

# Numerical Prediction of Periodically Convection in Vertical Channel Mounted by Heat Generating Blocks

P.V. Janardhana Reddy

Asst Professor, Dept. of Mathematics Humanities, MGIT, Hyderabad.

\*\*\*

## ABSTRACT:

A numerical study is attempted on the free convection flow of an incompressible viscous fluid through a vertical channel filled with a porous matrix bounded by parallel impermeable walls. The flow is assumed to be along the axis of the channel. The uniform temperature is considered in the study. The momentum conservation equations for the fully developed flow are based on the Brinkman model. The viscous and Darcy dissipation are taken into account in the energy equation as the modification of the heat flow. By using the finite element method, the fluid and the porous matrix are in local thermal equilibrium and the flow is unidirectional along the direction of the buoyancy. The non-linear equations governing the flow, heat and mass transfer have been analyzed. Various parameters such as the velocity, temperature, concentration, Nusselt number, and Sherwood number are analyzed and their behavior is interpreted in the paper.

**Keywords:** viscous fluid, Nusselt number, Sherwood Number, Galerkin finite element method.

## 1. INTRODUCTION

Components in electronic systems generate heat during their operation and, therefore, the reliability of such systems greatly depends on the thermal management techniques employed for the removal of heat. It has been reported by Doshy [1] that various failure mechanisms are thermally activated and failure rate increases exponentially with increase of operating temperature. Therefore, a principal objective of the thermal management techniques for electronic application systems is to maintain components at or below a specified maximum temperature to ensure the expected level of performance and reliability. Natural convection, a passive mode of heat transfer, is considered to be a promising option for cooling of electronic components in air because of the specific features of natural convection such as simplicity in design, low installation and maintenance cost, reliability and noiseless operation. Natural convection from protruding and flush mounted heat generating elements has received considerable attention in heat transfer literature, as they simulate heat generating electronic components such as resistors, capacitors, inductors, transformers, ICs and so on. An experimental study of natural convection from a finite thick heat source, mounted on a vertical and a horizontal plate made of Masonite is reported by Kang and Jaluria [2]. The heat source module is made from a highly polished stainless steel foil which is in close constant with three exposed surfaces of a stack of Bakelite strips. The study concluded that the flow and heat transfer characteristics have a strong dependence on the rate of energy input, heat source thickness and the interaction between the wakes generated by the three exposed surfaces of the heat source module. Separate correlations for average Nusselt number as a function of Grashof number, for horizontal and vertical plate, are also presented. Kelkar and Choudhury [3] carried out numerical computation of the periodically fully developed natural convection in a vertical channel with surface mounted heat generating blocks. They studied the effect of dimensionless length of the channel and module Rayleigh number on draft cooling of low power heat generating modules. Fujii et al. [4] studied, numerically and experimentally, natural convection cooling of an array of vertical finite thick parallel plates with discrete and protruding heat sources mounted on them. They proposed an optimum spacing between the plates and presented a correlation for local Nusselt number in terms of modified Grashof number.

Numerical modeling of natural convection cooling of heat generating devices mounted on a vertical wall of an upright open top slot has been reported by Desrayaud and Fichera [5]. Their study highlighted the influence of design parameters such as heat generation rate, size and position of heat generating devices and thermal conductivity of the substrate on the heat transfer characteristics. A numerical study of steady two-dimensional laminar natural convection from a single protruding heat source, mounted at mid-height of a substrate of finite thickness, was carried out by Desrayaud et al. [6]. They have conducted parametric study by varying the thermal conductivity of the substrate, thickness of the substrate and width of the module and the study concluded that the heat transfer from the heat source is considerably affected by the thermal conductivity of the substrate. Rajkumar, Venugopal and Anil Lal [7] presented the results of Natural convection with surface radiation from a planar heat generating element mounted freely in a vertical channel. Ermolaev and Zhbanov [8] presented the results of mixed convection in vertical channel with Discrete Heat Sources at the wall. Barletta and Nield [9] presented the results of combined forced and free convective flow in a vertical porous channel: The effects of viscous dissipation and pressure work. Dileep Singh Chauhan and Priyanka Rastogi [10] presented the results of Radiation effects on Natural convection MHD flow in a rotating vertical porous channel partially filled with a porous medium.

## 2. FORMULATION OF THE PROBLEM

We analyze the free convection flow of an incompressible viscous fluid through a vertical channel filled with a porous matrix bounded by parallel impermeable walls. The flow takes place along the axis of the channel. The surface of the walls is maintained at uniform temperature. The momentum conservation equations for the fully developed flow are based on the Brinkman model. The viscous and Darcy dissipation are taken into account in the energy equation as the modification of the heat flow. The fluid and the porous matrix are in local thermal equilibrium and the flow is unidirectional along the direction of the buoyancy.

We choose the Cartesian frame of reference  $0(x, y, z)$  such that the x-axis is in upward vertical direction against buoyancy and the vertical walls are parallel to  $(y, z)$  plane  $y = \pm b$ . Let  $(u, 0, 0)$  be the velocity field of the unidirectional flow along the channel and  $T$  the temperature in the flow field and to the ambient temperature. The equations governing the flow and heat transfer are

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} = \mathbf{0} \tag{2.1}$$

$$\mu \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{k} \right) + g\beta(T - T_0) + g\beta^*(C - C_0) = -v_0 \frac{\partial u}{\partial y} \tag{2.2}$$

$$\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\rho_0 v}{k_1} \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 \right] + \frac{\rho_0 v}{kk_1} u^2 - \frac{\partial}{\partial y} (vq_{\partial r}) = -\rho_0 c_p \frac{v_0}{k_1} \frac{\partial T}{\partial y} \tag{2.3}$$

$$D_1 \left( \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) + K_{11} \left( \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) = 0 \tag{2.4}$$

$$\rho = \rho_0 (1 - \beta(T - T_0) - \beta^*(C - C_0)) \tag{2.5}$$

where  $\rho_0$  is the density at the ambient temperature  $T_0$  and concentration  $C_0$  and  $\nu, k, \beta$  are the coefficients of Kinematic viscosity, thermal conductivity and thermal expansion of the fluid

respectively,  $\beta^*$  is the volumetric coefficient of expansion with mass fraction concentration,  $k$  is the permeability of the porous medium and  $C_p$  is the specific heat at constant pressure, this the molecular diffusivity and  $k_1$  is the cross diffusivity..

In view of the continuity equations, we take  $u = u(y, z)$

The boundary conditions are

$$u = 0 \text{ on } z = \pm b$$

$$T = T_1, \quad C = C_1 \tag{2.6}$$

$$\frac{\partial u}{\partial z} = 0, \quad \frac{\partial T}{\partial z} = 0 \quad \text{and} \quad \frac{\partial C}{\partial z} = 0 \text{ on } z = 0 \text{ in view of the symmetry.}$$

We introduce the following non-dimensional variables as follows.

$$z^* = \frac{z}{b}; \quad y^* = \frac{y}{b}; \quad \theta^* = \frac{T - T_0}{T_1 - T_0}, \quad C^* = \frac{C - C_0}{C_1 - C_0}$$

$$u^* = \frac{\nu u}{\beta g b^2 (T_1 - T_0)}$$

Substituting these in the governing equations the corresponding dimensionless equations under Boussinesq approximations (on dropping the asteriks) are

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - D^{-1}u + (\theta + NC) = -S \frac{\partial u}{\partial y} \tag{2.7}$$

$$\frac{\partial^2 \theta}{\partial y^2} + N_z \frac{\partial^2 \theta}{\partial z^2} + GPrEc[(\frac{\partial u}{\partial y})^2 + (\frac{\partial u}{\partial z})^2] + D^{-1}u^2 = -PrS \frac{\partial \theta}{\partial y} \tag{2.8}$$

$$(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}) + \frac{S_0 Sc}{N} (\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}) = 0 \tag{2.9}$$

