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Nonlinear Determination of the Effective Flexural Rigidity of Reinforced Concrete Beams

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Abstract - *Deflection control in reinforced concrete beams is an important design step to satisfy serviceability limit state. In most current Codes the deflection of RC beams is determine using the effective moment of inertia formula. The formula of the effective moment of inertia is approximately the same in most Codes. The other way for deflection calculation is the double integration of moment curvature curve along the beam length. This method needs more effort and time, but it gives more accurate results for deflection. In this paper a comparison between deflection values of reinforced concrete beams obtain for some beams in pervious experimental work, Code approach and moment curvature curve double integration is carried out. This comparison shows good agreement of the deflection values obtained by moment curvature double integration with the experimental results. While the values obtained using code approach were conservative. A parametric study has been performed to obtain the effective moment of inertia based on the results of the moment curvature double integration procedure. The parameters considered in this study were: concrete compressive strength, tension steel percentage, compression steel percentage, beam span to depth ratio. On the bases of the parametric study an empirical model is proposed for the determination of the effective moment of inertia for the calculation of the deflection of RC beams.*

Key words: Deflection, Moment-Curvature, Nonlinear analysis, RC Beams, Effective moment of inertia.

1. INTRODUCTION

Determination of RC beams' deflections in most current concrete design Codes (ACI 318-14 [1], ACI 318-19 [2], CSA A23.3-14 [3] and SBC-304 [4], is performed using a constant effective moment of inertia (Ie). This method is mainly leading to conservative results. The other method that can be used, is the integration of curvature along the span [5]. In this case, the corresponding moment of inertia to each beam section is used. Theoretically, this can give better results for the predicted deflection. However, the integration of curvature requires more computational effort.

Biscoff [5,6] carried out a study to compare the deflection predicted by ACI Codes [1,2] (using a constant average value for the effective moment of inertia) and by the integration of section curvature. It was concluded that an integration based on the equivalent moment of inertia gives a stiffer response that improves prediction of deflection as compared to a section based on constant effective moment of inertia (ACI Code approaches). Also, the use of an effective moment of inertia based on member stiffness at the critical section gives a reasonably conservative result. Obozov and Elgohary (2008) [7], carried out a study to compare the deflection calculated using Codes of different countries. It was found that the ACI-318-03 Code method gave the most conservative results compared with others. Also, in this study, an empirical formula for the determination of both gross and effective moment of inertia was suggested.

This study aims to obtain an empirical formula for the calculation of an equivalent moment of inertia based on the integration of beam curvature to minimize the computational effort of this method. The results obtained by the curvature integration method are compared with the results obtained by the ACI Codes equations along with some experimental results.

2. METHOD OF ANALYSIS

The method adopted in ACI-318-14,[1] for calculation of the effective moment of inertia is given as:

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \qquad (1)$$

Where I_g is the gross amount of inertia (without considering the steel);

I_{cr} is the transformed moment of inertia of the cracked section;

M_{cr} is the cracking moment, $M_{cr}=f_r I_g/(h/2)$ with $f_r=0.7(f_c')^{0.5}$;

and M_a is the maximum service-load moment occurring for the condition under consideration.

In ACI-318-19 [2], a modified formula is adopted for the effective moment of inertia is given in the following form:

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{(2/3)M_{cr}}{M_a}\right)^2 \qquad (2)$$

Chart 1 shows a comparison between the ratio I_e/I_g of equation (1) and Equation (2). The two equations give similar results when M_a is more than twice that of M_{cr} the same results. The modification adopted in ACI 318-19 has no change in the calculated effective moment of inertia except for the cases of $M_a=2/3M_{cr}$ and $M_a=M_{cr}$.

The method used in the current analysis is based on the integration of the curvature. The deflection-moment relationship has the form [8]:

$$\Delta = \iint \frac{M}{EI} dx = \iint \frac{1}{R} dx = \iint \phi \, dx \qquad (3)$$

Where

ϕ and R – are the curvature and radius of curvature, respectively.

The moment-curvature relationship will be determined for all models considered in this study, using Response 2000 software [9]. Then the deflection will be calculated using the double integration given by Equation (3).

