

Studying the Applications of Linear Programming in Basic Science and Engineering Curriculum

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Abstract – Linear Programming deals with the statistical data to maximize or minimize the given function of variables. It provides the complete structure of Linear Programming. Considering the assumptions we can determine the different test statistics in Linear programming models like transportation, Variation transportation, unbalance business supply and demand problems, Degeneracy and its resolution problems etc., LPP provides different methods of finding initial feasible solutions with given data. Graphical Method representations can be utilized for representation of test statistics for easy understanding. Simplex applications, Penalty methods, Least Cost Approximation Methods, Degeneracy methods are the major applications in Linear Programming Solutions. Properties of LPP and graphical method of solving a LPP with distinct variable cases Dual form, are also discussed in this Study of Research paper. Solving the Analytical problems using Matrix form of LP, Two Phase method approaches, Computational procedure of Simplex methods, Artificial Variable Techniques are also discussed in this Study of Linear Programming Problem.

Key Words: Artificial variable, Degeneracy, Dual form, Graphical method, Penalty method, Simplex method, Two phase method,

1. INTRODUCTION

Linear programming provides the optimisation of a function of variables known as objective functions. It consists set of linear equalities or inequalities. In evaluation methods we consider the approach as mathematical technique which involves the allocation of limited resources in a optimal manner. Properties of Linear Programming Problems with distinct approaches are discussed in this Research method. In majority of cases we use simplex method using the Linear Programming Problem. In case of more than two variables Graphical solution is not possible in such stages for further evaluation we use Simplex method. In general simplex method provides a systematic mathematical algorithm which consists of moving from one basic feasible solution to another basic feasible solution in a systematic manner so that the values of the objective functions is improved at each level.

One of the most important discoveries in the early development of linear programming was the concept of duality and its division into important stages. The optimal solution of given problem reveals information related to the optimal solution of the other. If we know the optimal solution result then the optimal solution of the other is readily available. Formulation of LPP in standard form, Graphical solution of two variables, Geometric properties of LPP, Standard form of LPP, Introduction of surplus and slack variables, matrix form of LPP, Applications of LPP and their Advantages, Artificial variable approach methods, Penalty methods are introduced in Linear Programming Optimization in Business Statistics and Science and Engineering Organizations.

2. FORMULATION OF LPP

In any given statistical Analysis if we consider products x_1, x_2 elements respectively in A, B Objective functions such that then the objective function can be represented in terms of Objective function $Z = x_1 + x_2$

Example-1 : If a company produces two types of chairs. Each chair of the first type requires twice as much labor time as the second type. If all chairs are of the second type only, the company can produce a total of 300 chairs a day. The market limits daily sales of the first and second type to 100 and 200 chairs. Assuming that the profits per chairs are Rs.10 for type A and Rs.6 for type B, formulate the problem as a linear programming model in order to determine the number of chairs to be produced of each type so as to maximize he profit.

Example-1: From the given statistics Objective function for profit :

$$\text{Max}(z) = 10x_1 + 6x_2$$

$$\text{Stc:- } 2x_1 + x_2 \leq 300 ; \text{ where } x_1 \leq 100 \text{ and } x_2 \leq 200$$

Considering there will not any negative quantities such that $x_1, x_2 \geq 0$

2.1 Decision Variables

The given variables which are required to be determined using distinct approach methods are said to be decision variables.

Example-2 : If $8x_1 + 10x_2 \leq 24$ is an equation then a Slack variable(s_1) must be added on LHS of inequality.

$8x_1 + 10x_2 + S_1 = 24$; where $x_1, x_2, S_1 \geq 0$

Exempl-3 : If $12x_1 + 15x_2 \geq 30$ is an equation then a Surplus variable(S_2) must be added on RHS of inequality.

i.e., $12x_1 + 15x_2 = 30 + S_2$; $12x_1 + 15x_2 - S_2 = 30$;

where $x_1, x_2, S_2 \geq 0$

2.2 Artificial Variables

The variables which should be added in case of minimization of Linear Programming Problem along with Surplus variables is known as artificial variables such conditions are extension form of Surplus and Slack variable conditions.

