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Effect of Radial Temperature Gradient on Stability of Narrow- Gap Taylor-Dean Flow

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Abstract - *This paper presents a linear stability analysis for the Taylor-Dean flow of a viscous fluid between two concentric horizontal cylinders with a constant azimuthal pressure gradient, keeping the cylinders at different temperatures, when the inner cylinder is rotating and outer one is stationary. Here, the ratio of representative pumping and rotation velocities* $\boldsymbol{\beta}$ *and both positive and negative values of temperature gradient parameter N are considered, where N depends on the temperature differences* $T_2 - T_1$ *between the inner and outer cylinder. The analytical solution of the eigen value problem is obtained by using the trigonometric series method, when the gap between the cylinders is narrow. The critical values of parameters a and T are computed, where a is the wave number and T is the Taylor number, determining the onset of stability from the obtained analytical expressions for the second and third approximations. The critical values of T obtained by the third approximation agree very well with the earlier results computed numerically by differential transform method using unit disturbance scheme along with shooting technique. This clearly indicates that for the better result one should obtain the numerical values by taking more terms in approximation. Also, the amplitude of the radial velocity and the cell-patterns are shown on the graphs for different values of the parameters.*

Key Words: Stability; Radial temperature gradient; Taylor-Dean flow; Trigonometric series method; Rotating cylinders.

Nomenclature

a Dimensionless wave number
d Difference between two radii of the cylinders
R_1, R_2 Radii of inner and outer cylinders respectively
r, θ, z Cylindrical co-ordinates
u, v, w Velocity components in r, θ and z directions respectively
T_a Taylor number
Pr Prandtl number
N Radial temperature gradient
T_1, T_2 Temperature of inner and outer cylinder respectively
V_R The average velocity due to rotation
V_P The average velocity due to pumping.

Greek symbols

α Thermal diffusivity of the fluid
β Ratio of the pumping and rotational velocity
η Ratio of radii (R_1/R_2)
Ω_1, Ω_2 Angular velocity of the inner and outer cylinders respectively
μ Ratio of angular velocities (Ω_2/Ω_1)
ρ Density of fluid
λ Wave number of disturbance
ν Kinematic viscosity.

1. INTRODUCTION

The simplest example of a steady-circular flow of a viscous fluid between two rotating concentric cylinders is the Taylor-Couetee flow for which the laminar basic state is the circular Couette flow. In the absence of viscosity, the first criterion of stability was given by Rayliegh [1920]. For the case of viscous Couetee flow, theoretical and experimental investigations were performed for first time by Taylor [1923] for the case of small gap d $(d = R_2 - R_1)$ between the rotating cylinders. In Taylor problem, the stability of the fluid motion is due to the rotational velocity of the cylinders. If both the concentric cylinders are stationary, and the flow is due to the pressure gradient acting round the curved channel, then the effect of small disturbances on the stability of such a motion, was first studied by Dean [1928], known as the Dean problem. Later, Reid [1958] and Hammerlin [1958] studied the Dean problem for the narrow-gap case, whereas Walowit et al. [1964] studied it for the wide gap case.

When rotation and an azimuthal pressure gradient are both present, the problem of instability has some dinstictive feature which are absent from either Taylor or Dean Instability. This problem is known as Taylor-Dean problem, is the one discovered by Brewster & Nissan [1958] and by Brewster et al. [1959]. The same problem was further studied by DiPrima [1955, 1959], Meister [1962], Kruzweg [1963], Hughes and Reid [1964] and Raney and Chang [1971] for different physical conditions.In all these papers, it was basically assumed that the two cylinders are at the same temperature and as a result of which radial temperature gradient does not exist. However, in many chemical, electrical and mechanical engineering applications the temperature of two cylinders cannot remain the same. Thus, due to the change in the temperature of two cylinders, there exist a temperature gradient and the stability of the fluid flow is affected by the temperature gradient. Hence, Chandrashekhar [1954] studied the effects of the presence of a radial temperature gradient on the onset of instability in the narrow-gap case, Walowit et al. [1964] under wide gap approximation and experimentally by Becker and kaye [1962].

Further, the effect of radial temperature gradient on the stability of Dean flow was investigated by Ali et al. [1998] under narrow gap approximation and the effect of radial temperature gradient on the circular Couette flow was analysed by Mutabazi et al. [2001]. Chang [2003] investigated the linear stability of Taylor–Dean flow between porous concentric rotating cylinders in the presence of radial flow. The Taylor–Dean flow through a curved duct of square cross section, in which walls of the duct except the outer wall rotate around the center of curvature and an azimuthal pressure gradient was imposed, was analyzed by Yamamoto et al. [2004].

