Rough Hyperideals in Join Hyperlattice

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Abstract:

In this paper, we consider a rough hyperideals in join hyperlattice. Moreover, we investigate some theorems and properties for rough hyperideals in join hyperlattice.

Keywords:

Rough hyperideals, Hyper congruences, Hyperlattices

Introduction:

In this section, we introduce the notion of rough hyperideals in hyperlattices and discuss some properties of them.

Given a hyperlattice L, by $P^*(L)$ we will denote the set of all nonempty subsets of L. If θ is an equivalence relation on

L, then, for every $a \in L$, $[a]_\theta$ stands for the equivalence class of a with the represent θ. For any nonempty subset A of L, we denote $[A]_\theta = \{[a]_\theta | a \in A\}$. For any $A, B \in P^*(L)$, we denote $A\bar{\theta}B$ if the following conditions hold:

(1) for all $a \in A, \exists b \in B$ such that $a\theta b$;

(2) for all $d \in B, \exists c \in A$ such that $c\theta d$.

Now, we can introduce the notion of hyper congruences on hyperlattices in the following manner.

Definition 1. Let $(L, \wedge, \vee)$ be a hyperlattice. An equivalence relation θ on L is called a hyper congruence on L if for all a, a', b, $b' \in$ L the following implication holds: a$\theta a'$, and b$\theta b'$

imply (a ∧ b) $\bar{\bar{\bar{\theta}}}$(a ∧ b) and (a ∨ b) $\bar{\bar{\bar{\theta}}}$ (a ∨ b).

Obviously, an equivalence relation θ on $(L, \wedge, \vee)$ is a hyper congruence if and only if for all a, b, x $\in L$, we have that $a\theta b$ implies (a ∧ x) $\bar{\theta}$ (b ∧ x) and (a ∨ x)$\bar{\theta}$ (b ∨ x).

Lemma 2. Let $(L, \wedge, \vee)$ be a hyperlattice, and let θ be a hyper congruence on L. For all $a, b, \in L$, then $[a]_\theta \wedge [b]_\theta \subseteq [a \wedge b]_\theta$.$[a]_\theta \vee [b]_\theta \subseteq [a \vee b]_\theta$.

Proof. Suppose that x $\in [a]_\theta \wedge [b]_\theta$, then there exist $x_1 \in [a]_\theta$ and $x_2 \in [b]_\theta$ such

That x $\in x_1 \wedge x_2 \in$. Since aθx_1, bθx_2, by Definition 1, we have (a ∧ b) θ $(x_1 \wedge x_2)$.

x $\in x_1 \wedge x_2$ implies that there exists y $\in$ a ∧ b such that $x\theta y$. Therefore, we have

x$\in [a \wedge b]_\theta$, which implies $[a]_\theta \wedge [b]_\theta \subseteq [a \wedge b]_\theta$. Similarly, we can prove that

$[a]_\theta \vee [b]_\theta \subseteq [a \vee b]_\theta$.

A hyper congruence relation θ on $(L, \wedge, \vee)$ is called ∧-*complete* if

$[a]_\theta \wedge [b]_\theta = [a \wedge b]_\theta$ for all a, b $\in L$. Similarly, θ is called ∨-*complete* if

$[a]_\theta \vee [b]_\theta \subseteq [a \vee b]_\theta$ for all a, b $\in L$. We call θ *complete* if it is both ∧-complete and ∨-Complete. Now, we briefly recall the rough set theory in Pawlak's sense.

Let θ be an equivalence relation on L, and let A be a nonempty subset of L.

Then, the sets $\theta(A) = \{x \in L \mid [x]_\theta \cap A \neq \emptyset\}$ and $\theta(A) = \{x \in L \mid [x]_\theta \subseteq A\}$ are called, respectively, the *upper and lower approximations* of A with respect to θ. $\theta(A) = (\theta(A), \bar{\theta}(A))$ is called a *rough set* with respect to θ.