Exempl-4 : If $5x_1 + 10x_2$ is a minimum function

Then there exists $\text{Min}(z) = 5x_1 + 10x_2$

Stc:- $2x_1 - 3x_2 \geq 9$ and $15x_1 + 25x_2 \leq -300$

\therefore Artificial Variables Introduction will be as follows :

Subject to the Constraints are : $2x_1 - 3x_2 - S_1 + A_1 = 9$ and

$$15x_1 + 25x_2 + S_2 = -300$$

Where S_1 = Surplus variable and S_2 is Slack variable

3. APPLICATIONS OF LPP

- Marketing Management – LP can be applied in life cycle marketing areas which combines the advertising campaign, advertising media and travelling salesman etc.
- Agricultural Applications – LP can be applied in agricultural planning for allocating the limited resources like water, labor working capital etc., as to maximize the profit.
- Military Sectors – LP model can be used for various military operations like wars, defense against communal clashes, terrorists, military hospitals, canteens etc., Which minimize the cost of operations.
- Transportation Problem – Linear Programming helps in determining the optimum transportation schedule with minimum total transportation cost of moving goods from various origins to various destinations.
- Assignment Problem – Linear Programming methods give the best assignment schedule with minimum total cost of assignment of various available resources to various activities on a one to one basis.
- Manpower Management – Linear programming can be applied to analysis personal policy combinations in terms of their appropriateness of people into the firm and out of the firm.
- Environmental – Linear programming can be used analyze alternatives for handling liquid waste material in order to satisfy antipollution requirements of different organizations.

- Airline Sectors – Linear Programming can be used to determine the most economic pattern and timing for flights.
- Financial Sectors – Linear Programming is most useful in financial sectors. Mix strategy problems can be solved using Linear Programming methods.

4. LIMITATIONS OF LPP

- Linear Programming deals with only single objective, whereas in real life situations problems come across with multiple objectives.
- Linear Programming problem does not take into consideration the effect of time and uncertainty.
- In few problems objective functions and constraints are not linear. In general Business and industrial problems constraints are not linearly treated to variables.
- In few problems large scale problems cannot be solved with LP techniques even when the computer facility is available. Such issues may be removed by decomposing the main problem into several small problems and then we can solve independently.
- There is no guarantee of getting integer valued solutions in real life situations like business and industrial problems.

5. GRAPHICAL SOLUTIONS OF LPP

We consider each inequality constraint as equation. Indicate given equation on the graph such that each equation represent a straight line. For the inequality constraint \geq representation, the region above the line in the first quadrant is shaded. The point lying in common area will satisfy all the given subject to the constraints. Consider arbitrary value (Zero) for the objective function. Using the Straight line extend the objective function line till the extreme points of the feasible region. Using this we can find the co ordinates of the extreme points. In this process we can find the maximum or minimum value of Objective function.

Example-5 : $\text{Max}(Z) = 12x_1 + 16x_2$ Solving the LP problem using the graphical method given subject to the constraints

$$10x_1 + 20x_2 \leq 120$$

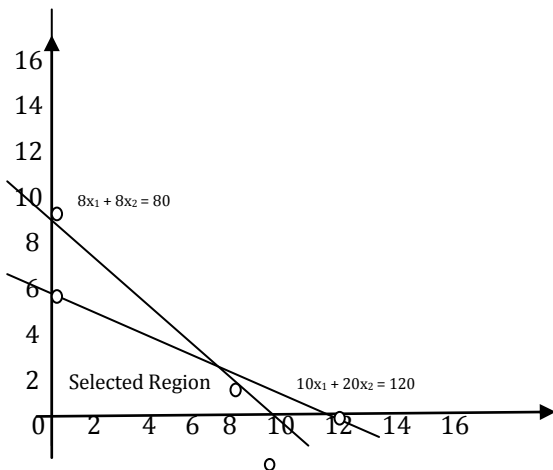
$$8x_1 + 8x_2 \leq 80 \text{ where } x_1, x_2 \geq 0$$