Later, Yamamoto et al. [2006] investigated experimentally, the secondary flow in a curved duct of square cross section, using a visualization method. After that, Soleimani and Sadeghy [2011] investigated numerically the stability of Bingham fluids in Taylor–Dean flow between two concentric cylinders at arbitrary gap spacing. Their results showed that the yield stress always has a stabilizing effect on the Taylor–Dean flow. Centrifugal instability of Bingham fluids was analyzed in Taylor–Dean flow when the gap size was large compared to the cylinder radii by Soleimani and Sadeghy [2011].The three dimensional linear stability analysis of Couette flow between two axial cylinders for shear-thinning fluids with and without yield stress was performed by Alibenyahia et al. [2012].

Recently, Mahapatra et al. [2013] studied the effect of radial temperature gradient on the stability of Taylor-Dean flow between two arbitrarily spaced concentric cylinders. They emphasized to the point if the two neutral stability curves crosses at some point for varying the radial temperature gradient parameter for given values of the ratio of pumping and rotation velocities of the cylinders. Stability of narrow-gap Taylor-Dean flow with radial heating is studied by Deka and Paul [2013]. In this paper we study the stability of narrow-gap Taylor-Dean flow, i.e. a viscous flow between concentric horizontal cylinders with a constant azimuthal pressure gradient, keeping the cylinders at different temperatures, when the inner cylinder is rotating and outer one is stationary. We have solved this problem by using the Trigonometric series method and the results are compared with those obtained by Deka and Paul [2013]. Also, the amplitude of the radial velocity and the cell-pattern are shown on graphs for different values of the parameters.

1. Mathematical Analysis

Consider the flow of an incompressible viscous fluid between two concentric horizontal cylinders of radii R_1 and R_2 (R_1, radius of the inner cylinder; R_2, radius of the outer cylinder), when the inner cylinder is rotating while the outer one is stationary, assuming that the inner and outer cylinders are maintained at two different temperatures T_1 and T_2 respectively and flow is due to a constant azimuthal pressure gradient.

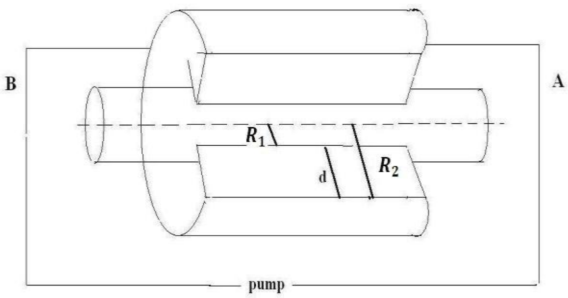

Fig.1. *Schematic of the Tayor-Dean system illustrating the Taylor-Dean flow*

Assuming stationary marginal state, the following differential equations have been obtained to govern the stability of Taylor-Dean flow of an incompressible viscous flow in a narrow-gap annular-space (Deka and Paul [2013]):

$$(D^2 - a^2)^2 u = -a^2 T_a \left[v r(x) + N\theta\big(r(x)\big)^2 \right] \quad (1)$$

$$(D^2 - a^2)\, v = [1 - \frac{\beta}{\alpha'}(1-2x)]u, \quad (2)$$

$$(D^2 - a^2)\theta = u. \quad (3)$$

with boundary conditions

$$u = Du = v = \theta = 0 \quad at\ x = 0\ and\ x = 1. \quad (4)$$

where,

$$d = R_2 - R_1, \qquad x = \frac{r - R_1}{d}, \qquad D = \frac{d}{dx},$$

$$a = \lambda d,\ \mu = \frac{\Omega_2}{\Omega_1}, \alpha' = 1-\mu,\ Pr = \frac{v}{\alpha},\ \eta = \frac{R_1}{R_2},$$

$$h(x) = x(1-x),\ \ g(x) = 1 - \alpha' x,$$

$$V_R = \frac{R_1\Omega_1(1+\mu)}{2}, V_P = -\left(\frac{\partial P}{\partial \theta}\right)\left(\frac{d^2}{12\nu R_1}\right),$$

$$\beta = \frac{3(1+\mu)V_P}{V_R}, \qquad r(x) = h(x) + \beta g(x),$$

$$T_a = -\frac{4A\Omega_1 d^4}{\nu^2} N = \frac{\alpha\Omega_1 \mathrm{Pr}(T_2 - T_1)}{4A ln\eta}, \theta = \frac{2A\bar{T}R_1 ln\eta}{Pr(T_2 - T_1)} \quad (5)$$

