NUMERICAL SOLUTIONS FOR THE FLOW OF CONDUCTING POWER – LAW FLUID IN A TRANSVERSE MAGNETIC FIELD AND WITH A PRESSURE GRADIENT USING PETROV-GALERKIN METHOD

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Abstract - *The paper deal the study of steady two-dimensional incompressible flow of a conducting Power-law fluid past a flat plate in the presence of transverse magnetic field under the influence of a pressure gradient is consider. The non linear ordinary differential equation converted in to linear ordinary differential equation by using the technique of Quasilinearization. Apply Petrov-Galerkin Method to solve the given differential equations. The energy equation for a special case for which solution exists is also considerable. Apply Petrov-Galerkin Method to check the skin friction value for given different parameters and also compare the values of skin friction with the exact solutions.*

Keywords*: *Non-Newtonian Fluids, Magnetic field, Pressure gradients, Quasilinearization, Petrov-Galerkin Method

1. INTRODUCTION

Different fluids main basely Newtonian and non Newtonian. Newtonian fluids are based on a linear relationship between the shear stress and the strain-rate. Non-Newtonian model which has been widely studied uses the following nonlinear relationship between the stress components and strain-rate components.

$$\tau_{ij} = \mu \left| \sum_{m=1}^{3} \sum_{l=1}^{3} e_{lm} e_{ml} \right|^{(n-1)/2} e_{ij}$$

Here μ and n are known as the consistency and flow behaviour indices of the fluid. Fluids are known as power-law fluids. n > 1 represents the fluid is described as dilatants, n < 1 represents pseudo- plastic and n = 1represents the Newtonian fluid.

Schowalter [4] applied the concept of boundary layer to power-law fluid. Kapur and Srivastava [5], [6] Lee and Ames obtained the similarity solution. Djukie [1] [2][3] employed Crocco equation to study the unsteady boundary layer flows with external velocity proportional to e^{at}.Sapunkov[8] derived equation governing the self similar solution. Cobble [7] later derived the same equation.

Exact solutions of the equations of motion of power-law non–Newtonian fluids are difficult to obtain. The difficulty arises not only due to the nonlinearities but also due to the order of the differential equations. The aim of this work is to solve the problem of a two-dimensional, steady, incompressible power-law non–Newtonian electrically conducting fluid past a continuously moving surface in the presence of a transverse magnetic field B(x) numerically by Petrov-Galerkin Method.

2. Formulation of the problem

Sapunkov [7], consider a power- law fluid past a semi-infinite flat plate under the effect of a pressure gradient and in the present of a transverse magnetic field with the field of magnetic intensity H. The continuous and momentum equation are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_\infty \frac{du_\infty}{dx} + \frac{K}{\rho}\frac{\partial}{\partial y}\left[\left(\frac{\partial u}{\partial y}\right)^{n-1}\frac{\partial u}{\partial y}\right] + \frac{\sigma\mu^2}{\sigma}(u_\infty - u)H^2$$

(2)

In the above equations x and y are known as Cartesian coordinates to the body surface.u and v are known as velocity profiles.k is known as the fluid consistency and n is the flow behaviour index.

The external velocity distribution is given below

$$u_\infty = cx^m$$

(3)

Here c and m are known as constant. The magnetic Reynolds number is supposing so small. Sapunkov [7] has shown that similarity solutions exist if

$$H = H_0 x^{(m-1)/2}$$

Let us consider the similarity variable given below

$$\eta = \alpha_1 y x^{\alpha_2} \quad (4)$$

$$\varphi = \frac{c}{\alpha_1} x^{r/(1+n)} F(\eta)$$

$$\alpha_2 = \frac{m(2-n)-1}{n+1}$$

$$r = (2n-1)m+1$$

$$\alpha_1 = [\frac{\rho c^{2-n}}{k} \frac{m(2n-1)+1}{n(n+1)}]^{\frac{1}{n+1}}$$

we obtain the following similarity equation:

$$f''' f''^{(n-1)} + ff'' + \beta(1-f'^2) + M(1-f') = 0 \quad (5)$$

Subject to the boundary condition

$$f(0)=0, f'(0)=0, f'(\infty)=1 \quad (6)$$

3. Numerical Solution

Due to Quasilinearization techniques the equation (5) and (6) can be written as

$$f''' + \left[ff''^{(1-n)} + (1-n)\left(f''\right)^{-n} ff'' + (1-n)\left(f''\right)^{-n}\beta(1-(f')^2 + (1-n)\left(f''\right)^{-n} M(1-f')\right] f'' +$$
$$\left[-2\beta f' f''^{(1-n)} - M\left(f''\right)^{(1-n)}\right] f' + \left[f''(f'')^{(1-n)}\right] f - 2ff''^{(2-n)} + f'f''^{(1-n)}\left[2\beta f' + M\right]$$
$$+ nf''^{(1-n)}\left[ff'' + \beta(1-f'^2) + M(1-f')\right]$$