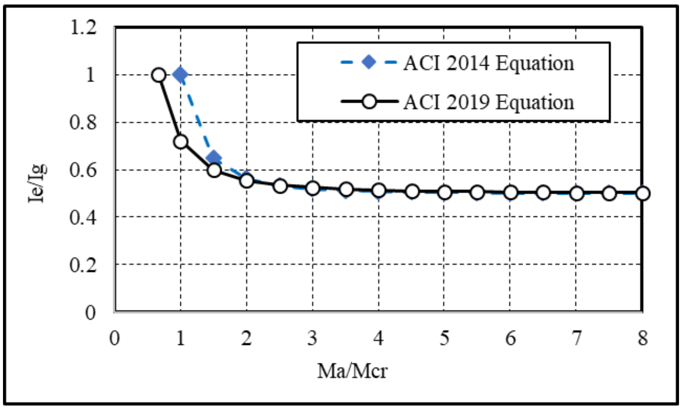

Chart -1: Comparison of the effective moment of inertia in ACI Code 2014 and 2019

3. VERIFICATION OF CURVATURE INTEGRATION METHOD

The results of experimental work carried out by Issa (2009) [10] were used to verify the results of the proposed nonlinear analysis method, and with the results obtained using the ACI-318-19 Code approach. Three models were selected for this comparison with properties shown in Table 1. The details of the experimental work and results for the selected models are presented in detail in [10].

Table -1: Selected models of experimental work (Issa (2009) [10])

Model	Section (bxh) mm	As	As'	fc'
B2	100X300	2ϕ16	2ϕ10	66
B4	100X300	2ϕ18	2ϕ10	65
B6	100X300	2ϕ16	2ϕ10	65

The results of the verification study are shown in Figs 2 to 4. The deflections obtained, using the double integration of moment curvature curve along the beam span, for the three selected models are very close to the test results [10], while the ACI-318-19 Code equation gives larger values. From these Charts (2 to 4), it is noted that, the deflection obtained by the ACI equation is about 1.45 the experimental results. While this ratio is about 0.96 for the results obtained by the double integration of moment curvature curve. The ratio of deflection determined by ACI equation and the results of the proposed nonlinear analysis is 1.5. The results of the proposed method are very close to the experimental ones.

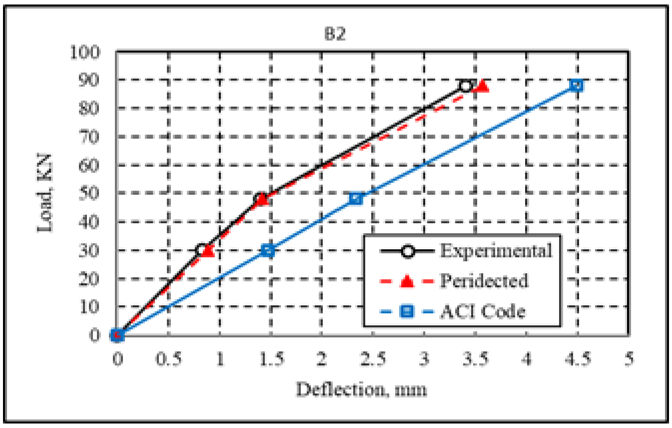

Chart -2: Comparison of Results for beam B2 [10].

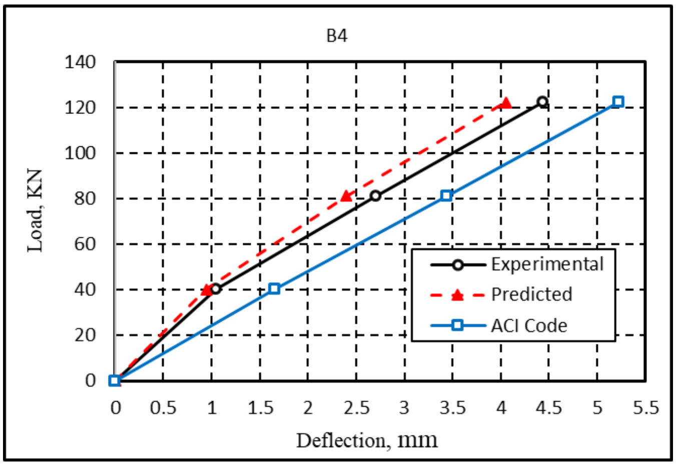

Chart -3: Comparison of Results for beam B4 [10].

