HYBRID ALGORITHM BASED ON KALMAN FILTER AND CROSS CORRENTROPY FOR GPS RECEVIER POSITION AUGMENTATION

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Abstract – The Extended Kalman Filter is required because the uncertainty in the Minimum Mean Square Error (MMSE) estimation can be minimized. In this article, the researcher carried out a new kinematic positioning algorithm named as Cross-Correntropy Kalman Filter (CCKF) to enhance the position accuracy and performance of the Global Positioning System (GPS) receiver. Performance of the two algorithms (EKF and CCKF) are evaluated by considering the data of a dual frequency GPS receiver located at IGS station: IISc Bangolore (lat/lon: 13.01⁰ N/77.56 ⁰ E). In this article, batch processing data of IGS station, IISc Bangalore, obtained from IGS network of Scrips Orbit Permanent Array Center(SOPAC) is given as input and output yields in East, North and Up directions of the receiver position.

Key Words: Cross-Correntropy, Extended Kalman Filter, Cross-Correntropy Kalman Filter, Fixed-Point algorithm, Global Positioning System.

1. INTRODUCTION

Global Positioning System (GPS) is a three dimensional positioning system using many artificial satellites and has been used extensively in navigation systems, surveying, target tracking, etc., [1]. Nevertheless, there are various problems with GPS. For example, the positioning estimation cannot occur if the GPS receiver cannot receive the signals from more than three satellites [2]. Further, where barriers such as tall buildings block the radio wave, it is not possible to receive the signals from the satellite because of the straight path of the radio wave used in the GPS and the reflected wave. However, there is still a positioning error even if the GPS receiver can receive signals from more than three satellites. Nonetheless, as the above-mentioned difficulties cannot be overcome entirely with these solutions, many are aimed at improving the GPS itself. Such problems [3] should be addressed by introducing an effective positioning algorithm to reliably estimate receiver position [4] while enhancing the system itself, such as increasing the number of satellites, which is impractical. Therefore, we concentrate in this paper on the implementation of a new positioning algorithm called the Cross-Correntropy Kalman Filter (CCKF), based mainly on the criterion of cross-correntropy [5-6] and on the variance estimation method.

2. Extended Kalman Filter

Replacing the nominal trajectory with the projected trajectory is a simple, efficient solution for the deviation problem. The Extended Kalman Filter [2][7-10] is equivalent to a Kalman Filter except that linearization happens on the projected trajectory of the stream, rather than a conditional trajectory pre-computed. The Extended Kalman Filter provides a major method for working with nonlinear processes. Consider a nonlinear plant defined by the equations of nonlinear state and linear measurement:

$$\boldsymbol{s}(n) = \boldsymbol{a}(n-1)\boldsymbol{s}(n-1) + \boldsymbol{p}(n-1) \quad \textbf{(1)}$$

$$\boldsymbol{m}(n) = \boldsymbol{B}(n)\boldsymbol{s}(n) + \boldsymbol{q}(n) \quad \textbf{(2)}$$

Where,

$\boldsymbol{s}(n)$: i-dimensional state vector.

$\boldsymbol{m}(n)$: j-dimensional measurement vector at instant ‘n’.

$\boldsymbol{a}$: Non linear system function

$\boldsymbol{B}$: Observation matrix.

$\boldsymbol{p}(n-1)$: Process noise.

$\boldsymbol{q}(n)$: Measurement noise.

$$\left.\begin{array}{r} E[\boldsymbol{p}(n-1)\boldsymbol{p}^T(n-1) = \boldsymbol{P}(n-1) \\ E[\boldsymbol{q}(n)\boldsymbol{q}^T(n)] = \boldsymbol{Q}(n) \end{array}\right\} \textbf{(3)}$$

Similar to the EKF, the EKF includes two steps also, namely prediction and correction. The only change needed is to substitute partial derivatives in assessments.

3. Cross-Correntropy Kalman Filter

The Extended Kalman Filter works admirably under Gaussian noises. However, its output can mainly breakdown under non-Gaussian noises, particularly, when impulsive noises disturb the basic system. EKF's underlying purpose is to be implemented according to the MMSE criterion [11], which collects only second-order error signal statistics and is vulnerable to significant deviations. In this article, a new Kalman filter is implemented using the cross-correntropy criterion that can be better performed in non-Gaussian noisy systems

[5] [12-13], with cross-correntropy being adapted to second and higher order error statistics [14-15].