With the boundary condition

$$f(0)=0, f'(0)=0, f'(\infty)=1 \quad (7)$$

Where $\beta = \frac{m(1+n)}{r}, M = \frac{\sigma\mu^2 H_0^2(1+n)}{c\sigma r}$ n=0.5 take and n=1.5 (two problem are exist for different n)

We take $f(\eta)$ as before to satisfy the boundary conditions and condition given in equation no.6 and take $\infty = 5$ to restricted the interval [0, 5].

The Residue is given by

$$f''' + \left[\left(\frac{\eta^2}{10}\right)\left(\frac{1}{5}\right)^{(1-n)} + (1-n)\left(\frac{1}{5}\right)^{-n}\left(\frac{\eta^2}{10}\right)\left(\frac{1}{5}\right) + (1-n)\left(\frac{1}{5}\right)^{-n}\beta\left(1-\frac{\eta^2}{25}\right) + (1-n)\left(\frac{1}{5}\right)^{-n} M(1-\frac{\eta}{5})\right] f'' +$$
$$\left[-2\beta\frac{\eta}{5}\left(\frac{1}{5}\right)^{(1-n)} - M\left(\frac{1}{5}\right)^{(1-n)}\right] f' + \left[\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)^{(1-n)}\right] f - 2\left(\frac{\eta^2}{10}\right)\left(\frac{1}{5}\right)^{(2-n)} + \left(\frac{\eta}{5}\right)\left(\frac{1}{5}\right)^{1-n}\left[2\beta\left(\frac{\eta}{5}\right) + M\right]$$
$$+ n\left(\frac{1}{5}\right)^{1-n}\left[\left(\frac{\eta^2}{10}\right)\left(\frac{1}{5}\right) + \beta\left(1-\frac{\eta^2}{25}\right) + M\left(1-\frac{\eta}{5}\right)\right]$$

(8)

So that, $\phi_1 = \left(\frac{\eta^2}{10}\right), \phi_2 = \left(\frac{\eta^3}{75}\right), \phi_3 = \left(\frac{\eta^4}{500}\right)$ (9)

$$f(\eta) = (0.0133\eta^3) + c_2(\eta^2 - 0.13333\eta^3) + c_4(\eta^4 - 6.66666\eta^3) + c_5(\eta^5 - 41.66666\eta^3)$$

(10)

$$f'(\eta) = (0.0399\eta^2) + c_2(2\eta - 0.3999\eta^2) + c_4(4\eta^3 - 20\eta^2) + c_5(5\eta^4 - 125\eta^2) \quad (11)$$

$$f''(\eta) = (0.0799\eta) + c_2(2 - 0.7999\eta) + c_4(12\eta^2 - 40\eta) + c_5(20\eta^3 - 250\eta) \quad (12)$$

$$f'''(\eta) = (0.0799) + c_2(-0.7999) + c_4(24\eta - 40) + c_5(60\eta^2 - 250)$$

(13)