Where

$$D = \frac{k}{b^2} \quad \text{is the Darcy parameter}$$

$$P = \frac{\mu C_p}{k_1} \quad \text{is the Prandtl number}$$

$$Ec = \frac{\beta g b}{C_p} \quad \text{is the Eckert number}$$

$$G = \frac{\beta g b^3 (T_1 - T_0)}{\nu^2} \quad \text{is the Grashof number}$$

$$S = \frac{\nu_0 b}{\nu} \quad \text{is the suction Reynolds number}$$

$$N = \frac{\beta^* (C_1 - C_0)}{\beta (T_1 - T_0)} \quad \text{is the buoyancy ratio}$$

$$S_c = \frac{\nu}{D_1} \quad \text{is the Schmidt number}$$

$$S_o = \frac{\beta^* K_{11}}{\beta \nu} \quad \text{is the Soret number}$$

$$N_1 = \frac{\infty \beta_R K_1}{4 \sigma T_e^3} \quad \text{is the radiation parameter}$$

$$N_2 = \frac{3N_1}{3N_1 + 4} \quad P_1 = PN_2 \quad \alpha_1 = \alpha N_2$$

The corresponding boundary conditions in the non-dimensional form are  $u = 0$ ,

$$\theta = 1, C = 1 \text{ on } z = 1$$

$$\frac{\partial u}{\partial z} = 0, \quad \frac{\partial \theta}{\partial z} = 0 \quad \text{and} \quad \frac{\partial C}{\partial z} = 0 \quad \text{On } Z = 0 \quad (2.10)$$

In view of the symmetry of the flow with respect to the mid plane of the channel, we investigate the flow in one half of the domain bounded by the impermeable wall to the right and the mid plane. The finite element analysis with quadratic approximation functions is carried out by using eight noded rectangular serendipity element in the normal cross sectional plane ( $y - z$ ) bounded by planes  $z = 0$  and  $1$ .

### 3. FINITE ELEMENT ANALYSIS OF THE PROBLEM

If  $u$  and  $\theta$  are the approximations of  $u$  and  $\theta$  we define the errors (residual)  $E_1$  and  $E_2$

as

$$\frac{\partial^2 u^i}{\partial y^2} + \frac{\partial^2 u^i}{\partial z^2} - D^{-1} u^i + (\theta^i + NC^i) + S \frac{\partial u^i}{\partial y} \quad (3.1)$$

$$E_2 = \frac{\partial^2 \theta^i}{\partial y^2} + N_2 \frac{\partial^2 \theta^i}{\partial z^2} + GR_1 Ec \left[ \left( \frac{\partial u^i}{\partial y} \right)^2 + \left( \frac{\partial u^i}{\partial z} \right)^2 \right] + D^{-1} (u^i)^2 + P_1 S \frac{\partial \theta^i}{\partial y} \quad (3.2)$$

$$E_3^i = \left( \frac{\partial^2 C^i}{\partial y^2} + \frac{\partial^2 C^i}{\partial z^2} \right) + \frac{S_u S_c}{N} \left( \frac{\partial^2 \theta^i}{\partial y^2} + \frac{\partial^2 \theta^i}{\partial z^2} \right) \quad (3.3)$$

where

$$u^i = \sum_{k=1}^8 u_k^i N_k^i \quad (3.4)$$

$$\theta^i = \sum_{k=1}^8 \theta_k^i N_k^i \quad (3.5)$$

$$C^i = \sum_{k=1}^8 C_k^i N_k^i \quad (3.6)$$

These errors are orthogonal to the weight function over the domain of  $\Omega_i$ . Under Galerkin we choose the approximation functions as the weight function. Multiply both sides of the equations (3.1) - (3.3) by the weight function i.e., each of the approximation function  $N_j^i$  and integrate over the surface  $\Omega_i$ , we obtain

$$\int_{\Omega_i} E_1^i N_j^i d\Omega_i = 0 \quad (j = 1, 2, \dots, 8) \quad (3.7)$$

$$\int_{\Omega_i} E_2^i N_j^i d\Omega_i = 0 \quad (j = 1, 2, \dots, 8) \quad (3.8)$$

$$\int_{\Omega_i} E_3^i N_j^i d\Omega_i = 0 \quad (j = 1, 2, \dots, 8) \quad (3.9)$$

$$\int_{\Omega_i} \left[ \frac{\partial^2 u^i}{\partial y^2} + \frac{\partial^2 u^i}{\partial z^2} - D^{-1} u^i + (\theta^i + N C^i) + S \frac{\partial u^i}{\partial y} \right] N_j^i d\Omega_i = 0 \quad (3.10)$$

$$\int_{\Omega_i} \left[ \frac{\partial^2 \theta^i}{\partial y^2} + N_2 \frac{\partial^2 \theta^i}{\partial z^2} + G P_1 E c \left[ \left( \frac{\partial u^i}{\partial y} \right)^2 + \left( \frac{\partial u^i}{\partial z} \right)^2 \right] + D^{-1} (u^i)^2 \right] + P_1 S \frac{\partial \theta^i}{\partial y} N_j^i d\Omega_i = 0 \quad (3.11)$$

$$\int_{\Omega_i} \left[ \left( \frac{\partial^2 C^i}{\partial y^2} + \frac{\partial^2 C^i}{\partial z^2} \right) + \frac{S_u S_c}{N} \left( \frac{\partial^2 \theta^i}{\partial y^2} + \frac{\partial^2 \theta^i}{\partial z^2} \right) \right] N_j^i d\Omega_i = 0 \quad (3.12)$$

$$\int_{\Omega_i} \left[ \frac{\partial N_j^i}{\partial y} \frac{\partial u^i}{\partial y} + \frac{\partial N_j^i}{\partial z} \frac{\partial u^i}{\partial z} - D^{-1} u^i N_j^i + (\theta^i + N C^i) N_j^i + S N_j^i \frac{\partial u^i}{\partial y} \right] d\Omega_i = \int_{\Gamma} \left[ N_j^i \frac{\partial u^i}{\partial y} n_y + N_j^i \frac{\partial u^i}{\partial z} n_z \right] d\Gamma_i \quad (3.13)$$

$$\int_{\Omega_i} \left[ \frac{\partial N_j^i}{\partial y} \frac{\partial \theta^i}{\partial y} + N_2 \frac{\partial N_j^i}{\partial z} \frac{\partial \theta^i}{\partial z} + G P_1 E c N_j^i \left[ \left( \frac{\partial u^i}{\partial y} \right)^2 + \left( \frac{\partial u^i}{\partial z} \right)^2 + D^{-1} (u^i)^2 \right] + P_1 S N_j^i \frac{\partial \theta^i}{\partial y} \right] d\Omega_i$$

$$= \int_{\Gamma} [N_j^i \frac{\partial \theta^i}{\partial y} n_y + N_j^i \frac{\partial \theta^i}{\partial z} n_z] d\Gamma_i \tag{3.14}$$

$$\int_{\Omega_i} [N^i \frac{\partial N_j^i}{\partial y} (N^i \frac{\partial C^i}{\partial y} + S_c S_o \frac{\partial \theta^i}{\partial y}) + \frac{\partial N_j^i}{\partial z} (N^i \frac{\partial C^i}{\partial z})] d\Omega_i$$

$$= \int_{\Gamma} [N_j^i (N^i \frac{\partial C^i}{\partial y} + S_c S_o \frac{\partial \theta^i}{\partial y}) n_y + N_j^i (N^i \frac{\partial C^i}{\partial z} + S_c S_o \frac{\partial \theta^i}{\partial z}) n_z] d\Gamma_i \tag{3.15}$$

where  $\Omega_i$  is the serendipity element bounded by  $\Gamma_i$ ,  $n_y, n_z$  are the direction cosines normal to  $\Gamma_i$ . Substituting (3.4), (3.5) & (3.6) in L.H.S of (3.13), (3.14) & (3.15) we get

$$\int_{\Omega_i} \sum_{k=1}^8 u_k^i [\frac{\partial N_j^i}{\partial y} \frac{\partial N_k^i}{\partial y} + \frac{\partial N_j^i}{\partial z} \frac{\partial N_k^i}{\partial z} - D^{-1} N_k^i N_j^i + S N_j^i \frac{\partial N_k^i}{\partial y}] d\Omega_i$$