3.1 Steps in computing the Cross-Correntropy Kalman Filter

Step 1:
Initialize kernel size 'σ' and 'ε' which is a small positive number. Take 'n' as 1 and set $\hat{\mathbf{s}}(0|0),\mathbf{C}(0|0)$ which represents the initial estimate and covariance matrix respectively.
Step 2:
Compute $\hat{\mathbf{s}}(n|n-1)$ and $\mathbf{C}(n|n-1)$ by using Eq. (8) and (9) and also determine $\mathbf{D}_c(n|n-1)$ with the cholesky decomposition.
Step 3:
Assume't' as 1 and $\hat{\mathbf{s}}(n|n)_t = \hat{\mathbf{s}}(n|n-1)$,
Where $\hat{\mathbf{s}}(n|n)_t$: Estimated state at the fixed-point iteration t;
Step 4:
Compute $\hat{\mathbf{s}}(n|n)_t$ by using the Equations from (26) to (32)

$$\hat{\mathbf{s}}(n|n)_t = \hat{\mathbf{s}}(n|n-1) + \widetilde{\mathbf{N}}(n)(m(n) - \mathbf{B}(n)\hat{\mathbf{s}}(n|n-1))\mathbf{(4)}$$

$$\widetilde{\mathbf{N}}(n) = \tilde{\mathbf{C}}(n|n-1)\mathbf{B}^T(n)(\mathbf{B}(n)\tilde{\mathbf{C}}(n|n-1)\mathbf{B}^T(n) + \widetilde{\mathbf{Q}}(n))^{-1} \mathbf{(5)}$$

$$\tilde{\mathbf{C}}(n|n-1) = \mathbf{D}_c(n|n-1)\widetilde{\mathbf{H}}_x^{-1}(n)\mathbf{D}_c^T(n|n-1) \mathbf{(6)}$$

$$\widetilde{\mathbf{Q}}(n) = \mathbf{D}_q(n)\widetilde{\mathbf{H}}_y^{-1}(n)\mathbf{D}_q^T(n) \mathbf{(7)}$$

$$\widetilde{\mathbf{H}}_x(n) = \mathrm{diag}\left(J_\sigma\left(\tilde{e}_1(n)\right), \dots\dots\dots J_\sigma\left(\tilde{e}_a(n)\right)\right) \mathbf{(8)}$$

$$\widetilde{\mathbf{H}}_b(n) = \mathrm{diag}\left(J_\sigma\left(\tilde{e}_{a+1}(n)\right), \dots\dots\dots J_\sigma\left(\tilde{e}_{a+b}(n)\right)\right) \mathbf{(9)}$$

$$\tilde{e}_i(n) = f_i(n) - \mathbf{v}_i(n)\hat{\mathbf{s}}(n|n)_{t-1} \mathbf{(10)}$$

Step 5:
If (11) holds, put $\hat{\mathbf{s}}(n|n) = \hat{\mathbf{s}}(n|n)_t$ and continue to step (6) in case of comparing the estimations of current step and last step. if not, t+1 → t and go back to step (4).

$$\frac{\|\hat{\mathbf{s}}(n|n)_t - \hat{\mathbf{s}}(n|n)_{t-1}\|}{\|\hat{\mathbf{s}}(n|n)_{t-1}\|} \le \varepsilon\ \mathbf{(11)}$$

Step 6:
By using Eq. (12) update the matrix of corrected covariance, $n+1 \to n$ and go back to step (2).

$$\mathbf{C}(n|n) = \left(\mathbf{I} - \widetilde{\mathbf{N}}(n)\mathbf{B}(n)\right)\mathbf{C}(n|n-1)\left(\mathbf{I} - \widetilde{\mathbf{N}}(n)\mathbf{B}(n)\right)^T + \widetilde{\mathbf{N}}(n)\mathbf{Q}(n)\widetilde{\mathbf{N}}^T(n) \mathbf{(12)}$$

4. RESULTS & DISCUSSION

This paper proposes a new approach called the Cross-Correntropy Kalman Filter (CCKF) based on the cross-correntropy criterion and the fixed-point iterative algorithm for the problem of GPS based position estimation. The actual RINEX pseudo-range measurement dataset has been collected on 1st January, 2014 by the GPS receiver positioned at IISc, Bangalore, south zone of Indian sub continent (Lat/Lon: 13.01⁰ N/77.56 ⁰ E). The true location coordinates for the receiver are X=1337936.309m, Y=6070317.116m, and Z=1427876.908m. Since this is a huge data set, estimated receiver position error measured for X, Y, Z coordinates over a period of 10 epochs (randomly selected & collected for 30 sec. each) is shown in Table 1, whereas the estimated receiver position & errors measured over 22hrs are depicted in Fig. 1, Fig.2, and Fig. 3 respectively.