According to Geometric series method, we take a sine series for θ in order to satisfy the boundary conditions given by Eq. (4) as follows:

$$\theta = \sum_{m=1}^{\infty} A_m \sin(m\pi x). \quad (6)$$

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Substituting Eq. (6) in Eq. (3) and then with the help of Eq. (2) and (3), we obtain the value of v. Using these values of $\theta$and v in Eq. (1), we have obtained the general solution for u as follows:

$$u = -a^2T_a \sum_{m=1}^{\infty} A_m\left[(A_1^{(m)} + xA_2^{(m)})sinh(ax) + (A_3^{(m)} + xA_4^{(m)})cosh(ax) + K_{38}x^4 sinh(ax) + K_{39}x^3 sinh(ax) + K_{40}x^2 sinh(ax) + K_{41}x^4 cosh(ax) + K_{42}x^3.cosh(ax) + K_{43}x^2 cosh(ax) + K_{44}x^4 sin(m\pi x) + K_{45}x^3 sin(m\pi x) + K_{46}x^2 sin(m\pi x) + K_{47}x sin(m\pi x) + K_{48} sin(m\pi x) + K_{49}x^3 cos(m\pi x) + K_{50}x^2 cos(m\pi x) + K_{51}x cos(m\pi x) + K_{52} cos(m\pi x)\right]. \quad (7)$$

where,

$K_1 = (m^2\pi^2 + a^2)$, $K_2 = \frac{\beta}{\alpha'}$, $K_3 = \frac{4m\pi K_2}{K_1}$,

$K_4 = \frac{cosha - (-1)^m}{sinh(a)}$, $K_5 = \frac{(1-K_2)+N}{{K_1}^2}$,

$K_6 = \frac{2(K_2 - \alpha' N) - \alpha'(1-K_2)}{{K_1}^2}$, $K_7 = \frac{\alpha'(N\alpha' - 2K_2)}{{K_1}^2}$,

$K_8 = \frac{K_3(aK_4 - \alpha')}{8a^3}$, $K_9 = \frac{K_3K_4\alpha'}{24a^2}$, $K_{10} = \frac{K_3(\alpha' K_4 - a)}{8a^3}$,

$K_{11} = \frac{\alpha' K_3}{24a^2} K = K_1 - 6m^2\pi^2, K_{12} = K_5 + K + \frac{4m\pi K_3\alpha'}{{K_1}^3}$,

$K_{13} = K_6$, $K_{14} = \frac{4K_6 m\pi}{K_1} + \frac{K_3}{{K_1}^2}$, $K_{15} = \frac{8K_7 m\pi}{K_1} - \frac{\alpha' K_3}{{K_1}^2}$,

$K_{16} = -\frac{4(1+\alpha')K}{{K_1}^2}$, $K_{17} = \frac{K_3(aK_4 - 2)}{24a^3}$, $K_{21} = -\frac{K_3K_4}{48a^2}$

$K_{18} = \frac{K_3(3 - 2aK_4)}{16a^4}$, $K_{19} = \frac{K_3(2K_4 - a)}{24a^3}$,

$K_{20} = \frac{K_3(2a - 3K_4)}{16a^4}$, $S_1 = \frac{1-K_2}{{K_1}^2}$, $K_{22} = \frac{K_3}{48a^2}$,

$S_2 = \frac{2K_2(1+N\alpha')}{{K_1}^2}$, $K_{25} = \frac{3K_2 - 1 + N\alpha'^{K_2}}{{K_1}^2}$,

$P = 3K_1 - 8m^2\pi^2$, $K_{23} = S_1 + \frac{8K_3 m\pi}{{K_1}^3} - \frac{12S_2K}{{K_1}^2}$,

$K_{28} = K_{25} + \frac{24N\alpha' K_2 K}{{K_1}^4}$, $K_{31} = \frac{16K_2 m\pi\alpha' N}{{K_1}^3}$,

$K_{24} = \frac{4S_1 m\pi}{K_1} - \frac{4K_3K}{{K_1}^4} - \frac{24m\pi S_2 P}{{K_1}^3}$, $K_{32} = \frac{N\alpha' K_2}{{K_1}^2}$,

$K_{26} = \frac{4S_1 m\pi}{K_1} - \frac{4K_3K}{{K_1}^4} - \frac{24m\pi S_2 P}{{K_1}^3}$,

$K_{34} = \frac{4m\pi}{K_1} + \frac{24m\pi\alpha' K}{{K_1}^3}$, $K_{35} = \frac{8m\pi K_{36}}{K_1} = \frac{12\alpha' m\pi}{K_1}$,

$K_{27} = -\frac{4K_3 m\pi}{{K_1}^3} + \frac{4K_{25}K}{{K_1}^2} + \frac{8N\alpha' K_2(9{K_1}^2 - 144K_1m^2\pi^2 + 240m^4\pi^4)}{{K_1}^6}$,

$K_{29} = -S_2$, $K_{30} = -\frac{12S_2 m\pi}{K_1} - \frac{K_3}{{K_1}^3}$, $K_{33} = 1 + \frac{12\alpha' K}{{K_1}^2}$,