Equation (10) to (13) putting in eq. (7) then it becomes

$$R(\eta)$$
$$= [(0.0799) + c_2(-0.7999) + c_4(24\eta - 40) + c_5(60\eta^2 - 250)] +$$
$$\left[\left(\frac{\eta^2}{10}\right)\left(\frac{1}{5}\right)^{(1-n)} + (1-n)\left(\frac{1}{5}\right)^{-n}\left(\frac{\eta^2}{10}\right)\left(\frac{1}{5}\right) + (1-n)\left(\frac{1}{5}\right)^{-n}\beta\left(1-\frac{\eta^2}{25}\right) + (1-n)\left(\frac{1}{5}\right)^{-n} M(1-\frac{\eta}{5})\right]$$
$$\left((0.0799\eta) + c_2(2 - 0.7999\eta) + c_4(12\eta^2 - 40\eta) + c_5(20\eta^3 - 250\eta)\right)$$
$$+ \left[-2\beta\frac{\eta}{5}\left(\frac{1}{5}\right)^{(1-n)} - M\left(\frac{1}{5}\right)^{(1-n)}\right]\left((0.0399\eta^2) + c_2(2\eta - 0.3999\eta^2) + c_4(4\eta^3 - 20\eta^2) + c_5(5\eta^4 - 125\eta^2)\right)$$
$$+ \left[\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)^{(1-n)}\right]\left((0.0133\eta^3) + c_2(\eta^2 - 0.13333\eta^3) + c_4(\eta^4 - 6.66666\eta^3) + c_5(\eta^5 - 41.66666\eta^3)\right) - 2\left(\frac{\eta^2}{10}\right)\left(\frac{1}{5}\right)^{(2-n)}$$
$$+ \left(\frac{\eta}{5}\right)\left(\frac{1}{5}\right)^{1-n}\left[2\beta\left(\frac{\eta}{5}\right) + M\right] + n\left(\frac{1}{5}\right)^{1-n}\left[\left(\frac{\eta^2}{10}\right)\left(\frac{1}{5}\right) + \beta\left(1-\frac{\eta^2}{25}\right) + M\left(1-\frac{\eta}{5}\right)\right]$$

(14)

$R(\eta)$ of equation (14) integrate with below equation with

$$\int_0^5 R(\eta)\phi_1(\eta)d\eta \quad \int_0^5 R(\eta)\phi_2(\eta)d\eta \quad \int_0^5 R(\eta)\phi_3(\eta)d\eta$$

Where $\phi_1(\eta)$ $\phi_2(\eta)$ $\phi_3(\eta)$ given in (9) using in above integral and using the MATLAB we obtain the equation with unknown constants c_2, c_4, c_5 and β ,m and the value of n.

Table -1: Table values for skin friction for n=0.5

β	M	*F"(0)* (Crocco)	*F"(0)*(Petrov - Galerkin)
0.0	0.0	0.2583	0.273 5
	2.0	1.3666	1.375 9
	4.0	2.1241	2.131 0
	6.0	2.7939	2.769 8
	8.0	3.3366	3.341 8
	10.0	3.8637	3.868 5
0.5	0.0	0.7175	0.729 8
	2.0	1.6389	1.646 8

	4.0	2.3473	2.353 7
	6.0	2.9612	2.966 7
	8.0	3.5168	3.521 8
	10.0	4.0316	4.036 2
1.0	0.0	1.0684	1.077 6
	2.0	1.8901	1.897 1
	4.0	2.5604	2.566 2
	6.0	3.1520	3.157 3
	8.0	3.6925	3.697 4
	10.0	4.1960	4.200 5

Table -2: Table values for skin friction for n=1.5

β	M	$F''(0)$ (Crocco)	$F''(0)$(Petrov-Galerkin)
0.0	0.0	0.5838	0.618 9
	2.0	1.4867	1.499 2
	4.0	1.9324	1.940 2
	6.0	2.2613	2.267 2
	8.0	2.5307	2.535 6
	10.0	2.7628	2.767 1
0.5	0.0	1.0194	1.040 2
	2.0	1.6560	1.665 9
	4.0	2.0511	2.058 0
	6.0	2.3564	2.361 8
	8.0	2.6116	2.616 2
	10.0	2.8341	2.838 1
1.0	0.0	1.2859	1.299 8
	2.0	1.8026	1.811 0
	4.0	2.1603	2.166 5
	6.0	2.4461	2.451 l
	8.0	2.6890	2.693 3
	10.0	2.9028	2.906 6