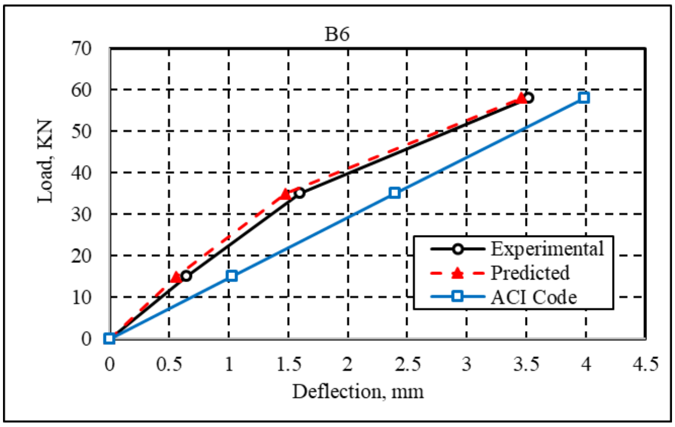

Chart -4: Comparison of Results for beam B6 [10].

4. PARAMETRIC STUDIES

Two parametric studies had been carried out in this research. The first study was performed to obtain empirical expression to predict the gross moment of inertia for the transformed beam section. In this case, a wide number of beam sections with various values of beam height equals 400; 500; 600; 700; 800; 900; and 1000 mm. The beam width was chosen to be 250 mm for all models. The tension-steel percentage was taken in the range from 0.25% to 2.0% with an interval of 0.25%. The compression steel ratio was chosen to be, 0.0; 0.1; 0.2; 0.3; 0.4; and 0.5 of the tension steels. Concrete cylinder compressive strength (fc') had the values 21; 25; 28; 32; 34; 40 MPa. For all these values, the gross moment of inertia of the transformed concrete section was determined.

The second parametric study had been carried out on a group of simply supported beams with spans of 6; 8; 10 m. The span to depth ratio was chosen to be 10 and 12. Beam section width is 250 mm for all cases. The tensile reinforcement percentage used in the parametric study was chosen to start with value 0.33 (minimum for flexure); 1.16; 1.8; and 2.125% (maximum for flexure). The compression steel used in the study was chosen as a ratio of tensile steel and had three values, 0.0; 0.25; and 0.5. In all models, the concrete compressive strength was 28 MPa and steel yield stress was 420 MPa. The deflection of all models was determined using the proposed nonlinear analysis along with the ACI Code equation.

5. ANALYSIS OF RESULTS

With the comparison between the results obtained using curvature integration method and the results obtained using the ACI Code formula, the relation between deflection and tension steel percentage is presented in Chart 5, for the samples with span to depth ratio, equals 10. From Figure 5, it can be observed that the ratio of the deflection values calculated using ACI Code equations and the values obtained by the proposed method has is about 1.96 (approximately two). Figure 5 also shows that the deflection of reinforced concrete beams decreases with the increase of the tensile steel percentage at the same load. The deflection decreases sharply at the lower values and becomes steady at the large values of reinforcement ratios (as also shown in Chart 6, for the cases of span to depth ratio equals 12).

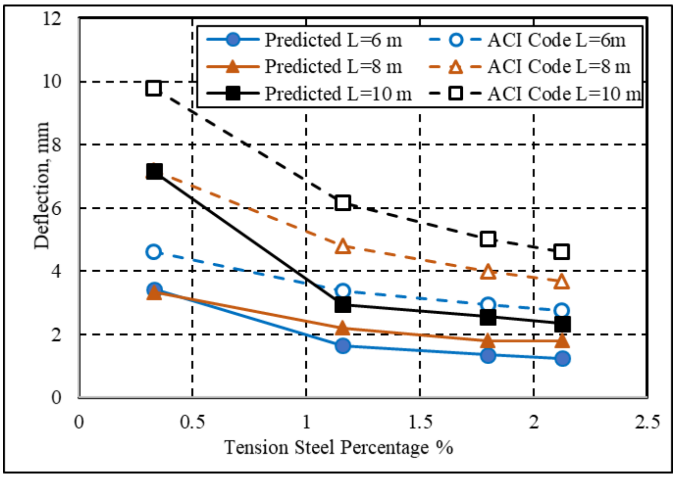

Chart -5: Effect of Tension Steel Percentage on the Deflection Predicted by ACI and the Proposed Method (L/h=10)

