam Element	**Voltage**
1	0.21
2	0.215
3	0.218
4	0.22
5	0.222
6	0.223
7	**0.2231**
8	0.2227
9	0.2221
10	0.22
11	0.2193
12	0.2174
13	0.2151
14	0.2125
15	0.2097

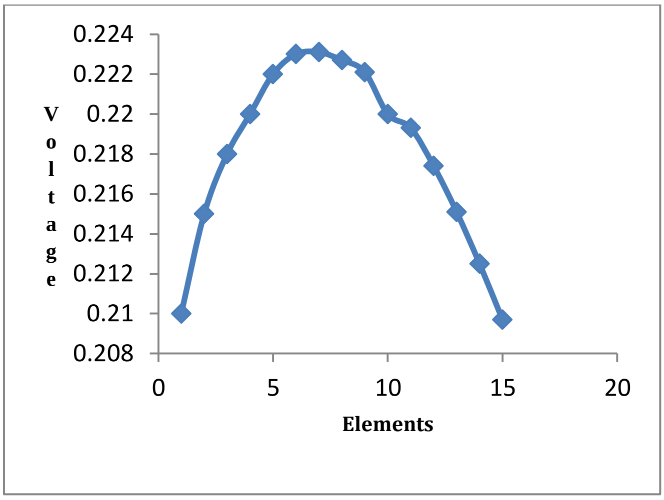

Figure 8. PZT on Individual Element

Figure 8 shows the variation of volts verses elements. Here the PZT patch is bounded on each individually element at 100 ohms. The maximum volt 0.2231 is obtained when the PZT patch is on 7th element.