4. Graphical Solution of Velocity Profile:

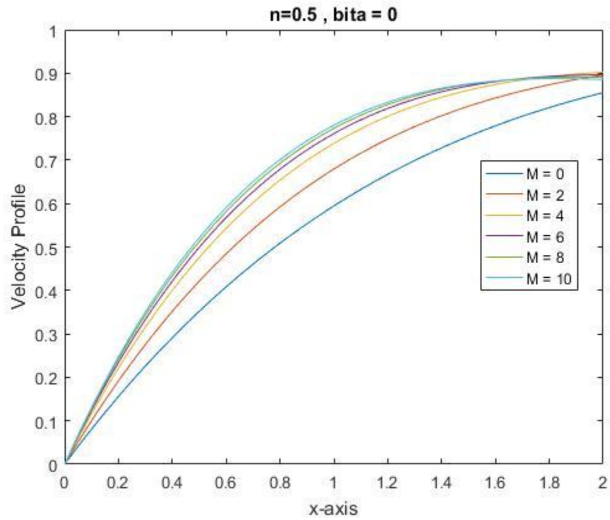

Chart -1: Velocity profiles for flat plate flow (beta=0.0) of a pseudo plastic fluid (n=0.5) with M=0, 2, 4, and 6,8,10

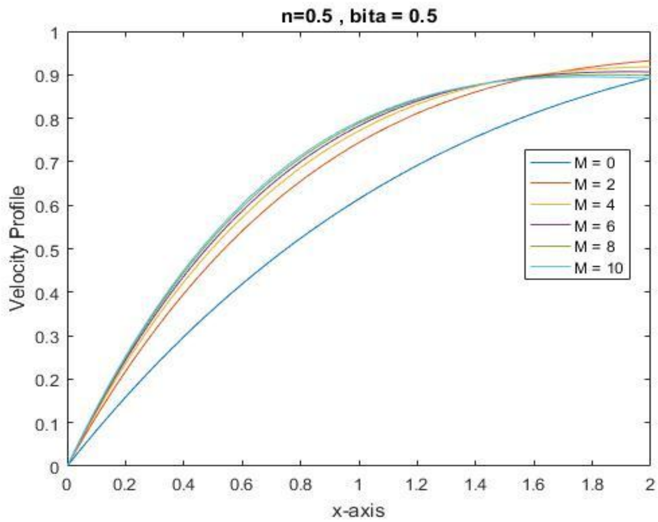

Chart -2: Velocity profiles for flat plate flow (beta=0.5) of a pseudo plastic fluid (n=0.5) with M=0, 2, 4, and 6,8,10

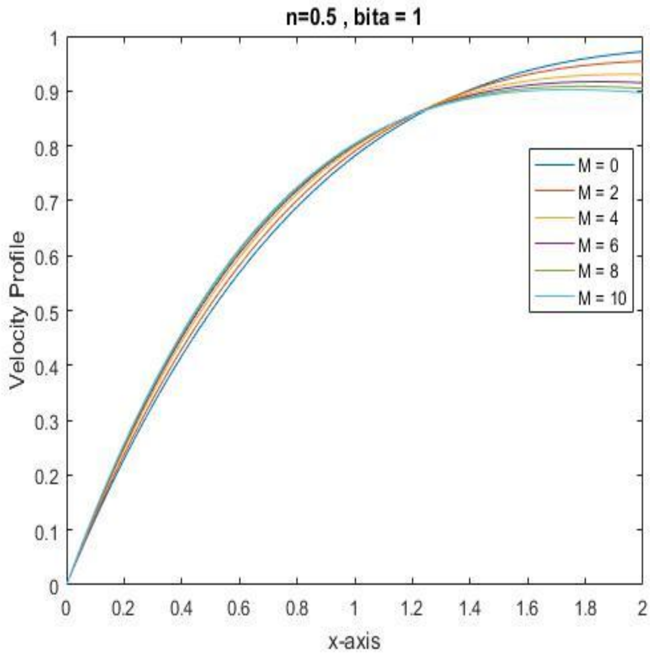

Chart -3: Velocity profiles for flat plate flow (beta=0.5) of a pseudo plastic fluid (n=0.5) with M=0, 2, 4, and 6,8,10