Proposition 3. *Let θ be a hyper congruence on a hyperlattice $(L, \wedge, ⍰)$. If A, B are two nonempty subsets of L, then*

(i) $\bar{\theta}(A) \wedge \bar{\theta}(B) \subseteq \bar{\theta}(A \wedge B)$. In particular, if θ is a $\wedge$ −complete, then $\bar{\theta}(A) \wedge \bar{\theta}(B) = \bar{\theta}(A \wedge B)$.

(ii) $\bar{\theta}(A) \vee \bar{\theta}(B) \subseteq \bar{\theta}(A⍰B)$. In particular, if θ is a $\vee$ −complete, then $\bar{\theta}(A) \vee \bar{\theta}(B) = \bar{\theta}(A⍰B)$.

Proof:

Suppose that $x \in \bar{\bar{\theta}}(A) \wedge \bar{\theta}(B)$. There exist $x_1 \in \bar{\theta}(A)$ and $x_2 \in \bar{\theta}(B)$ such that $x \in x_1 \wedge x_2$.

It follows that there exists a, b ∈ L such that a ∈ $[x_1]_\theta \cap$ A and b ∈ $[x_2]_\theta \cap$ B. Since θ is a hyper congruence on L, we have $a \wedge b \subseteq [x_1]_\theta \wedge [x_2]_\theta \subseteq [x_1 \wedge x_2]_\theta$ by lemma 2.

On the other hand, since $a \wedge b \subseteq A \wedge B$, we obtain $a \wedge b \subseteq [x_1 \wedge x_2]_\theta \cap$

$(A \wedge B)$, which implies $x \in x_1 \wedge x_2 \subseteq \bar{\bar{\theta}}(A \wedge B)$. Therefore $\bar{\theta}(A) \wedge \bar{\theta}(B) \subseteq \bar{\theta}(A \wedge B)$.

If θ is $\wedge$ −complete, let $x \in \bar{\bar{\theta}}(A \wedge B)$, then $[x]_\theta \cap (A \wedge B) \neq \emptyset$. Therefore, there exists $y \in [x]_\theta \cap (A \wedge B)$, and so for some a ∈ A and b ∈ B, we have $y \in a \wedge b$. Since θ is a $\wedge$ −complete, we can obtain $x \in [y]_\theta \subseteq [a \wedge b]_\theta = [a]_\theta \wedge [b]_\theta$.

Thus, there exists $x_1 \in [a]_\theta$ and $x_2 \in [b]_\theta$ such that $x \in x_1 \wedge x_2$.

It follows that $a \in [x_1]_\theta \cap A$ and $b \in [x_2]_\theta \cap B$. Hence, $x_1 \in \bar{\bar{\theta}}(A)$ and $x_2 \in \bar{\bar{\theta}}(B)$, and we have $x \in x_1 \wedge x_2 \subseteq \bar{\theta}(A) \wedge \bar{\theta}(B)$. Therefore, $\bar{\theta}(A) \wedge \bar{\theta}(B) = \bar{\bar{\theta}}(A \wedge B)$.

(2) is similar to that of (1).

Proposition 4: Let θ *be a hyper congruence on a hyperlattice $(L, \wedge, ⍰)$ and A, B are two nonempty subsets of L, then*

(i) If A and B are two $\wedge$-hyperideals of L, then $\bar{\theta}(A) \wedge \bar{\theta}(B) = \bar{\theta}(A \wedge B)$.

(ii) If A and B are two $\vee$-hyperideals of L, then $\bar{\theta}(A) \vee \bar{\theta}(B) = \bar{\theta}(A \vee B)$.

(1) Let $x \in \bar{\theta}(A \wedge B)$, then there exist a ∈ A and b ∈ B such that $[x]_\theta \cap (a \wedge b) \neq \emptyset$, which implies that there exists $t \in a \wedge b$ such that $x\ \theta\ t$. Since A is a $\wedge$-hyperideal of L, we have $a \wedge b \subseteq A$. It follows that t ∈ A. Hence, we obtain that $[x]_\theta \cap A = [t]_\theta \cap A \neq \emptyset$, which implies $x \in \bar{\theta}(A)$. In a similar way, we have $x \in \bar{\theta}(B)$. Thus, $x \in x \wedge x \subseteq \bar{\theta}(A) \wedge \bar{\theta}(B)$.