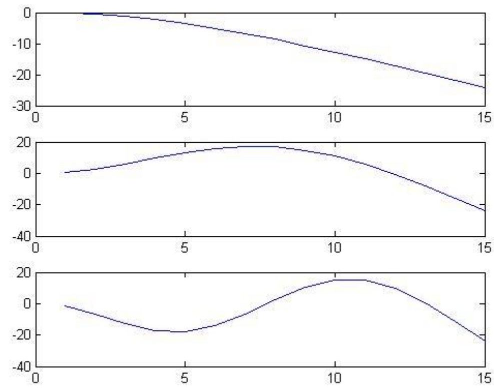

Figure 9. Mode Shapes PZT on 1st element

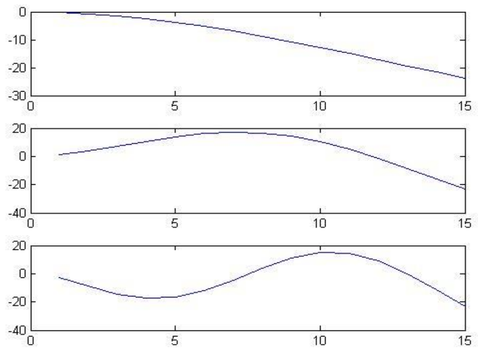

Figure 10. Mode Shapes PZT on 2nd element

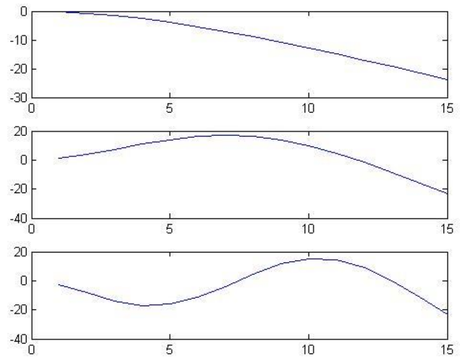

Figure 11. . Mode Shapes PZT on 3rdelement

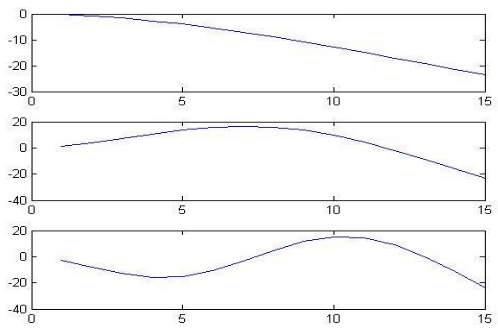

Figure 12. Mode Shapes PZT on 4th element

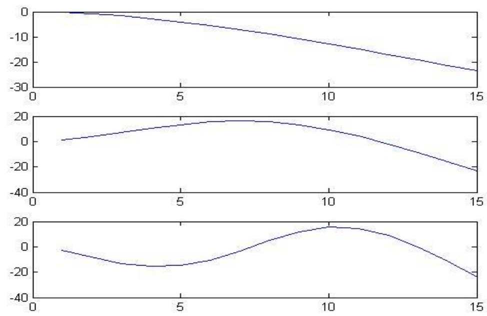

Figure 13. Mode Shapes PZT on 5th element

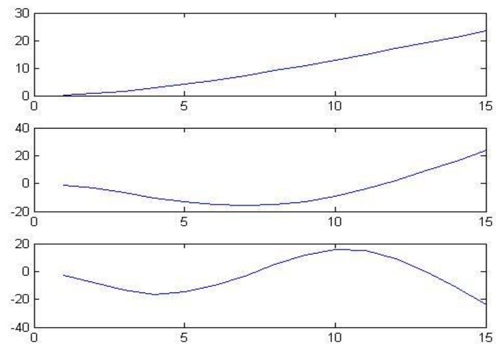

Figure 14. Mode Shapes PZT on 6th element

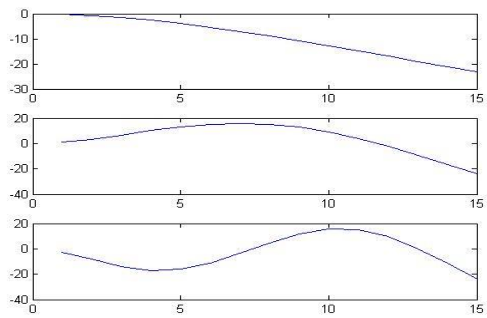

Figure 15. Mode Shapes PZT on 7th element

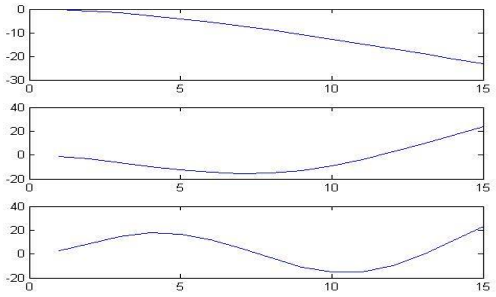

Figure 16. Mode Shapes PZT on 8th element

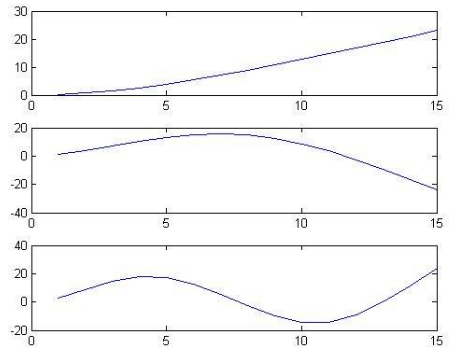

Figure 17. Mode Shapes PZT on 9th element

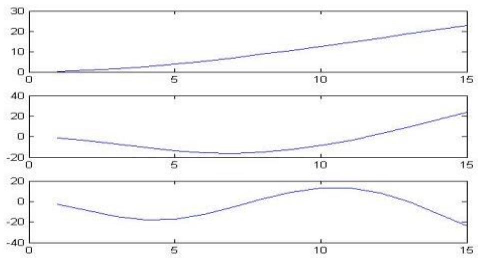

Figure 18. Mode Shapes PZT on 10th element

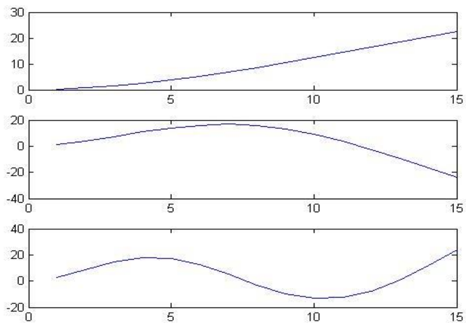

Figure 19. Mode Shapes PZT on 11th element

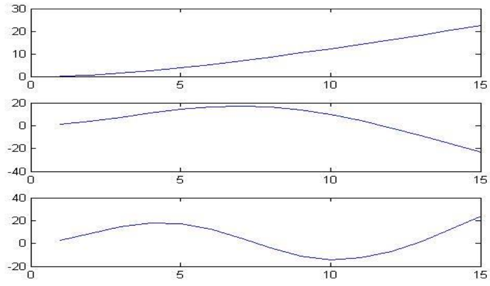

Figure 20. Mode Shapes PZT on 12th element