$$\text{Sol : Solving STC } 10x_1 + 20x_2 = 120 \text{ \& } 8x_1 + 8x_2 = 80$$

$$(10x_1 + 20x_2) / 120 = (x_1 / 12) + (x_2 / 6) = 1 \text{ Eqn(1)}$$

$$(8x_1 + 8x_2) / 80 = (x_1 / 10) + (x_2 / 10) = 1 \text{ Eqn(2)}$$

Plotting each equation in x, y axis for given LPP graphical representation and for further simplification process



Corresponding Coordinates: (0,0), (0,6), (10,0), (8,2)

By Solving the STC $x_1 = 8$ and $x_2 = 2$

Objective function with respect to the above co ordinates

$$\text{Max}(Z) = 12x_1 + 16x_2 \text{ (given)}$$

$$\therefore \text{for } (0,0) = \text{Max}(z) = 0 \text{ similarly}$$

$$\text{for } (0,6) = \text{Max}(z) = 96$$

$$\text{for } (10,0) = \text{Max}(z) = 120$$

$$\text{for } (8,2) = \text{Max}(z) = 128 \rightarrow \text{Max. Value}$$

\therefore Optimum Solution of LPP exists at $x_1 = 8$ and $x_2 = 2$

$$\text{Max}(Z) = 12x_1 + 16x_2 \rightarrow Z = 128$$

6. SOLVING LPP USING SIMPLEX METHOD

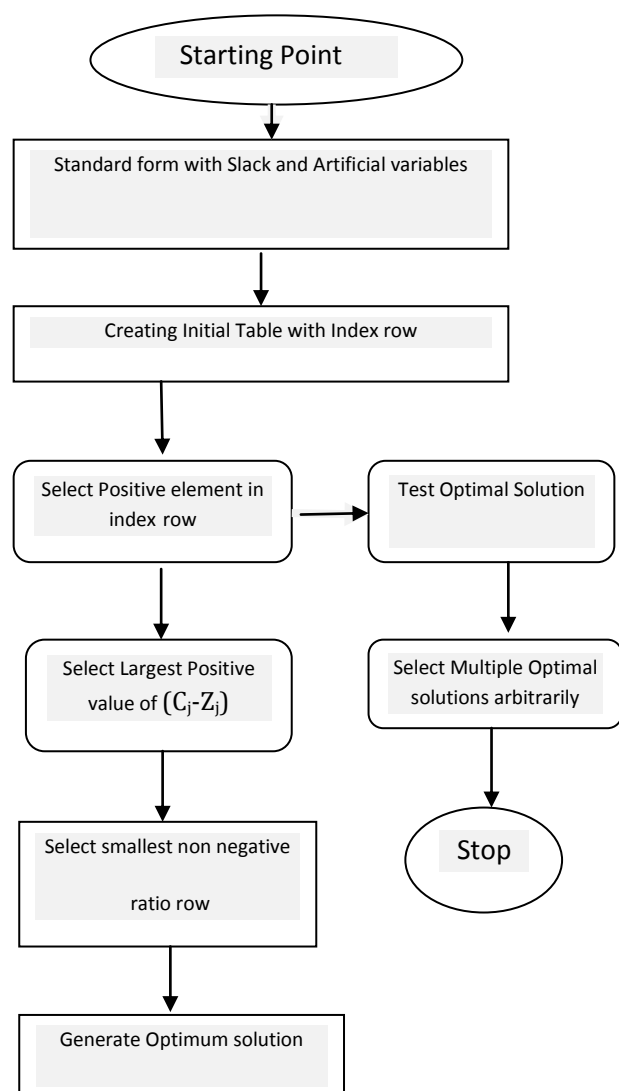
In extension of graphical method to overcome few drawbacks we use simplex method. This is suitable method to provide an algorithm which have its basis related to fundamental concept of LPP. The entire simplification process is a systematic algorithm approach moves from one basic solution to another basic solution. It consists of Minimization and maximization approaches towards the profit and loss of distinct sectors.