$K_{36} = -(1+\alpha')K_{38} = \alpha' K_2 K_{21}$, $K_{39} = -K_9 + \alpha' K_2 K_{17}$,
$K_{40} = K_8 + \alpha' K_2 K_{20}$, $K_{41} = \alpha' K_2 K_{22}$, $K_{49} = \alpha' K_2 K_{31}$,
$K_{42} = K_{11} + \alpha' K_2 K_{19}$, $K_{43} = K_{10} + \alpha' K_2 K_{18}$,
$K_{44} = \alpha' K_2 K_{32}$, $K_{45} = \alpha'(2K_{32} + K_2 K_{29})$,
$K_{46} = K_7 + \alpha' K_2 K_{28} + 2K_{36}K_{32}$, $K_{50} = \alpha' K_2 K_{30} + 2K_{32}K_{37}$
$K_{47} = K_{13} + \alpha' K_2 K_{23} + 2K_{33}K_{32}$, $M_1 = K_{38} + K_{39} + K_{40}$,
$K_{48} = K_{12} + \alpha' K_2 K_{27} + 2K_{32}K_{16}$, $K_{51} = K_{15} + \alpha'$.
$K_2 K_{26} + 2K_{35}K_{32}$, $K_{52} = K_{14} + \alpha' K_2 K_{24} + 2K_{34}K_{32}$,
$M_2 = K_{41} + K_{42} + K_{43}$, $M_3 = K_{49} + K_{50} + K_{51} + K_{52}$,
$M_4 = 4K_{38} + 3K_{39} + 2K_{40} + a(K_{41} + K_{42} + K_{43})$,
$M_5 = a(K_{38} + K_{39} + K_{40}) + 4K_{41} + 3K_{42} + 2K_{43}$,
$M_6 = m\pi(K_{44} + K_{45} + K_{46} + K_{47} + K_{48}) + 3K_{49} + 2K_{50} + K_{51}$,
$M_7 = M_2 - K_{52} - K_{51} - K_{48}.m\pi$, $M_8 = M_4 - a(K_{48}m\pi + aK_{51} + aK_{52})$, $M_9 = M_5 - K_{48}m\pi - K_{51}$,
$M_{10} = M_6 - M_3$, $M_{11} = M_9 - M_7 - aM_1$,
$M_{12} = M_8 - M_1$, $M_{13} = sinh^2 a - a^2$.

Using the boundary conditions (4), the constants of integration $A_1^{(m)}$, $A_2^{(m)}, A_3^{(m)}, A_4^{(m)}$ are as follows

$$A_1^{(m)} = \frac{1}{M_{13}}[M_{11} sinh(a)cosh(a) + M_{12} sinh^2 a + (-1)^m(M_{10} sinh(a) - aM_3.cosh(a)) - aM_7 cosh^2 a],$$

$$A_2^{(m)} = -\frac{1}{sinh(a)}[M_1 sinh(a) + M_7 cosh(a) + M_3(-1)^m + A_1^{(m)}(sinh(a) - acosh(a))],$$

$$A_3^{(m)} = -K_{52}, A_4^{(m)} = -[K_{48}m\pi + K_{51} + aA_1^{m}].$$

By inserting the mathematical expressions of $\theta$and u from Eqs. (6) and (7) respectively, in Eq. (3), we have,

$$\sum_{n=1}^{\infty} A_n K_1 sin(n\pi x) = a^2T_a \sum_{m=1}^{\infty} A_m[(A_1^{(m)} + xA_2^{(m)}). sinh(ax) + (A_3^{(m)} + xA_4^{(m)})cosh(ax) + K_{38}x^4 sinh(ax) + K_{39}x^3 sinh(ax) + K_{40}x^2 sinh(ax) + K_{41}x^4 cosh(ax) + K_{42}x^3 cosh(ax) + K_{43}x^2 cosh(ax) + K_{44}x^4 sin(m\pi x) + K_{45}x^3 sin(m\pi x) + K_{46}x^2 sin(m\pi x) + K_{47}x sin(m\pi x) + K_{48} sin(m\pi x) + K_{49}x^3 cos(m\pi x) + K_{50}x^2 cos(m\pi x) + K_{51}x cos(m\pi x) + K_{52} cos(m\pi x)]. \quad (9)$$