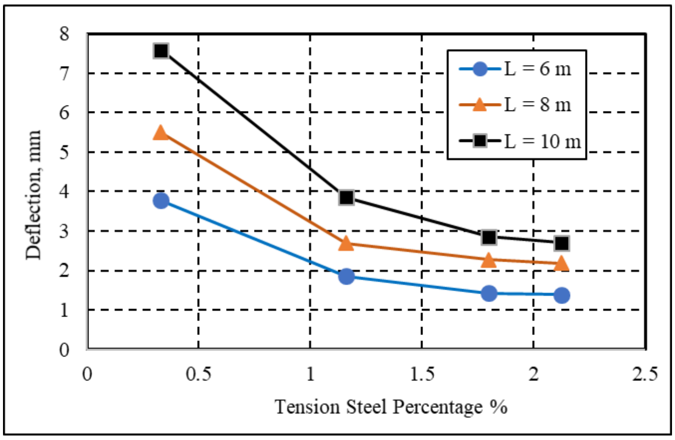

Chart -6: Effect of Tension Steel Percentage on the Deflection Predicted by ACI and the Proposed Method (L/h=12)

The effect of compression steel is shown in Chart 7. The effect of the compression steel on the deflection is not significant and can be considered linear.

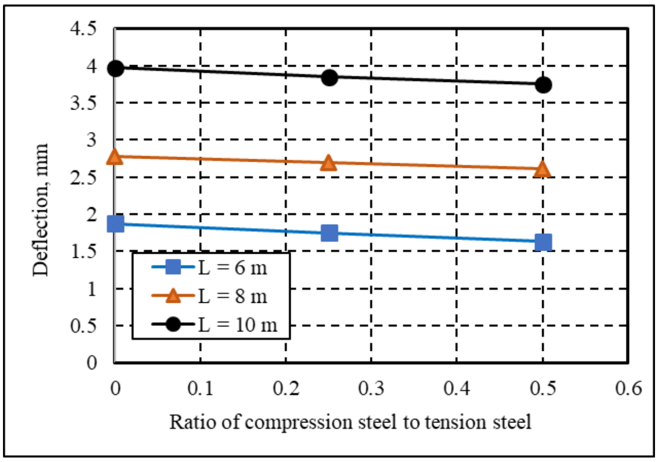

Chart -7: Effect of Compression Steel Percentage on the predicted deflection

The equivalent moment of inertia based on the integration of curvature can be determined from deflection value at any stage from the equation:

$$I_e = \frac{5\,W\,L^4}{384\,E_c\,\delta} \qquad (4)$$

6. PROPOSED EQUATION FOR THE EFFECTIVE MOMENT OF INERTIA

The main variable is the gross moment of inertia of the transformed section and the indepented variables are tension steel percentage (ranges from 0.25 to 2%,), compression steel percentage (ranges from 0.0 to 0.5 of the tension steel percentage) and the modular ratio (concrete compressive strength 21- 40MPa). Scatter plot of the relation between the main variable and indepented variables shows that the relations are not linear, and the main variable increases with the increase of any of the individual variables. For that, nonlinear regression analysis is applied to the results to get the best fitting [11]. An expression for the gross moment of inertia for the concrete transformed section is obtained and has the form:

$$I_g = 0.881 \times \rho^{0.087} \times (\rho')^{0.075} \times n^{0.204}\left(\frac{bh^3}{12}\right) \qquad (5)$$

where

$$\rho = \frac{A_s}{bd}\% - \textit{tension steel percentage}$$

$$\rho' = \frac{A_s'}{bd}\% - \textit{compression steel percentage}$$

$$n = \frac{E_s}{E_c} - \textit{modular ratio}$$

Chart 8 shows the ratio of the gross moment of inertia of the transformed section calculated using the proposed empirical equation (5) and the exact values. The average ratio is 1.0014, with a maximum ratio of 1.052 and a minimum ratio of 0.87.