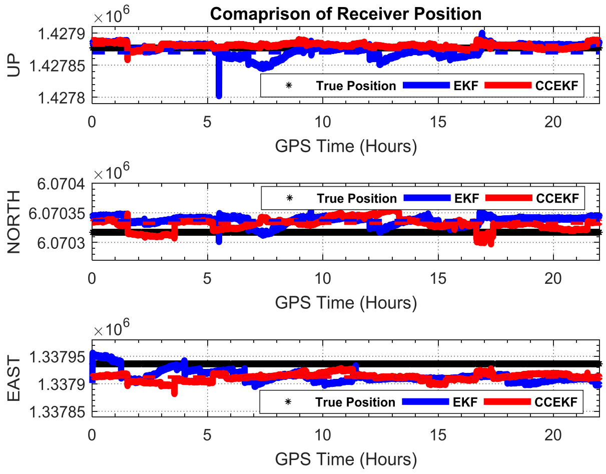

Fig-1: Comparison of Estimated Receiver Position

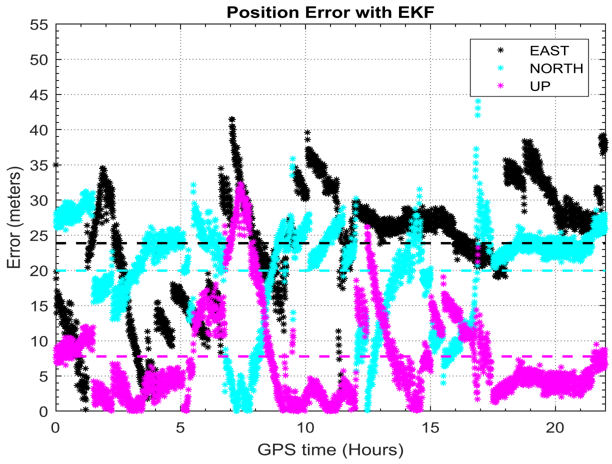

Fig-2: Comparison of Position Error with EKF

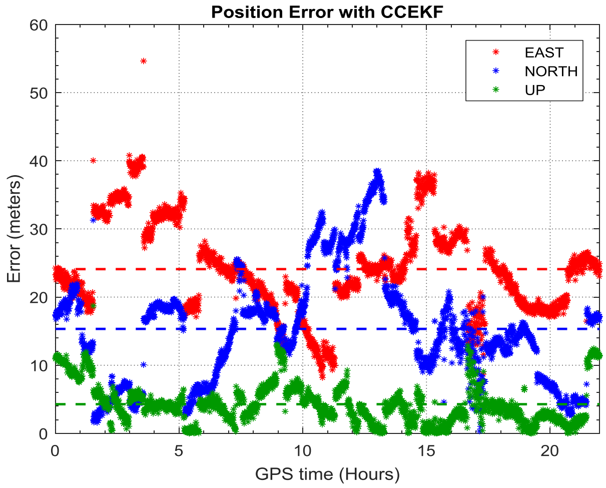

Fig-3: Comparison of Position Error with CCEKF

Table-1: Comparison of Position Error

GPS Time (Hours)	Error in Receiver Position					
	EAST		NORTH		UP	
	EKF (meters)	CCEKF (meters)	EKF (meters)	CCEKF (meters)	EKF (meters)	CCEKF (meters)
13.5297	25.75	19.11	25.09	6.70	19.26	3.87
13.5381	26.39	19.66	25.10	7.38	18.58	3.11
13.5464	25.39	19.27	25.11	6.38	19.58	3.88
13.5547	26.28	19.39	24.60	5.38	20.58	4.80
13.5631	25.28	19.61	24.78	4.38	21.58	3.63
13.5714	26.20	18.76	25.42	8.93	17.19	4.09
13.5797	25.20	18.76	25.42	7.93	18.19	4.09
13.5881	26.74	18.76	25.42	6.93	19.19	4.09
13.5964	25.74	20.71	23.82	5.93	20.19	4.80
13.6047	26.32	20.00	23.79	7.71	18.40	4.20

Table 2 provides information on EKF and CCKF's 3 error pdf’s over IISc, Bangalore GPS receiver data. These parameters are used to compute the accuracy measures of the 2-D and 3-D positions shown in Table 3. Accuracy metrics are the statistical approaches used to characterize the efficiency of the algorithm for GPS receiver position estimation. The results of CCKF show that the accuracy and precision are better than that of the Extended Kalman Filter algorithm.