Multiplying Eq. (9) by $sin(m\pi x)$ and then integrating over the range $0 \le x \le 1$, we obtain a system of linear homogeneous equations for the constants and the requirement that these constants are to all zero leads to the following secular equation:

$$\left\|\left(K_{48} - \frac{K_1}{a^2T_a}\right)\delta_{mn} + A_1^{(m)}I_1 + A_2^{(m)}I_2 + A_3^{(m)}I_3 + A_4^{(m)}I_4 + K_{38}I_5 + K_{39}I_6 + K_{40}I_7 + K_{41}I_8 + K_{42}I_9 + K_{43}I_{10} + K_{44}I_{11} + K_{45}I_{12} + K_{46}I_{13} + K_{47}I_{14} + K_{49}I_{15} + K_{50}I_{16} + K_{51}I_{17} + K_{52}I_{18}\right\| = 0. \quad (10)$$

where,

$R = n^2\pi^2 + a^2$, $R_1 = \pi^2(m^2 + n^2)$, $R_2 = (3 + mn\pi^2)$,
$R_4 = (1 + mn\pi^2)$, $R_3 = (3 - mn\pi^2)$,
$R_5 = (1 - mn\pi^2)$, $I_1 = \frac{n\pi}{R}(-1)^{n+1} sinh(a)$,
$I_2 = \frac{n\pi}{R}[(-1)^{n+1} sinh(a) - \frac{2a}{R}((-1)^{n+1} cosh(a) + 1)]$

$I_3 = \frac{n\pi}{R}[(-1)^{n+1}cosh(a) + 1],$

$I_4 = \frac{n\pi(-1)^{n+1}}{R}[cosh(a) - \frac{2a}{R}sinh(a)],$

$I_5 = \frac{n\pi}{R}(-1)^n[-\sinh(a) + \frac{12\sinh(a)}{R^2}(n^2\pi^2 - 3a^2) + \frac{8\,a\cosh(a)}{R} + \frac{96\cosh(a)}{R^3}(a^2 - n^2\pi^2) + \frac{24\sinh(a)}{R^4}\{n^4\pi^4 - 5a^2(a^2 - 2n^2\pi^2)\}],$

$I_6 = \frac{n\pi}{R}[(-1)^n\{-\sinh(a) + \frac{6\,a\cosh(a)}{R} + \frac{6\sinh(a)}{R^2}.(n^2\pi^2 - 3a^2) + \frac{24a\cosh(a)}{R^3}(a^2 - n^2\pi^2)\} - \frac{24a}{R^3}(a^2 - n^2\pi^2),$

$I_7 = \frac{n\pi}{R}(-1)^n[\sinh(a) + \frac{2\sinh(a)}{R^2}(n^2\pi^2 - 3a^2) + \frac{4\,a\cosh(a)}{R}],$

$I_8 = \frac{1}{R^5}[24\{5a^2n\pi(a^2 - 2n^2\pi^2) + n^5\pi^5\} - n\pi(-1)^n \cosh(a)\{a^6(a^2 + 4(9 + n^2\pi^2)) + n^4\pi^4(24 - 12n^2\pi^2 + n^4\pi^4) + 4a^2n^2\pi^2(-60 + 3n^2\pi^2 + n^4\pi^4) + 6a^4(20 + 10n^2\pi^2 + n^4\pi^4)\} + 8n\pi a(-1)^n\{a^6(-12 + n^2\pi^2) + 3a^4 + (4 + n^2\pi^2)\}\sinh(a)].$

$I_9 = \frac{n\pi}{R}(-1)^n[-\cosh(a) + \frac{24a\sinh(a)}{R^3}(a^2 - n^2\pi^2) + \frac{6\,a\sinh(a)}{R} + \frac{6\,a\cosh(a)}{R^2}.(n^2\pi^2 - 3a)]\ ,$

$I_{10} = \frac{n\pi}{R}(-1)^n[-\cosh(a) + \frac{2\cosh(a)}{R^2}(n^2\pi^2 - 3a^2) + \frac{4\,a\sinh(a)}{R}] + \frac{2n\pi}{R^3}(3a^2 - n^2\pi^2),$

$$I_{11} = \begin{cases} \frac{1}{10} + \frac{3 - 2m^2\pi^2}{4m^4\pi^4}; & if\ m = n \\ \frac{2}{\pi^4}\left[\frac{(-1)^{m-n}(R_1 - 2R_2)}{(m-n)^4} - \frac{(-1)^{m+n}(R_1 - 2R_3)}{(m+n)^4}\right]; & if\ m \neq n \end{cases}$$

$$I_{12} = \begin{cases} \frac{1}{8}\left(1 - \frac{3}{m^2\pi^2}\right); & if\ m = n \\ \frac{3}{2\pi^4}\left[\frac{2 + (-1)^{m-n}(R_1 - 2R_4)}{(m-n)^4} - \frac{2 + (-1)^{m+n}(R_1 - 2R_5)}{(m+n)^4}\right]; & if\ m \neq n \end{cases}$$