$$+ \int_{\Omega_i} \sum_{k=1}^8 (\theta_k^i + N C_k^i) N_j^i N_k^i d\Omega_i = Q_j^i \tag{3.16}$$

$$\int_{\Omega_i} \sum_{k=1}^8 \theta_k^i [\frac{\partial N_j^i}{\partial y} \frac{\partial N_k^i}{\partial y} + N_2 \frac{\partial N_j^i}{\partial z} \frac{\partial N_k^i}{\partial z} + P_1 S N_j^i \frac{\partial N_k^i}{\partial y}] d\Omega_i +$$

$$\int_{\Omega_i} \sum_{k=1}^8 u_k^i G P_1 E c [(\frac{\partial N_k^i}{\partial y})^2 + (\frac{\partial N_k^i}{\partial z})^2 + D^{-1} (N_k^i)^2] d\Omega_i = (Q^T)^i_j \tag{3.17}$$

$$\int_{\Omega_i} [\sum_{k=1}^8 C_k^i \{ \frac{\partial N_k^i}{\partial y} \frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z} \frac{\partial N_j^i}{\partial z} \}] d\Omega_i - \int_{\Omega_i} N_2 S c \sum_{k=1}^8 u_k^i (N_j^i N_k^i) d\Omega_i$$

$$(3.18) \quad \text{where}$$

$$- S_c S_o \int_{\Omega_i} \sum_{k=1}^8 \theta_k^i \{ \frac{\partial N_k^i}{\partial y} \frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z} \frac{\partial N_j^i}{\partial z} \} d\Omega_i = (Q^c)^i_j$$

$$Q_j^i = \int_{\Gamma_i} (N_j^i) (\frac{\partial u^i}{\partial y}) n_y + N_j^i \frac{\partial u^i}{\partial z} n_z d\Gamma_i$$

$$(j = 1, 2, \dots, 8) (Q^T)^i_j = \int_{\Gamma_i} (N_j^i) (\frac{\partial T^i}{\partial y}) n_y + N_j^i \frac{\partial T^i}{\partial z} n_z d\Gamma_i$$

$$\begin{aligned}
 (Q^c)_j^i = \int_{\Gamma_i} [N_j^i \{ N \frac{\partial C^i}{\partial y} + S_0 S_c \frac{\partial \theta^i}{\partial y} \} n_y + N_j^i \{ N \frac{\partial C^i}{\partial z} + \\
 + S_0 S_c \frac{\partial \theta^i}{\partial z} \} n_z] d\Gamma_i \quad j=1,2,\dots,8.
 \end{aligned}$$

Choosing different  $N_k^i$ 's corresponding to each element  $e_i$  (3.11) results in sixteen equations

for two sets of unknown  $(u_k^i)$  and  $(\theta_k^i)$  viz

$$(a_{kj}^i)(u_k^i) = Q_j^i \tag{3.19}$$

$$(b_{kj}^i)(\theta_k^i) + (c_{kj}^i)u_k^i = (Q^T)_j^i \quad (j = 1, 2, \dots, 8) \tag{3.20}$$

$$(m_{kj}^i)(C_k^i) + (l_{kj}^i)(u_k^i) = (n_{kj}^i)(\theta_k^i) + (Q_j^c)^i \quad (j, k = 1, 2, \dots, 8) \tag{3.21}$$

where  $(a_{kj}^i), (b_{kj}^i), (c_{kj}^i), (m_{kj}^i), (n_{kj}^i)$  and  $(l_{kj}^i)$  are  $8 \times 8$  stiffness matrices and

$Q_j^i, (Q^T)_j^i$  and  $(Q_j^c)^i$  are  $8 \times 1$  column matrices. Repeating the process with each of  $m$

elements and making conditions as well as the boundary conditions to assemble the element matrices, we obtain global matrices for the unknowns  $u, \theta$  and  $C$  at the respective global nodes which ultimately determine them on solving the matrix equation.

For computational purpose, we choose a serendipity element with (0,0), (0,1) (1,0) and (1,1) as its vertices. The eight nodes of the element are shown in Fig.(a) and the quadratic interpolation functions at these nodes are

$$\begin{aligned}
 N_1 &= -2(y-1)(z-1)(z+y-\frac{1}{2}) & ; & \quad N_2 = -4z(z-1)(y-1) \\
 N_3 &= -2z(y-1)(z-y+\frac{1}{2}) & ; & \quad N_4 = -4yz(y-1) \\
 N_5 &= 2yz(z+y-\frac{3}{2}) & ; & \quad N_6 = -4yz(z-1) \\
 N_7 &= 2y(z-1)(z-y+\frac{1}{2}) & ; & \quad N_8 = -4y(z-1)(y-1)
 \end{aligned}$$

Substituting these shape functions in (3.19) and integrating over the element domain the matrix for the global nodes of  $u$  viz.  $u_i$  ( $i=1,2,\dots$ ) reduces to a  $8 \times 8$  matrix equations.

This  $8 \times 8$  matrix equations can be partitioned in the form

$$\begin{bmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{bmatrix} \begin{bmatrix} \Delta_U^1 \\ \Delta_U^2 \end{bmatrix} = \begin{bmatrix} F_U^1 \\ F_U^2 \end{bmatrix} \tag{3.22}$$

Where  $\Delta_U^1, \Delta_U^2, F_U^1, F_U^2$  are column matrices given by

$$\Delta_U^1 = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}, \quad \Delta_U^2 = \begin{bmatrix} U_5 \\ U_6 \\ U_7 \\ U_8 \end{bmatrix}$$

Equation (3.20) yields the following two equations in terms of the partitioned matrices.

$$\begin{bmatrix} S^{11} \\ S^{21} \end{bmatrix} \begin{bmatrix} \Delta_U^1 \\ \Delta_U^2 \end{bmatrix} + \begin{bmatrix} S^{12} \\ S^{22} \end{bmatrix} \begin{bmatrix} \Delta_U^2 \\ \Delta_U^1 \end{bmatrix} = \begin{bmatrix} F_U^1 \\ F_U^2 \end{bmatrix} \tag{3.23}$$

$$\begin{bmatrix} S^{21} \\ S^{22} \end{bmatrix} \begin{bmatrix} \Delta_U^1 \\ \Delta_U^2 \end{bmatrix} + \begin{bmatrix} S^{12} \\ S^{11} \end{bmatrix} \begin{bmatrix} \Delta_U^2 \\ \Delta_U^1 \end{bmatrix} = \begin{bmatrix} F_U^2 \\ F_U^1 \end{bmatrix}$$

Similarly the 8 × 8 matrix equations for  $\Delta_j, C_j$  ( $j = 1, 2, \dots, 8$ ) in the partitioned form are

$$\begin{bmatrix} B^{11} & B^{12} \\ B^{21} & B^{22} \end{bmatrix} \begin{bmatrix} \Delta_\theta^1 \\ \Delta_\theta^2 \end{bmatrix} = \begin{bmatrix} F_\theta^1 \\ F_\theta^2 \end{bmatrix} \tag{3.24}$$

$$\begin{bmatrix} L^{11} & L^{12} \\ L^{21} & L^{22} \end{bmatrix} \begin{bmatrix} \Delta_C^1 \\ \Delta_C^2 \end{bmatrix} = \begin{bmatrix} F_C^1 \\ F_C^2 \end{bmatrix} \tag{3.25}$$

Where  $\Delta_\theta^1, \Delta_\theta^2, F_\theta^1, F_\theta^2, \Delta_C^1, \Delta_C^2, F_C^1, F_C^2$  are column matrices given by

$$\Delta_\theta^1 = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}; \quad \Delta_\theta^2 = \begin{bmatrix} \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \end{bmatrix}; \quad \Delta_C^1 = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}; \quad \Delta_C^2 = \begin{bmatrix} C_5 \\ C_6 \\ C_7 \\ C_8 \end{bmatrix}$$

The boundary conditions (essential) on the primary variables are

$$u_3 = u_4 = u_5 = 0;$$

$$v_3 = v_4 = v_5 = 1 \text{ and}$$

$$C_3 = C_4 = C_5 = 1 \text{ on } y=1 \tag{3.26}$$

In view of the symmetry conditions we obtain  $Q_1 =$

$$Q_2 = Q_6 = Q_7 = Q_8 = 0$$

$$Q_1^T = Q_2^T = Q_6^T = Q_7^T = Q_8^T = 0 \tag{3.27}$$



$$Q_1^C = Q_2^C = Q_3^C = Q_4^C = Q_8^C = 0$$

Solving the ultimate 8 × 8 matrices we determine the unknown global nodal values of  $u_i$ ,  $\theta_i$  (  $i = 1, 2, \dots, 8$ ).