In simplex method we use certain steps to solve the given problem :

- ✚ In formation of Standard Objective function we Convert inequalities into equalities, adding slack variables, surplus variables and artificial variables based on type of inequality in the given problem.
- ✚ From the coefficient table and find the initial basic feasible solution.
- ✚ Calculation Z_j for given each variables

- ✚ Calculating $(C_j - Z_j)$ for given each variable.
- ✚ We consider Objective function(Z) is optimal if $\Delta_j = (C_j - Z_j) \leq 0$
- ✚ If $(C_j - Z_j) \leq 0$ does not exists then select maximum value from Δ_j , with at least one coefficient to this variable is positive.
- ✚ By calculating the ratios in each column and select the least non negative ratio.
- ✚ Select the row of departing variable and column of the entering variable and key element.
- ✚ Divide the row of the departing variable by the key element and set suitable transformation to make the other entering elements becomes zero. With this find the initial basic feasible solution.

7. SIMPLEX METHOD FLOW CHART



Example- 6 : Solving LPP using Simplex $Max(z)=3p_1 + 2p_2$

Subject to the constraints $p_1 + p_2 \leq 4$;
 $p_1 - p_2 \leq 2$;
 where $p_1, p_2 \geq 0$

Sol:- Let the Standard form:

$$Max(Z) = 3p_1 + 2p_2 + 0S_1 + 0S_2$$

Subject to the Constraints : $p_1 + p_2 + S_1 + 0S_2 = 4$;
 $- p_2 + p_1 + 0S_1 + S_2 = 2$;
 where each individual variables are ≥ 0

Assume that $AX=B$

$$A = \begin{pmatrix} p_1 & p_2 & S_1 & S_2 \\ 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{pmatrix}; X = \begin{pmatrix} p_1 \\ p_2 \\ S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

Initial Table with Index Values :

		C _j	3	2	0	0	
C _B	B	X _B	p ₁	p ₂	S ₁	S ₂	Min X _B /p ₁
0	1S ₁	4	1	1	1	0	4
0	1S ₂	2	1	-1	0	1	2 →
	Z _j	0	0	0	0	0	
	Z _j -C _j		-3	-2	0	0	



We can observe in the above simplification $Z_j - C_j \leq 0$, hence current basic feasible solution is not optimum, Continuing the simplex process....

Applying Row transformation $R_2 - R_1$,
 X_1 entered S_2 removed

		C _j	3	2	0	
C _B	B	X _B	p ₁	p ₂	S ₁	Min X _B /p ₂
0	S ₁	2	0	2	1	2 / 2 = 1 →
3	p ₁	2	1	-1	0	
	Z _j	6	3	-3	0	
	Z _j -C _j	6	0	-5	0	



We can observe in the above simplification $Z_j - C_j \leq 0$, hence current basic feasible solution is not optimum, Continuing the simplex process....

Applying $R_1 : (R_1 / 2)$ to Key Row & $R_2 : R_2 + R_1$. New

		C _j	3	2
C _B	B	X _B	p ₁	p ₂
2	p ₂	1	0	1
3	p ₁	3	1	0
	Z _j	11	3	2
	Z _j -C _j		0	0

$Z_j - C_j \geq 0$, Hence given is having Optimal solution at
 $p_1 = 3$; $p_2 = 1$
 $\therefore Max(Z) = 2(3) + 3(1) = 9$

Example- 7 : Solving LPP using Simplex $Min(z)=2x_2 - 3x_3 + 5x_5$; Stc:- $3x_2 - x_3 + 2x_5 \leq 7$; $-2x_2 + 4x_3 + 2x_5 \leq 12$; $-4x_2 + 3x_3 + 8x_5 \leq 10$; where All the given decision variables $x_2, x_3, x_5 \geq 0$