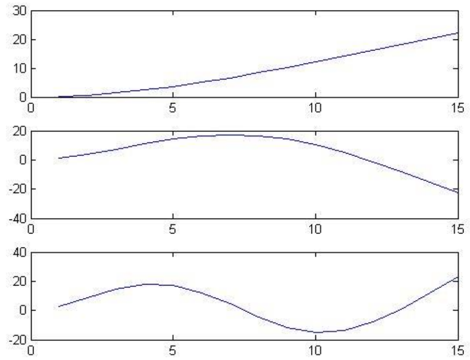

Figure 21. Mode Shapes PZT on 13th element

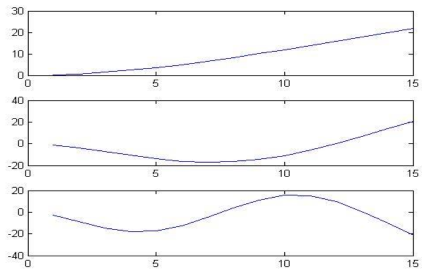

Figure 22. Mode Shapes PZT on 14th element

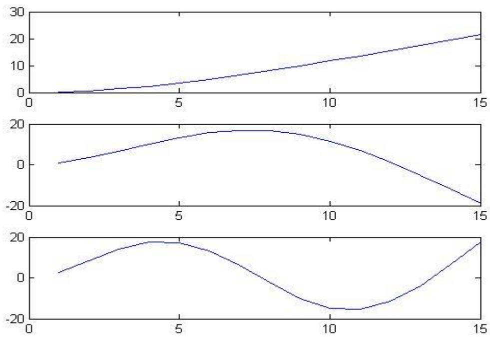

Figure 23. Mode Shapes PZT on 15th element

Figure 9-23 represents the First, Second and third mode shape, when PZT is bounded on individual element from 1 to 15th element respectively.

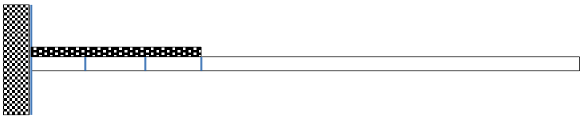

Figure 24. Cantilever beam with PZT length increased from fixed end

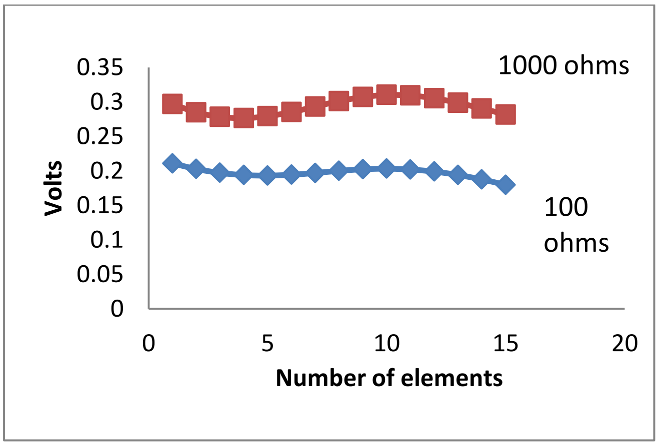

Figure 25. Voltage for 100 and 1000ohms

Figure 25. Shows Plot of voltage generated for the resistance of 100 ohms and 1000 ohms, Here the PZT bounded on substructure is increased from fixed end with the elements 1 to 15 as shown in Figure 24.From the graph, it shows that when the PZT is on first element the voltage is 0.211V, which is maximum when the resistance is 100 ohms and for 1000ohms the maximum voltage generated is 0.3102V, when the PZT is bounded from first to 10th element