$$I_{13} = \begin{cases} \frac{1}{6} - \frac{1}{4m^2\pi^2}; & if\ m = n \\ \frac{4mn(-1)^{m+n}}{\pi^2(m^2 - n^2)^2}; & if\ m \neq n \end{cases}$$

$$I_{14} = \begin{cases} \frac{1}{4}; & if\ m = n \\ \frac{2mn((-1)^{m+n} - 1))}{\pi^2(m^2 - n^2)^2}; & if\ m \neq n \end{cases}$$

$$I_{15} = \begin{cases} \frac{3}{8m^3\pi^3} - \frac{1}{4m\pi}; & if\ m = n \\ \frac{1}{2\pi^3}\left[\frac{(-1)^{m-n}(R_1 - 2R_2)}{(m-n)^3} - \frac{(-1)^{m+n}(R_1 - 2R_3)}{(m+n)^3}\right]; & if\ m \neq n \end{cases}$$

$$I_{16} = \begin{cases} -\frac{1}{4m\pi}; & if\ m = n \\ \frac{1}{2\pi^3}\left[\frac{2 + (-1)^{m-n}(M_6 - 2M_7)}{(m-n)^3} - \frac{2 + (-1)^{m+n}(M_6 - 2M_8)}{(m+n)^3}\right]; & if\ m \neq n \end{cases}$$

$$I_{17} = \begin{cases} -\frac{1}{4m\pi}; & if\ m = n \\ \frac{n(-1)^{m+n}}{\pi(m^2 - n^2)}; & if\ m \neq n \end{cases}$$

$$I_{18} = \begin{cases} 0; & if\ m = n \\ \frac{n((-1)^{m+1} - 1))}{\pi(m^2 - n^2)}; & if\ m \neq n \end{cases}$$

$$\delta_{mn} = \begin{cases} 1; & if\ m = n \\ 0; & if\ m \neq n \end{cases}$$

2. Results and Discussion

Table 1.1 Values of critical Taylor and wave numbers for different values of $\boldsymbol{\beta}$ for the case N = 0

	Branch I			
$\boldsymbol{\beta}$	a_c	T_{c_2}	T_{c_3}	T_c
1.0	3.0999	2432.09	2433.80	2433.85
0.0	3.126	3385.86	3389.84	3389.84
-1.0	3.233	5416.22	5416.80	5416.93
-3.0	6.418	40940.02	40941.43	40942.82
-3.25	6.959	51244.99	51246.66	51246.74
-3.5	7.493	62879.15	62880.15	62880.16
-3.6	7.768	68001.16	68002.17	68002.17
-3.65	7.974	70785.82	70785.83	70785.84
-3.666	8.061	71762.23	71762.99	71763.24
-3.667	8.064	71787.11	71788.12	71787.12
-3.7	8.283	73844.88	73846.66	73845.85

Table 1.2 Values of critical Taylor and wave numbers for different values of $\boldsymbol{\beta}$ for the case N = 0

	Branch II			
$\boldsymbol{\beta}$	a_c	T_{c_2}	T_{c_3}	T_c
-3.65	5.0311	76362.77	76364.8	76364.8
-3.666	5.407	71765.35	71766.2	71766.3

-3.667	5.413	71615.66	71618.3	71618.3
-3.7	5.700	66653.66	66654.7	66654.7
-3.75	5.727	61752.22	61753.2	61753.2
-3.8	5.697	57781.14	57782.1	57782.1
-3.9	5.630	511138.7	51140.8	51140.8
-4.0	5.570	45634.88	45634.8	45634.8

Table 2.1 Values of a_c and T_c for different values of N at $\beta^* = -3.666$

	Branch I			
N	a_c	T_{c_2}	T_{c_3}	T_c
-1.0	7.541	81934.00	81935.08	81935.09
-0.9	7.529	80589.28	80591.22	80590.28
-0.8	7.521	79288.18	79290.11	79290.10
-0.7	7.513	78035.44	78036.44	78036.49
-0.6	7.508	76831.22	76832.22	76832.29
-0.5	7.509	75684.30	75684.35	75684.33
-0.4	7.520	74601.99	74602.99	74602.98
-0.3	7.550	73609.66	73611.88	73610.81
-0.2	7.625	72749.77	72751.66	72751.66
-0.1	7.788	72104.33	72104.44	72104.34
0.0	8.059	71762.16	71763.17	71763.17
0.1	8.401	71744.24	71745.88	71746.24
0.2	8.469	71802.85	71803.85	71803.86

Table 2.2 Values of a_c and T_c for different values of N at $\beta^* = -3.666$

	Branch II			
N	a_c	T_{c_2}	T_{c_3}	T_c
-0.1	5.352	76824.12	76825.11	76825.13
0.0	5.405	71764.73	71766.77	71766.73
0.1	5.533	66414.00	66617.09	66417.10
0.2	5.627	61375.86	61377.99	61378.87