Sol:- Let the Standard form: $Min(Z) = Max(-Z)$

$Max(Z) = -2x_2 + 3x_3 - 5x_5 + 0S_1 + 0S_2 + 0S_3$
 Subject to the Constraints : $3x_2 - x_3 + 2x_5 + S_1 = 7$;
 $2x_2 + 4x_3 + 2x_5 + S_2 = 12$;
 $-4x_2 + 3x_3 + 8x_5 + S_3 = 10$;
 where $x_2, x_3, x_5, S_1, S_2, S_3 \geq 0$

		C _j	-2	3	-5	0	0	0	
C _B	B	X _B	X ₂	X ₃	X ₅	S ₁	S ₂	S ₃	Min X _B /X ₃
0	1S ₁	7	3	-1	2	1	0	0	
0	1S ₂	12	-2	4	0	0	1	0	12/4 →
0	1S ₃	10	-4	3	8	0	0	1	10/3
	Z _j	0	0	0	0	0	0	0	
	Z _j -C _j		2	-3	5	0	0	0	



We can observe in the above simplification $Z_j - C_j \leq 0$, hence current basic feasible solution is not optimum, Continuing the simplex process....

Shifting X_3 into S_2 and also Coefficient basis for Key Row
 Applying Row transformation $R_2 : R_2 / 4$ & $R_1 : R_1 + R_2$

		C _j	-2	3	-5	0	0
C _B	B	X _B	X ₂	X ₃	X ₅	S ₁	S ₃
0	1S ₁	10	5/2	0	2	1	0
3	1X ₃	3	-1/2	1	0	0	0
0	1S ₃	10	-4	3	8	0	1
	Z _j	9	-3/2	3	0	0	0
	Z _j -C _j		1/2	0	5	0	0

$Z_j - C_j \geq 0$, Hence given is having Optimal solution at
 $x_3 = 3$; $x_2 = 0$; $x_5 = 0$

$$\therefore \text{Min}(Z) = \text{Max}(-z) = 2x_2 - 3x_3 + 5x_5$$

$$= 2(0) + -3(3) + 5(0) = 9$$

8. DEGENERACY IN LPP

In LPP, we can observe the degeneracy where one or more of the basic variables assume the values of zero. The number of variables in the solution is the same as the number of constraints. Based on the nature we can declare that when the number of positive variables in the solution is less than the number of constraints then degeneracy occurs. In this condition all basic variables are required to have positive values. If one or more feasible solution values are zero then its is difficult to find out optimum solution of the given problem-Such type of conditions is known as degeneracy in LPP.

- Min of the ratio :
(Elements of the first column of the unit matrix) /
(Corresponding elements of key column)
- Min of the ratio :
(Objects of the Second column of the unit matrix) /
(Corresponding elements of key column)
- Min of the ratio :
(Elements of the Third column of the unit matrix) /
(Corresponding elements of key column)

Example- 8 : $\text{Max}(Z) = 4x_1 + 10x_2$

Subject to the Constraints : $x_1 + 4x_2 \leq 8$
 $1x_1 + 2x_2 \leq 4, x_1, x_2 \geq 0$

Sol :-) Standard form w.r.t Slack variables

$\text{Max}(z) = 4x_1 + 10x_2 + 0S_1 + 0S_2$
 Subject to the Constraints : $x_1 + 4x_2 + S_1 = 8$
 $x_1 + 2x_2 + S_2 = 4, \text{ where } x_1, x_2, S_1, S_2 \geq 0$

		C _j	4	10	0	0	Min.
C _B	B	x _B	x ₁	x ₂	1S ₁	1S ₂	Ratio
0	1S ₁	8	1	4	1	0	8/4=2
0	1S ₂	4	1	2	0	1	4/2=2
	Z _j	0	0	0	0	0	
	Z _j -C _j		-4	-10	0	0	



Ratio is Unique will solve the degeneracy in the given LPP.