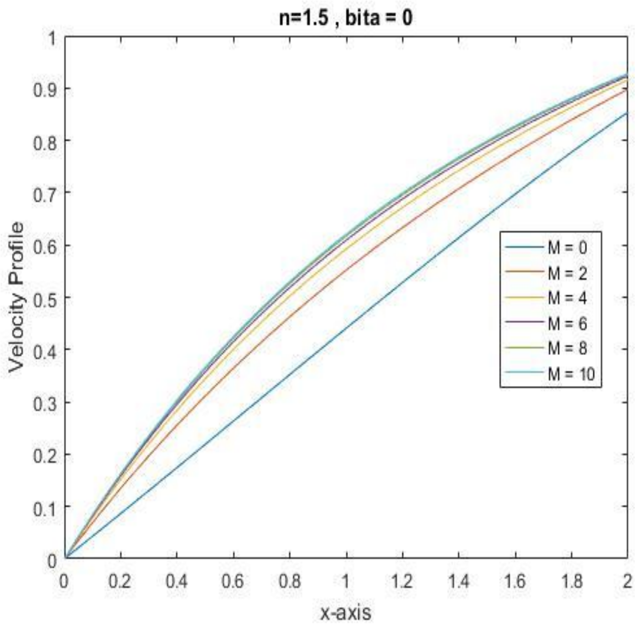

Chart -4: Velocity profiles for flat plate flow (beta=0.0) of a diltant plastic fluid (n=1.5) with M=0, 2, 4, and 6,8,10

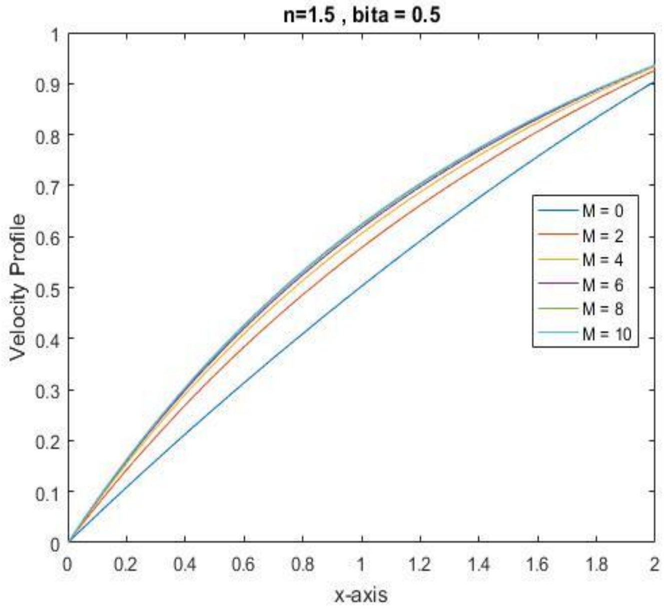

Chart -5: Velocity profiles for flat plate flow (beta=0.5) of a dilatants plastic fluid (n=1.5) with M=0, 2, 4, and 6,8,10

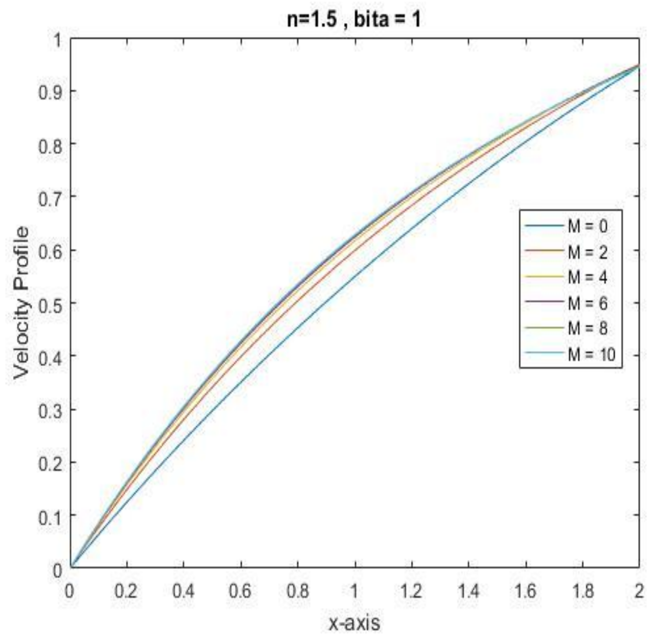

Chart -6: Velocity profiles for flat plate flow (beta=0.5) of a dilatants plastic fluid (n=1.5) with M=0, 2, 4, and 6,8,10

5. Solution for the energy equation

Viscous dissipation is absence and Joule heating is also absence then energy equation takes the simplification form and it is given by

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=k\frac{\partial^2 T}{\partial y^2} \quad (14)$$

An equation can be transformed in to the form of

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=k\frac{\partial^2\theta}{\partial y^2} \quad (15)$$