The numerical value of Ta_c computed from Eq. (10) corresponding to the second and third approximations are listed in Table 1 for different values of $\boldsymbol{\beta}$ for the case N = 0 and in Table 2 for different values of N at $\beta^* = -3.666$. In these tables Ta_2, Ta_3 represent the numerical values corresponding to the second and third approximations, while Ta_c is the values obtained by Deka and Paul [2013]. From these tables, we found that the values of Ta_c obtained by the third approximation agree very well with the values obtained numerically by Deka and Paul [2013] using the shooting method. From Table 1, we observe that for some fix value of N as we increase the value of $\boldsymbol{\beta}$ the numerical values of Ta_c decrease i.e. it destabilizes the flow.

Table 2 presents the effect of radial temperature gradient parameter N on the critical wave number and Taylor number. We observe that for some fix value of $\boldsymbol{\beta}$, when N is –ve (i.e the temperature of inner cylinder is higher than that of outer cylinder), as the temperature of inner cylinder increases the numerical values of Ta_c increases i.e. it stabilizes the channel flow and when N is +ve (i.e the temperature of outer cylinder is higher than that of inner cylinder), as N increases the numerical values of Ta_c drcreases i.e. it destabilizes the channel flow.

Other interesting phenomenon is to know the behaviour of the amplitude of the radial velocity and the corresponding cell-patterns. So, for a set of values of a_c and Ta_c, the values of $A_2^{(m)}/A_1^{(m)}$, $A_3^{(m)}/A_1^{(m)}$ are determined from Eq. (4). The eigenfunctions thus obtained are normalised so that the amplitude of the radial component of the velocity perturbation is unity. These eigenfunctions u(x) and the corresponding cell-pattern for the stream function $\Psi = u(x)\cos(a_c z)$ at the onset of instability for different values of β and N are shown in Figs. 2-6.

In Fig. 2, the cell patterns are shown for +ve values of N (for $N = 0.1$ and $N = 0.2$) at $\boldsymbol{\beta} = 1.0$. The most important conclusion we have from these figures is that as we increase the +ve value of N, the cells are shifted towards the inner cylinder. Physically, it is true because the convection currents are moving from the outer to the inner cylinder; thereby the left-handed edges of the cells start to become closer toward the innermost cell. Also the left-handed edge of the innermost cell is straightened and the corners are formed at the upper and lower ends of the innermost edge of the cell, and if the temperature is raised further such that the value of N increase beyond some particular value of N, the cells will start breaking through corners, and this confirms the destabilization of flow as N increases.

In Fig. 3, the cell patterns are shown for -ve values of N (for $N = -0.1$ and $N = -0.2$) at $\boldsymbol{\beta} = 1.0$. From these figures we found that when the temperature of inner cylinder is raised such that N changes from -0.1 to -0.2, the cells have shifted towards the outer cylinder. Physically it is true because the convection currents are moving from the inner to the outer cylinder; thereby the right-handed edges of the cells start to become closer toward the outermost cell. Also the right-handed edge of the outermost cell is straightened and the corners are formed at the upper and lower ends of the outermost edge of the cell, and if the temperature of inner cylinder is raised further, the cells will start breaking through corners, and this confirms the destabilization of flow.

In Fig. 4, the cell patterns are shown for $\boldsymbol{\beta}$ = 1.0, -1.0 at constant N= -0.4. From these figures we found that the cells have shifted towards the outer cylinder with increase

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in the value of $\boldsymbol{\beta}$. This confirms the destabilization of flow as we increase the value of $\boldsymbol{\beta}$.

In Fig. 5 and Fig. 6 u(x) is shown for $\boldsymbol{\beta}$ = 1.0, 0.0 and -0.1 corresponding to positive and negative values of N respectively. From these figures we find that for $\boldsymbol{\beta}$ = 1.0, the maximum of u(x) shifts toward the outer cylinder as compared to the case of $\beta = 0$, whereas for $\beta = -1.0$ the maximum of u(x) shifts toward the inner cylinder. From Fig. 5 we observe that as we increase the positive value of N, the maximum value of u(x) shifted toward the inner cylinder, whereas in the case when N is negative (Fig. 6), as we increase the negative the value of N, the maximum value of u(x) shifted towards the outer cylinder.