The solution for  $u$ ,  $\theta$  may now be represented as

$$u = \sum_{k=1}^8 u_k N_k, \quad \theta = \sum_{j=1}^8 \theta_j N_j \quad \text{and} \quad C = \sum_{i=1}^8 C_i N_i$$

#### 4. DISCUSSION OF NUMERICAL RESULTS

Figures 1,2, shows the velocity profile along the normal planes  $y=0$  &  $1$  while figures 3, 4 depicts the velocity profile along the parallel planes  $Z=0$  &  $Z=1/2$ . For variations in, the velocity on  $Z=0$  &  $Z=1/2$  indicates that the flow always moves in the upward direction as it is moved in the normal direction from  $y=0$  to  $1$ . In all the cases however the maximum is attained at  $y=0.4$  while at  $Z=1/2$  level the maximum across at  $y=1$ . This conveys the different flow pattern in planes parallel to the channel walls as we across the walls along the said direction this variation we studied in view of the finite element method adopted. It is observed that as we move from mid plane to the boundary we find that the velocity is decreases with increase in  $D^{-1}$ . Thus lesser is the permeability of the porous medium smaller the velocity in the fluid region. The variation of  $u$  along the normal planes  $y= 0$  &  $1$  shows that  $u$  decreases with  $D^{-1}$  (Figs 1, 2). This situation in the stud gives the fact that lesser the permeability of the porous medium smaller the magnitude of  $u$ . The values of  $u$  at  $y=0$  are higher than those at  $y=1$  level for all variations in parameters. Figures 5, 6,9,10 represents the variation of non-dimensional temperature at the horizontal levels  $y=0$  &  $1/2$  with respect to  $\alpha$ , and  $N1$ .

The influence of heat source on  $\theta$  is exhibited in figures 5&6. We find a marginal depreciation in  $\theta$  with increase in the heat source parameter  $\alpha$ . The variation of  $\theta$  with radiation parameter  $N1$  shows that an increase in  $N1$  leads to an enhancement in the actual temperature at  $y=0$  &  $1$  levels. The variation of  $\theta$  with  $\alpha$ , and  $N$ , at the vertical levels  $Z=0$  &  $1/2$  is shown in figures 7,8,11,12. From figures 7&8 we find that the actual temperature experiences depreciation with increase in the heat source parameter  $\alpha$ . This indicates that the presence of the heat generating source in the fluid region leads to a reduction in the actual temperature at both the vertical levels. The variation of  $\theta$  with radiation parameter  $N1$  causes an enhancement in tendency of  $N$  with radiative heat flux in figures 11&12. In general we find that the actual temperature at the horizontal level is lesser than that the vertical levels.

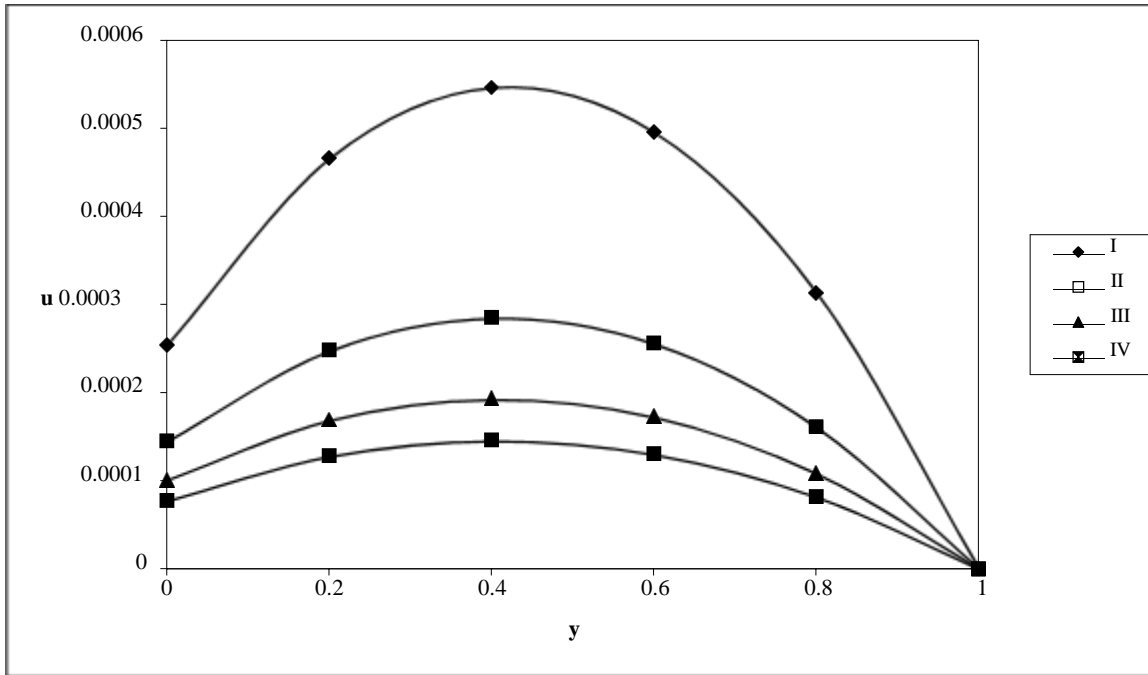


Fig. 1:

Variation of u with  $D$  at  $y=0$  level

$M=5; G=200; N1=0.5; S=0.8; k=0.5; P=0.71; \beta=2; D^{-1}=2000; z=0.5$

	I	II	III	IV
$D^{-1}$	$2 \times 10^3$	$4 \times 10^3$	$6 \times 10^3$	$8 \times 10^3$

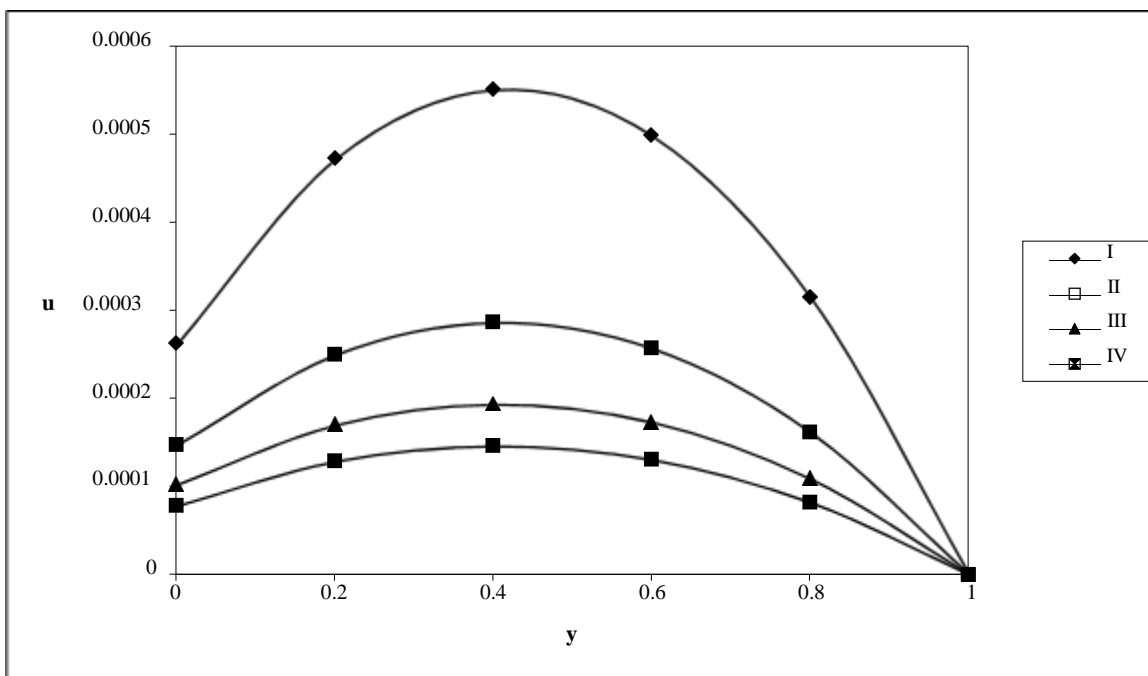


Fig. 2:

Variation of u with  $\mathcal{D}$  at  $y=0.5$  level

$M=5; G=200; N1=0.5; S=0.8; k=0.5; P=0.71; \mathcal{D}=2000; z=0.5$

	I	II	III	IV
$\mathcal{D}^{-1}$	$2 \times 10^3$	$4 \times 10^3$	$6 \times 10^3$	$8 \times 10^3$
		3		

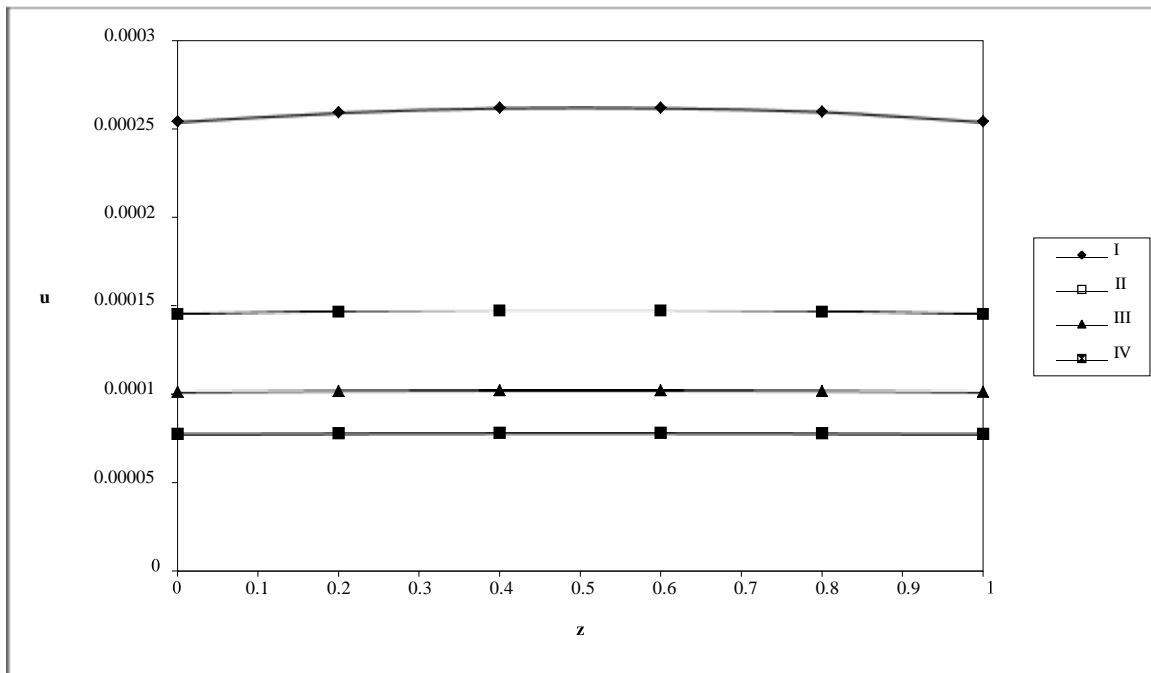


Fig. 3:

Variation of u with  $\mathcal{D}$  at  $z=0$  level

$M=5; G=200; N1=0.5; S=0.8; k=0.5; P=0.71; \mathcal{D}=2000; z=0.5$

	I	II	III	IV
$\mathcal{D}^{-1}$	$2 \times 10^3$	$4 \times 10^3$	$6 \times 10^3$	$8 \times 10^3$
		3		

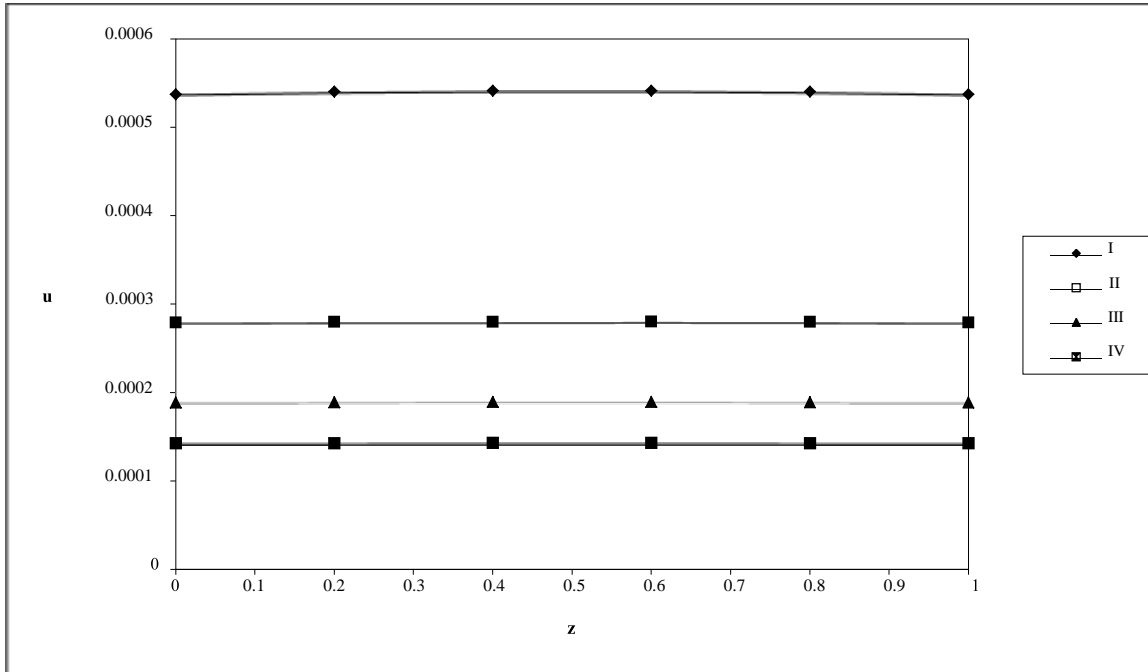


Fig. 4:

Variation of u with  $\bar{D}$  at  $z=0.5$  level

$M=5; G=200; cn=0.5; S=0.8; k=0.5; P=0.71; \bar{D}=2000; z=0.5$

I	II	III	IV
$2 \times 10^3$	$4 \times 10^3$	$6 \times 10^3$	$8 \times 10^3$

D-1

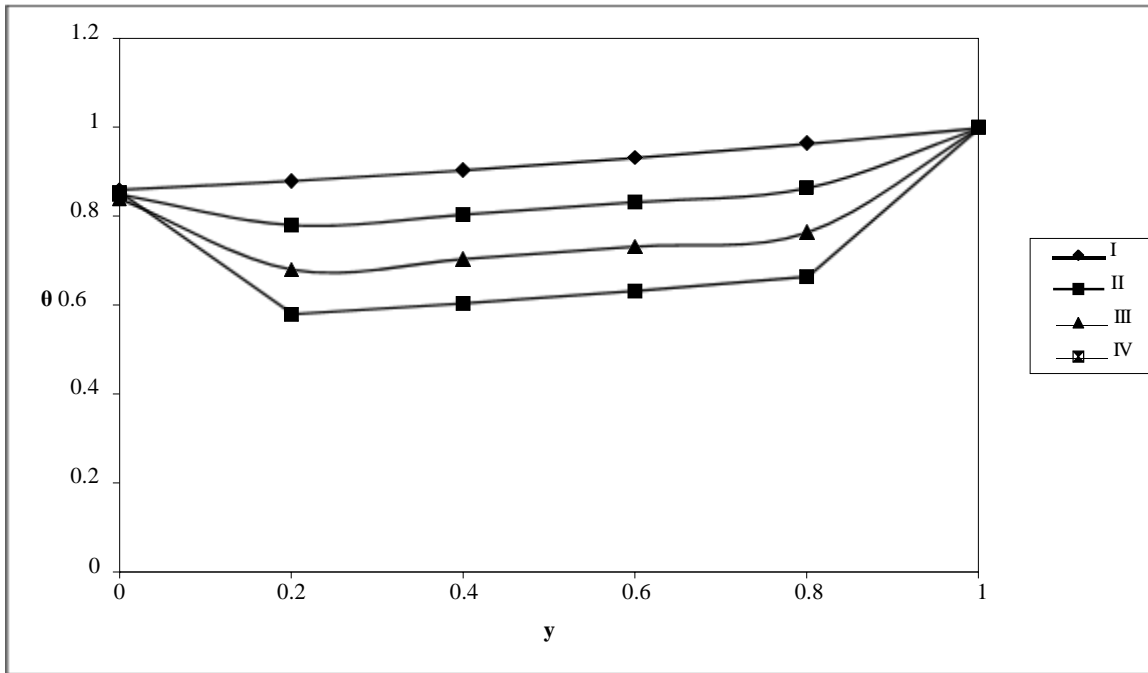


Fig. 5:

Variation of  $\theta$  with  $\alpha$  at  $y=0$  level

$M=5; G=200; N1=0.5; S=0.8; k=0.5; P=0.71; \beta=2; D^{-1}=2000; z=0.5$

	I	II	III	IV
$\alpha$	0	2	4	10

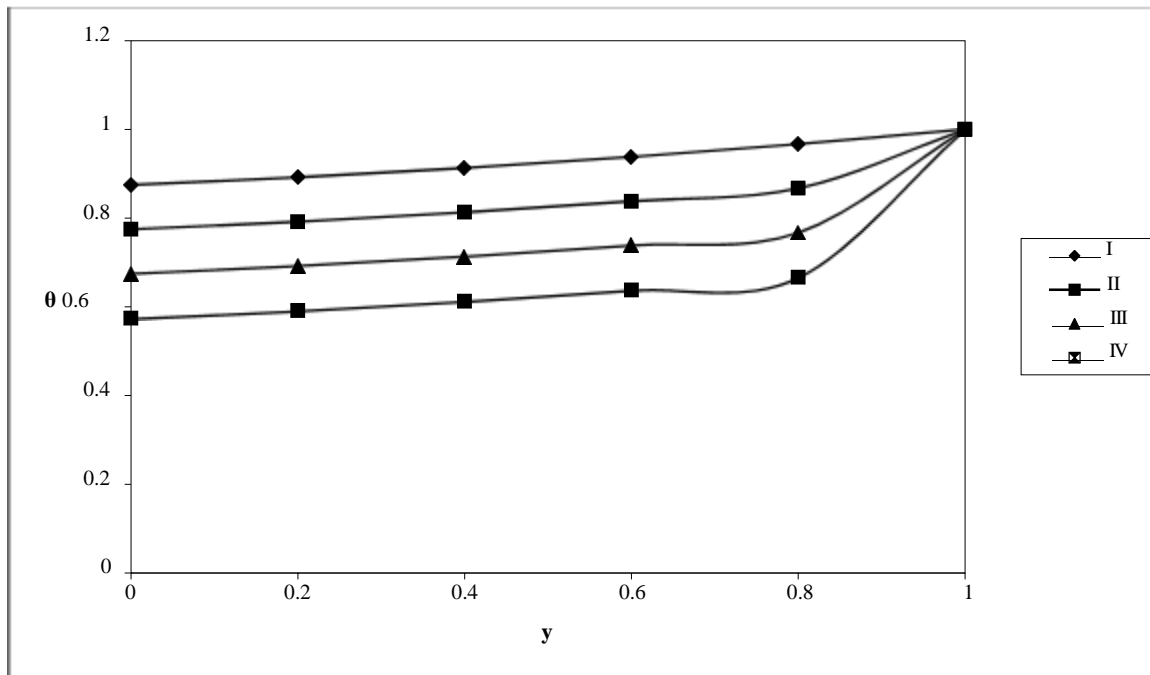


Fig. 6:

Variation of  $\theta$  with  $\alpha$  at  $y=0.5$  level

$M=5; G=200; N1=0.5; S=0.8; k=0.5; P=0.71; \beta=2; D^{-1}=2000; z=0.5$

	I	II	III	IV	
$\alpha$	0	2	4	10	

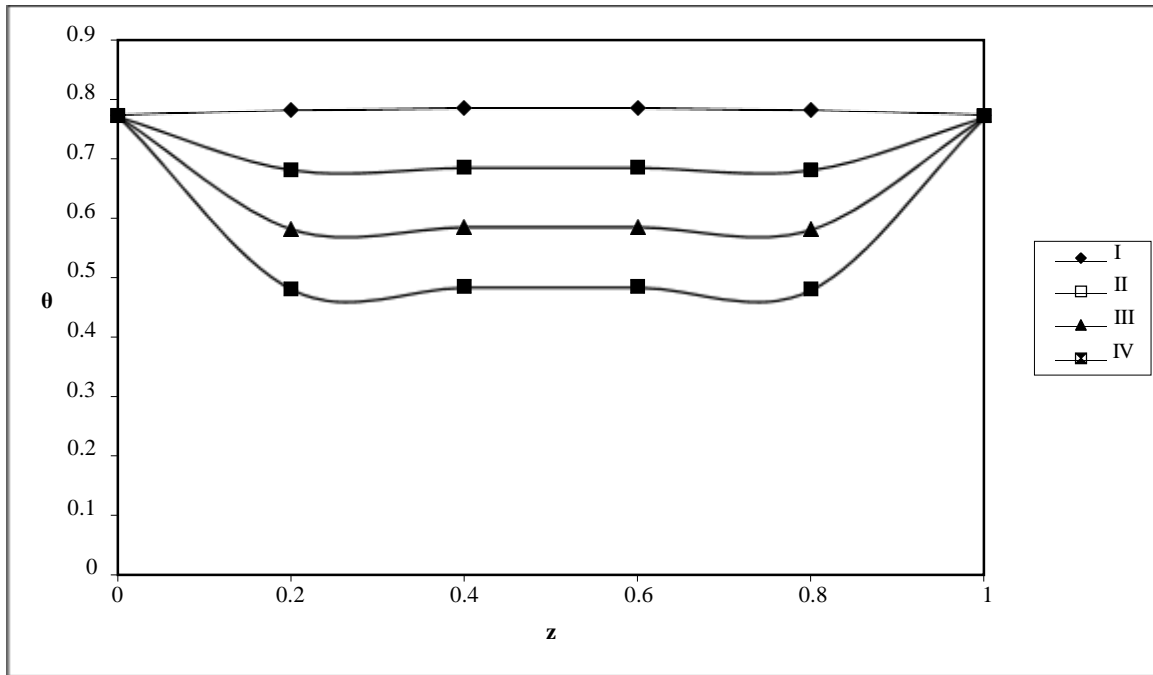


Fig. 7:

Variation of  $\theta$  with  $\alpha$  at  $z=0$  level

$M=5; G=200; N1=0.5; S=0.8; k=0.5; P=0.71; \beta=2; D^{-1}=2000; z=0.5$

	I	II	III	IV
$\alpha$	0	2	4	10

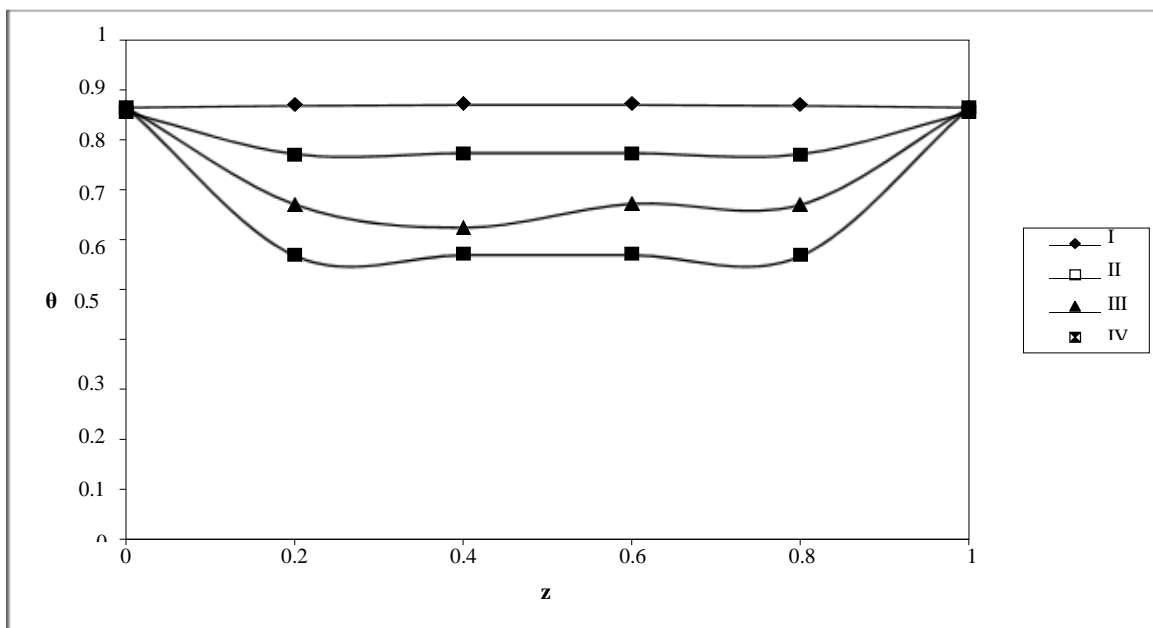


Fig.

Variation of  $\theta$  with  $\alpha$  at  $z=0.5$  level  
 $M=5; G=200; N1=0.5; S=0.8; k=0.5; P=0.71; D=2000; z=0.5$

	I	II	III	IV
$\alpha$	0	2	4	10

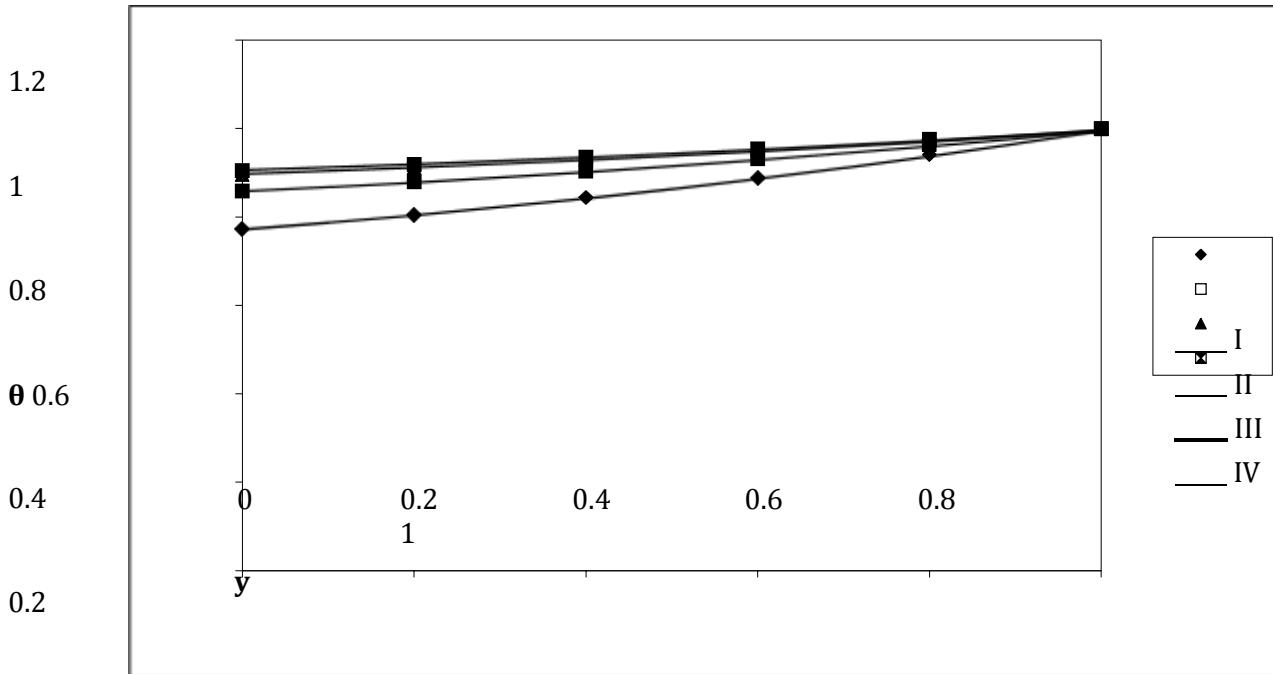


Fig. 9:

Variation of  $\theta$  with  $N1$  at  $y=0$  level  
 $M=5; G=200; S=0.8; k=0.5; P=0.71; \alpha=2; D=2000; z=0.5$

	I	II	III	IV
$N1$	0.5	1.5	5	10



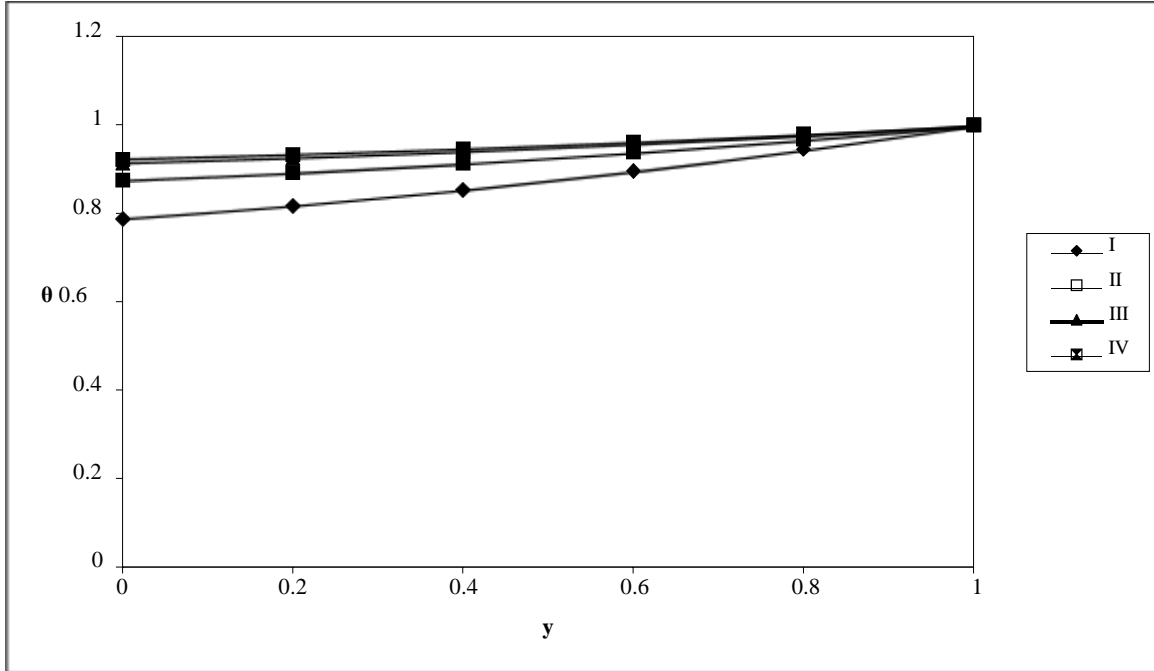


Fig. 10:  
 Variation of  $\theta$  with  $N1$  at  $y=0.5$  level  $M=5$ ;  
 $G=200$ ;  $S=0.8$ ;  $k=0.5$ ;  $P=0.71$ ;  $D=2000$ ;  $z=0.5$