Table-2: Comparison of Error Probability Measures

Error Probability Measures	EAST (meters)		NORTH (meters)		UP (meters)	
	EKF	CCEKF	EKF	CCEKF	EKF	CCEKF
Mean	23.88	24.10	19.8	15.33	7.78	4.28
Standard Deviation	8.28	6.54	7.19	8.17	6.85	2.93
Variance	68.55	42.79	51.74	66.72	46.92	8.59
Maximum error	41.49	54.64	44.06	38.51	75	18.71
Minimum error	0.24	8.19	0.03	0.34	0.00	0.00

The parameters for error analysis are shown in the above tables and are calculated for the entire data range. Finally, Compared the CCKF algorithm performance with the EKF algorithm, from the results, CCKF is observed to surpass the EKF algorithm by providing high accuracy and low variance in position estimation. The CCKF showed a mean difference of position of $E = 0.22\text{m}, N = 4.65\text{m}$ $U = 3.54\text{m}$ and position variance difference of $\text{var}(E) = 25.76\text{m}$, $\text{var}(N) = 14.98\text{m}$ and $\text{var}(U) = 38.33\text{m}$ over the EKF algorithm. The Statistical Accuracy Measures (SAM) [4][12] of the estimated receiver position is used to characterize position accuracy with a single value. These measures define accuracy for the position estimation with a single value contrary to the statistical moments of the mean and deviation allocated separately in the direction of East, North and Up. Table 3 provides a list that the most prevalently used accuracy measures of the 2-D and 3-D GPS receiver position.

Table-3: Statistical Accuracy Measures

Statistical Accuracy Measure (SAM)		EKF (meters)	CCEKF (meters)
2-D	DRMS(65%)	10.97	10.46
	2DRMS(95%)	21.93	20.93
	CEP (50%)	9.09	8.73
3-D	SEP (50%)	11.38	9.0
	MRSE(61%)	12.93	10.87
	SAS (99%)	25.04	19.79

From Table 3, it can be found that 50% of EKF's estimated horizontal point positions (i.e., E, N) are within 9.09m of the true position and 50% of the estimated 3-D point positions are within 11.38m, while CCKF is 8.73m and 9.0m respectively. In addition to that, the estimated error with the two algorithms (EKF & CCKF) in the receiver position is shown in Fig. 2 & 3. The position error scatter plot of EKF and CCKF algorithms represented with CEP value is shown in Fig. 4.

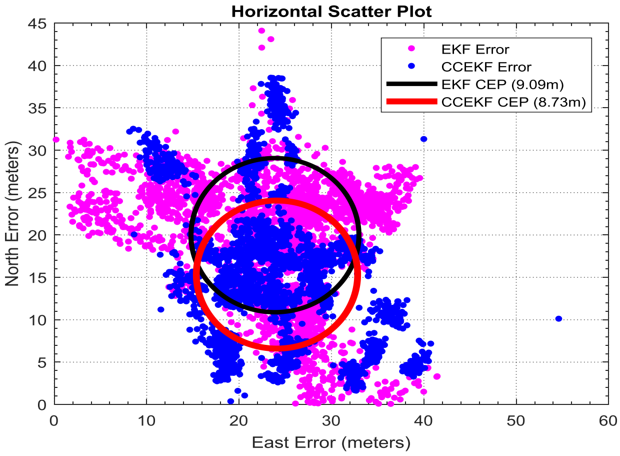

Fig-4: Horizontal Position Scatter Plot

Fig.4 reveals that the CEP circle CCKF is smaller than the CEP circle of EKF and the receiver position is nearer to the actual receiver position, 50 percent of the time relative to the EKF algorithm.

5. Conclusion

A new Kalman filter based on the criterion of correntorpy is proposed in this paper and tested with real-time GPS data. The results of the simulation show that the proposed filter out perform the Extended Kalman Filter. It is also evident that the CCKF results in estimates of the low variance position are compared to the EKF with a disparity in position deviation of $\text{std}(E) = 1.74\text{m}$, $\text{std}(N) = 0.98\text{m}$ and $\text{std}(U) = 3.92\text{m}$. Based on the results obtained in this paper, it can be concluded that the proposed CCKF algorithm can be used in precise positioning GPS applications, such as CAT I aircraft landing and geodesy.

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