Fig.2. *Comparison of the cell pattern at the onset of instability for N = 0.1 (shown by continuous curve) and N = 0.2(shown by broken curve) at constant* $\boldsymbol{\beta}$ *= 1.0.*

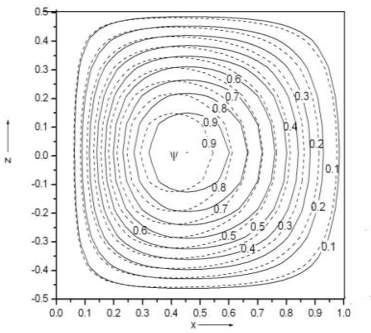


Fig.3. *Comparison of the cell pattern at the onset of instability for N = -0.1 (shown by continuous curve) and N = -0.2(shown by broken curve) at constant* $\boldsymbol{\beta}$ *= 1.0.*

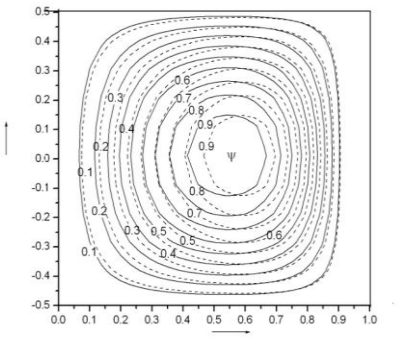


Fig.4. *Comparison of the cell pattern at the onset of instability for* $\boldsymbol{\beta}$ *= 1.0 (shown by continuous curve) and* $\boldsymbol{\beta}$ *= -1.0(shown by broken curve) at constant N = -0.4.*

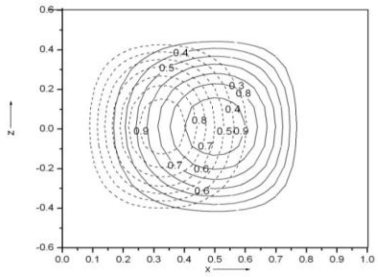


Fig.5. *The radial eigenfunction u(x) for N = 0.1 (shown by broken curve) and for N =0.2 (shown by continuous curve) for different values of* $\boldsymbol{\beta}$*.*

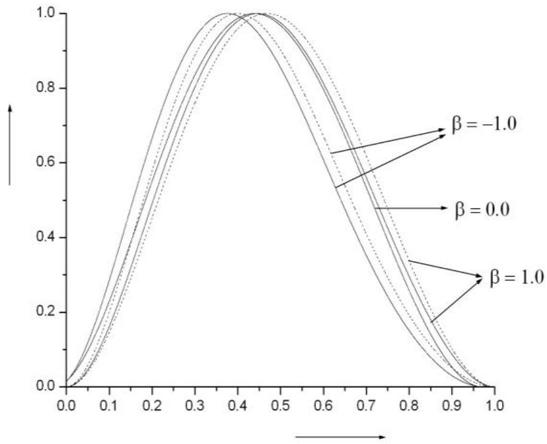
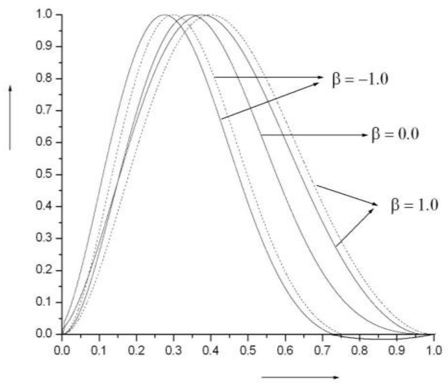


Fig.6. *The radial eigenfunction u(x) for N = - 0.2 (shown by broken curve) and for N = - 0.1 (shown by continuous curve) for different values of* $\boldsymbol{\beta}$*.*



3. CONCLUSIONS

The stability of the flow of a Newtonian viscous liquid between two coaxial horizontal cylinders has been investigated, when the inner one is rotating and the outer one is stationary, in the presence of a constant azimuthal pressure gradient, keeping the two cylinders at different temperatures. The following conditions have been obtained from the analysis:-

(1) When N is positive i.e. outer cylinder is at higher temperature than the inner cylinder, the maximum value of u(x) shifted more and more toward the outer cylinder as we increase the value of N.

(2) When N is negative i.e. inner cylinder is at higher temperature than the outer cylinder, the maximum value of u(x) shifted more and more toward the inner cylinder as we increase the temperature of inner cylinder.

(3) When $\boldsymbol{\beta}$ is positive, the maximum of u(x) shifts toward the outer cylinder as compared to the case of $\beta = 0$, whereas for negative values of β the maximum of u(x) shifts toward the inner cylinder.

(4) For some fix value of $\boldsymbol{\beta}$, as we increase the +ve value of N then the cells are shifted towards the inner cylinder and as we increase the –ve value of N then the cells are shifted towards the outer cylinder.

(5) For some fix value of N, as the value of $\boldsymbol{\beta}$ increase then the cells start shifted towards the outer cylinder.

(6) The channel flow is more and more stable when $\boldsymbol{\beta}$ and N both are negative.

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