Combining proposition 3, we have $\bar{\theta}(A) \wedge \bar{\theta}(B) = \bar{\theta}(A \wedge B)$.

(2) The proof is similar to that of (1).

Proposition 5: Let θ be a hypercongurence relation on a hyperlattice (L, $\wedge$, ⍰). If A and B are $\wedge$-hyperideals (⍰ -hyperideals) of L, then $\bar{\theta}(A \cap B) = \bar{\theta}(A) \cap \bar{\theta}(B)$.

Proof: Let $x \in \bar{\theta}(A) \cap \bar{\theta}(B)$, we have $[x]_\theta \cap A \neq \emptyset$ and $[x]_\theta \cap B \neq \emptyset$. Then, there exist $x_1 \in A$ and $x_2 \in B$ such that $x_1 \theta x$ and $x_2 \theta x$. It follows from θ which is a hyper congruence relation that $x_1 \wedge x_2 \bar{\theta} x \wedge x$, which implies that there exists $t \in x_1 \wedge x_2$ such that $t\ \theta x$. Since A and B are $\wedge$-hyperideals of L, we have $x_1 \wedge x_2 \subseteq A \cap B$. So, $t \in A \cap B$. It follows that $[x]_\theta \cap (A \cap B) = [t]_\theta \cap (A \cap B) \neq \emptyset$, which implies $x \in \bar{\theta}(A \cap B)$. Hence, $\bar{\theta}(A) \cap \bar{\theta}(B) \subseteq \bar{\theta}(A \cap B)$. On the other hand, it is clear that $\bar{\theta}(A \cap B) \subseteq \bar{\theta}(A) \cap \bar{\theta}(B)$. Therefore, $\bar{\theta}(A \cap B) = \bar{\theta}(A) \cap \bar{\theta}(B)$. In a similar way, if A and B are ⍰ -hyperideals of L, we can also obtain $\bar{\theta}(A \cap B) = \bar{\theta}(A) \cap \bar{\theta}(B)$.

Next, we will introduce and investigate a new algebraic structure called rough hyperideals in join hyper lattices. Let us begin with introducing the following definitions.

Definition 6: Let θ be a hypercongruence on a hyperlattice (L, ∧, ∨), and let A be a non empty subset of L. A is called a lower (an upper)rough sub hyperlattice of L if θ (A)($\bar{\theta}(A)$) is a sub hyperlattice of L. A is called a rough sub hyperlattice of L if A is both a lower rough sub hyperlattice and an upper rough sub hyperlattice of L.

Similarly, A is called a lower (an upper) rough ∧ -hyperideal of L if (A)($\bar{\theta}(A)$) is a ∧-hyperideal of L. And we call A as rough ∧-hyperideal of L if A is both a lower rough ∧-hyperideal and an upper rough ∧-hyperideal of L. In a similar way, a rough ∨ -hyperideal of L can be defined.

Example 7: Let L = {a, b, c, d} be the hyperlattice. Let θ be a hyper congruence relation on the hyperlattice L with the following equivalent classes: $[a]_\theta$ = {a, b}, $[c]_\theta$ = {c, d}. Considering A = {a, b, c}, we can obtain that (A) = {a, b} $\bar{\theta}(A)$ = L.

Notice that {a, b} and L are ∧-hyperideals, so A is a rough ∧-hyperideal of L. If A = {b, c, d}, we have that (A) = {c, d} and $\bar{\theta}(A)$ = L. we obtain that {c, d} and L are ∨-hyperideals, so A is a rough ∨-hyperideal of L.

Example 8: In example 7, A = {a, b, c} is a rough ∧-hyperideal of (L, ∧, ∨), but A is not a ∧-hyperideal of L.

Conclusion:

Hence, we have successfully introduced the Rough hyperideals in join hyperlattice. And we investigated some of their properties.

Reference:

[1] https://www.researchgate.net/publication/340109245_Fuzzy_Soft_Hyperideals_In_Join_Hyperlattices

[2]https://www.researchgate.net/search?context=publicSearchHeader&q=