Now taking

$$\theta=\frac{T-T_\infty}{T_w-T_\infty} \quad (16)$$

T_∞ =uniform temperature of free stream

T_w =wall temperature

(15) is obtained similarity solution for only $\beta=0.5$ and flow past a right angled wedge and that place no magnetic field. And in this case the energy equation can be reduced to an ordinary differential equation

$$\frac{d^2\theta}{d\eta^2}+\left(\frac{3}{2}\right)^{\frac{1-n}{1+n}}P_rF(\eta)\frac{d\theta}{d\eta}=0 \quad (17)$$

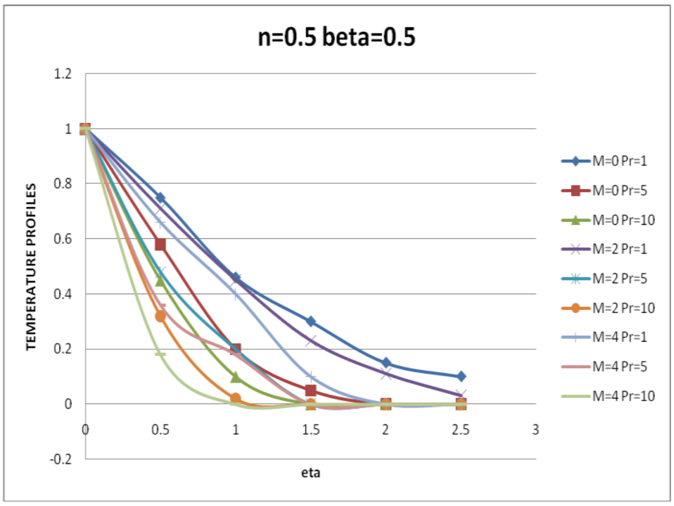

Chart -7: Temperature profile for the flow of a pseudo plastic fluid with beta=0.5 for Pr=1, 5, 10 and M=0, 2, and 4

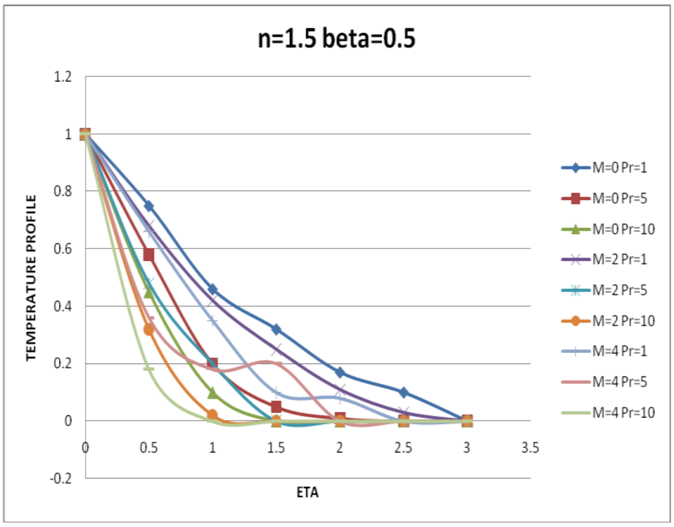

Chart -8: Temperature profile for the flow of a dilatants fluid with beta=0.5 for Pr=1, 5, 10 and M=0, 2, and 4

Prandtl number P_r is defined by

$$P_r=\frac{1}{k}\left(\frac{k}{\sigma}\right)^{2/(n+1)}\ell^{3(n-1)/(n+1)}$$

And $f(\eta)$ is the solution of (5). the boundary condition of (17) are

$$\theta(0)=1 \quad \theta(5)=1 \quad (18)$$

Again apply Petrov-Galerkin method to (17) and (18) obtain Temperature profile are given in chart (7) and (8) .for given η and β the magnetic field parameter decrease the thickness of the boundary layer. The effect of magnetic field parameter is felt more in the case of a pseudo plastic fluid than that of a dilatants fluid.

6. Conclusions

The tables (1) and (2) can be calculated for the dilatants fluid and pseudo plastic fluid. From the both the tables we conclude that the value of skin friction is increase as the increased in the magnetic field parameter as well as increased the power law fluid also.

We have presented the results of the calculation for the linear third order ordinary differential equation of the flow of a power law fluid in the presence of a transverse magnetic field and an external velocity distribution . The method of Petrov-Galerkin method used and easily accurate way of calculating the skin friction. The general effect of the magnetic field was seen to decrease the thickness of both the velocity and the thermal boundary layers.

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