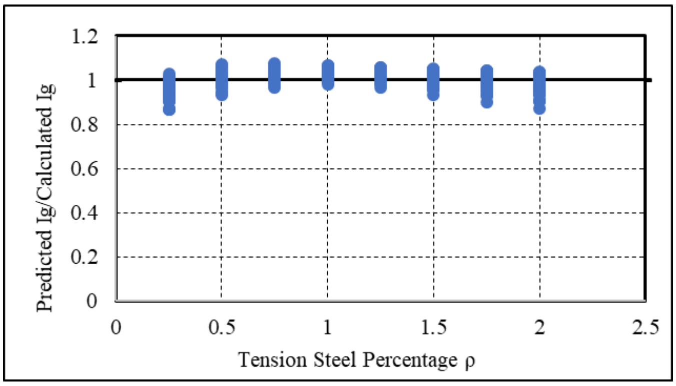

Chart -8: Ratio of the Predicted Gross Moment of Inertia to the Calculated Value

Nonlinear regression analysis had been applied to the results of the second parametric study and two relations for the determination of the effective moment of inertia of the RC beams based on curvature integration had been obtained. The first formula relates the effective moment of inertia with the gross moment of inertia in equation (3) and has the form:

$$I_e = 1.18 \times \rho^{0.399} \times (1 - 0.01 \times \rho')\left(\frac{M_{cr}}{M_a}\right)^{0.246} \times I_g \qquad (6)$$

The ratio of the value of the effective moment of inertia determined using the empirical equation (6) and the value obtained from the integration of the curvature is shown in Chart 9. The average ratio is 0.997; the maximum ratio is 1.155 and the minimum is 0.849.

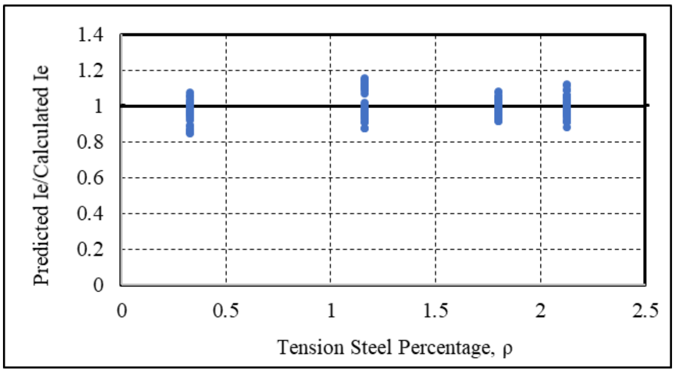

Chart -9: Ratio of Predicted Effective Moment of Inertia by Eq. (6) to the Calculated Value

The second equation is relating the effective moment of inertia to the gross moment of the concrete section only and has the form:

$$I_e = 1.351 \times \rho^{0.489} \times (1 + 0.157 \times \rho')\left(\frac{M_{cr}}{M_a}\right)^{0.241} \times \left(\frac{bh^3}{12}\right) \qquad (7)$$

The values of the effective moment of inertia obtained using Equation (7) are compared with the values obtained from the second parametric study, and the comparison is shown in Chart 10. The average ratio obtained is 1.012, while the maximum ratio is 1.163 and the minimum is 0.859.

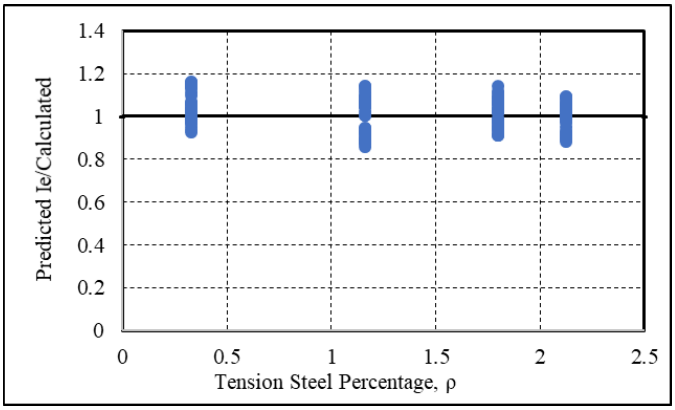

Chart -10: Ratio of Predicted Effective Moment of Inertia by Eq. (7) to the Calculated Value

7. CONCLUSIONS

1. The modification adopted in ACI-318-19 for the calculation of effective moment of inertia gives the same results as the formula used in the previous

version of the ACI 318-14 for the all cases of acting moment greater than twice cracking moment.

2. Generally, ACI approach gives conservative values for deflection of reinforced concrete beams about 1.45 the experimental results.
3. The proposed method for deflection calculations, integration of the beam curvature, gives more relevant results to experimental ones, than those values calculated by ACI equations.
4. Based on the results of the parametric study, two equations are proposed for the calculation of the effective moment of inertia.

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