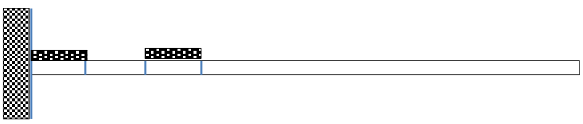

Figure 26. Cantilever beam with Pzt on First and Third element

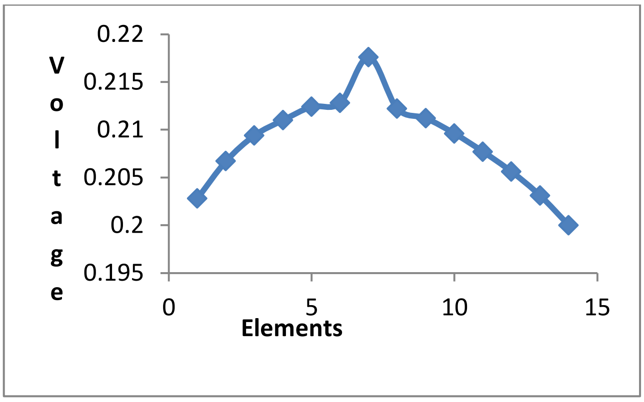

Figure 27.PZT on 2 elements

Figure 27.Gives the voltage generated, when the PZT patch is on both First element along with Second, First element along third similarly upto 15th element. Maximum voltage 0.2176V is obtained when the PZT is on First and eighth element.

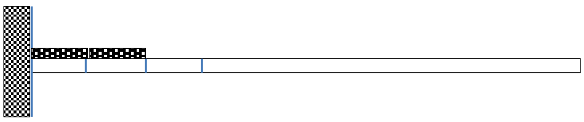

Figure 28. Cantilever beam with PZT on Successive two elements

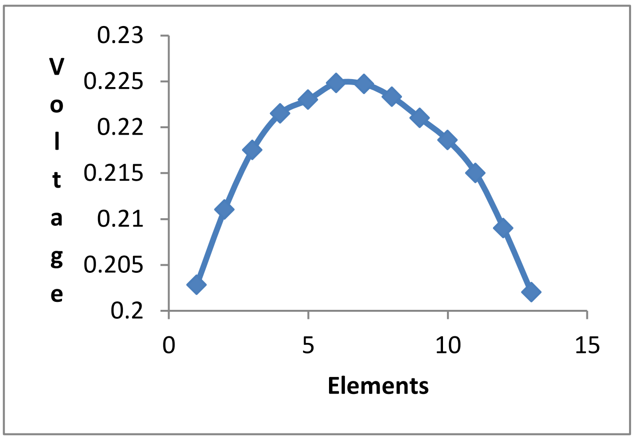

Figure 29. PZT on combination of 2 successive elements

Figure 29.Gives the voltage generated, when the PZT patch is on pair of 2 successive elements. Maximum voltage 0.2248V is obtained when the PZT is on 6th and 7th element.

Beam bounded with PZT throughout the length of base beam, generates 0.1798V of voltage.

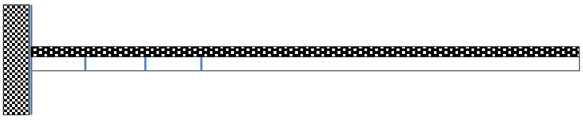

Figure 30. Cantilever beam with Pzt along the length of base beam

5. CONCLUSIONS

The Finite element modeling of the cantilever beam bounded with piezoelectric patch is carried out in Matlab to study the Frequency convergence with varying number of elements and voltage generated when the PZT is bounded continuously from fixed end to free end, PZT mounted on individual element and 2 PZT elements bounded on base beam.

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