	I	II	III	IV
$N1$	0.5	1.5	5	10

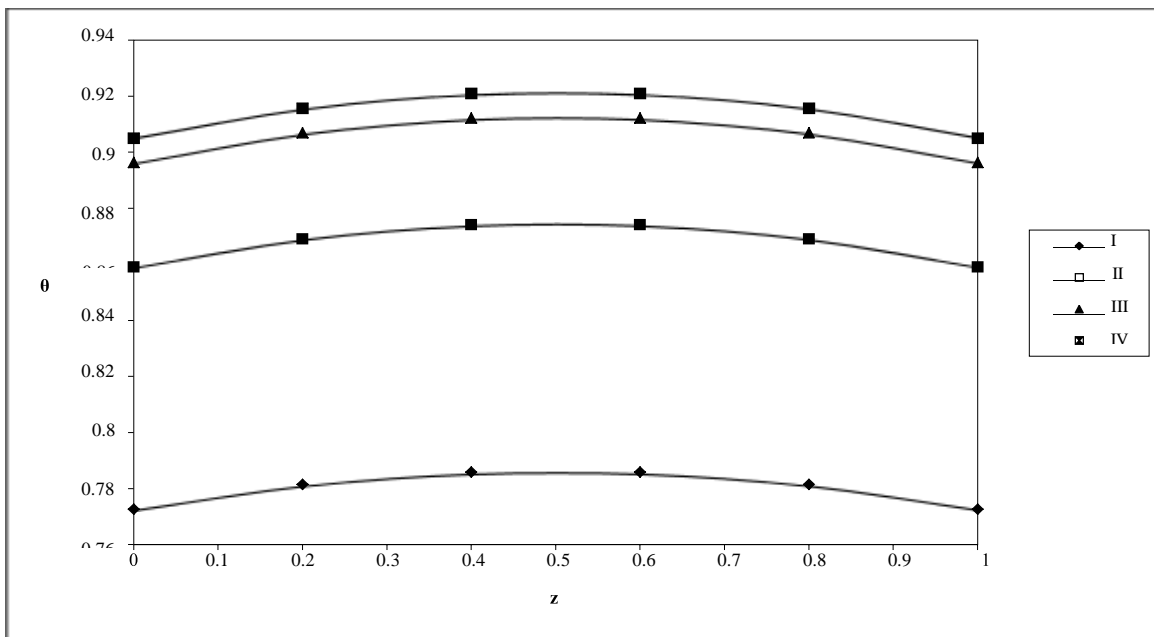


Fig. 11:

Variation of  $\theta$  with N1 at z=0 level

M=5; G=200; S=0.8; k=0.5; P=0.71; =2; D<sup>-1</sup> =2000; z=0.5

	$\beta$			
	I	II	III	IV
N1	0.5	1.5	5	10

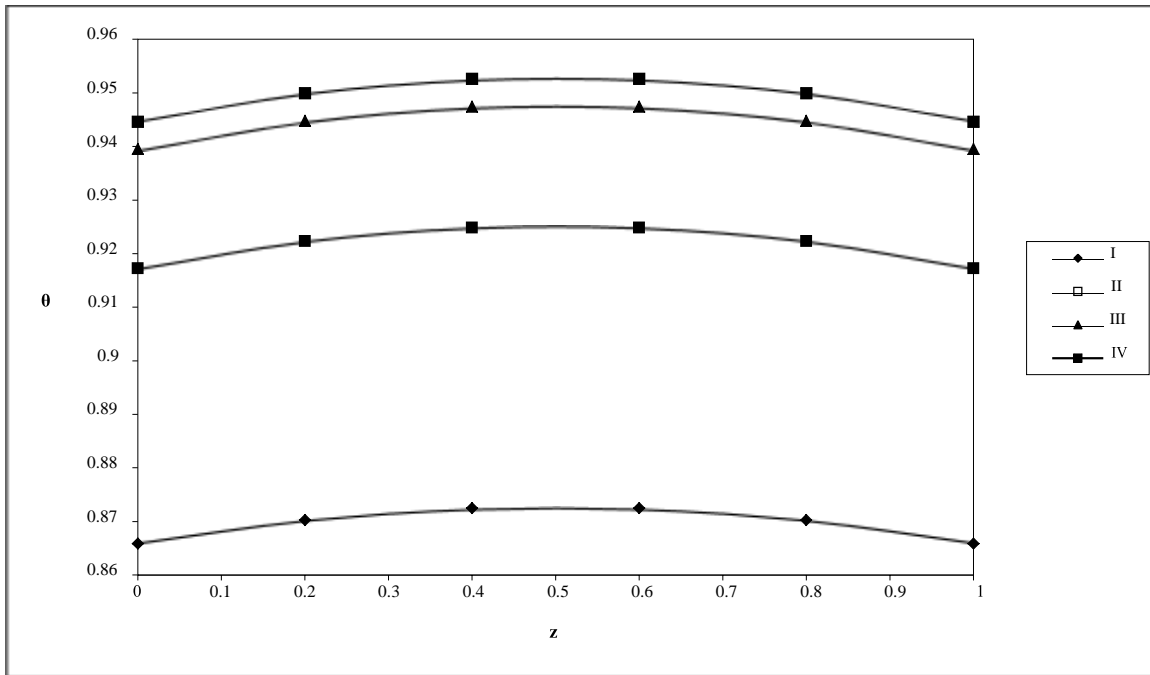


Fig. 12:  
 Variation of  $\theta$  with N1 at z=0.5 level M=5;  
 G=200; S=0.8; k=0.5; P=0.71; D=2000; z=0.5

	I	II	III	IV
N1	0.5	1.5	5	10

## 5. REFERENCES

- 1) Doshi, I: Reliability impact of thermal design. In: Technical conference-IEPS, 4th annual international electronic packaging conference, pp 307–317,(1984).
- 2) Kang, BH, Jaluria, Y: Natural convection heat transfer characteristics of a protruding thermal source located on horizontal and vertical surface. *Int J Heat Mass Transfer* 33, pp 1347–1357, (1990).
- 3) elkar, M K, & Choudhury, D: Numerical prediction of periodically fully developed natural convection in vertical channel with surface mounted heat generating blocks. *Int J Heat Mass Transfer* 36, pp 1133–1145, (1993).
- 4) Fujii, M, Gima S, Tomimura T, Zhang, X: Natural convection to air from an array of vertical parallel plates with discrete and protruding heat sources. *Int J Heat Fluid Flow* 17, pp 483–490, (1996).
- 5) Desrayaud, G, & Fichera, A : On Natural convective heat transfer in vertical channels with a single surface mounted heat flux module. *ASME J Heat Transfer* 125, pp 734–739 (2003).
- 6) Desrayaud G, Fichera A, Lauriat G : Natural convection air cooling of a substrate mounted protruding heat source in a stack of parallel boards. *Int J Heat Fluid Flow* 28, pp 469–482, (2007).
- 7) Rajkumar, M.R., Venugopal, G and Anil Lal ,S: Natural convection with surface radiation from a planar heat generating element mounted freely in a vertical channel. *Elliptic duct. International Journal of Heat Mass Transfer* DOI 10.1007/s0023-010-0734-z, (2011).
- 8) Ermolaev, A and Zhanov, A.I.: Mixed convection in vertical channel with Discrete Heat Sources at the wall. *Fluid Dynamics*, Vol.44, No.4, pp. 511-516. (2009).
- 9) Barletta and Nield, D.A.: Combined forced and free convective flow in a vertical porous channel: The effects of viscous dissipation and pressure work. *Transp Porous Med* 79:319-334, (2009).
- 10) Dileep Singh Chauhan and Priyanka Rastogi: Radiation effects on Natural convection MHD flow in a rotating vertical porous channel partially filled with a porous medium. *Applied Mathematical Sciences*, Vol.4, No.13, pp.643-655, (2010).