		C _j	4	10	0	0	M.Ratio
C _B	B	X _B	X ₁	X ₂	S ₁	S ₂	S ₁ /X ₂
0	S ₁	8	1	4	1	0	1/4
0	S ₂	4	1	2	0	1	0/2
	Z _j	0	0	0	0	0	
	Z _j -C _j		-4	-10	0	0	

Min(Elements of first column/Corresponding elements of key column)

$$\text{Min} (\frac{1}{4}, 0/2) = 0$$

S₂ leaves the basis and the key elements is 2.

		C _j	4	10	0	0
C _B	B	X _B	X ₁	X ₂	S ₁	S ₂
0	S ₁	0	-1	0	1	-2
10	X ₂	2	1/2	1	0	1/2
	Z _j	20	5	10	0	5
	Z _j -C _j	20	1	0	0	5

Z_j-C_j ≥ 0, solution is optimal.

$$\text{Max}(Z) = 20 ; \text{ where } X_1 = 0 ; X_2 = 2 ;$$

9. DUALITY IN LPP

If the given LPP is having the function in maximization form then converting into minimization or if the given is in the form of minimization then convert it into maximizations using the inequalities ≥ , ≤ by multiplying with -1.

Example - 9 : a) maximize(Z) = 100x₁ + 300x₂
 stc 2x₁ + x₂ ≤ 350 ; 3x₁ + 4x₂ ≤ 510 ;
 4x₁ + 7x₂ ≤ 900 ; where x₁, x₂ ≥ 0

Sol:-) Dual form :

Min(Z) = 350y₁ + 510y₂ + 900y₃
 stc 2y₁ + 3y₂ + 4y₃ ≥ 100 ;
 y₁ + 4y₂ + 7y₃ ≥ 300 ;
 y₁, y₂, y₃ ≥ 0

10. TWO PHASE METHOD

If we consider artificial variables in LPP then the representation consists two distinct approaches called named as Two Phase Method.

Phase - 1 :

- Let (-1) to each artificial variable and a cost to '0' at all other variables and get a new objective functions Z^* .
- Rewrite the LPP in maximized subject to the given set of constraints.
- Solve the LPP by simplex algorithm such that $Max(z^*) < 0$ and at least one artificial variable appears at positive level. OR
- $Max(Z^*) = 0$; at least one artificial variable appears at zero level.
- $Max(Z^*) = 0$; No artificial variables appears.

Phase - 2 :

- Allocate the actual costs to the given variable in the objective function and zero cost to every artificial variable in the basis at zero level.
- Remove the artificial(A_i) variable column from the table which is eliminated from the basis in Phase-1.
- Continue the simplex method to the current simplex table obtained at the end of phase-1 till an optimum basic feasible solution is obtained.

Example-10 : Solve the Linear Programming Problem using Two Phase Method where $Max(z) = 2l + m + n$;

Subject to the Constraints : $4l + 6m + 3n \leq 8$;
 $3l - 6m - 4n \leq 1$;
 $2l + 3m - 5n \geq 4$;
 Where $(l, m, n) \geq 0$

Sol:-) Standard form $Max(z) = 2l + m + n$

Let $l = x_1, m = x_2, n = x_3$

Subject to the Constraints :

$4x_1 + x_2 + 3x_3 + S_1 + 0S_2 + 0S_3 = 8$;
 $3x_1 - 6x_2 - 4x_3 + 0S_1 + S_2 + 0S_3 = 1$;
 $2x_1 + 3x_2 - 5x_3 + 0S_1 + 0S_2 - 0S_3 + A_1 = 4$
 we can observe that $S_1=8; S_2=1; A_1=4$

x_1, x_2, x_3, S_3 are non basic ; i.e., all are zero's.

Phase - 1

$Max(Z^*) = A_1$

		C _j	0	0	0	0	0	0	-1	Ratio
C _B	Y _B	X _B	l	m	n	S ₁	S ₂	S ₃	A ₁	
0	S ₁	8	4	6	3	1	0	0	0	8/6 →
0	S ₂	1	3	-6	-4	0	1	0	0	1/-6 ▼
-1	A ₁	4	2	3	-5	0	0	-1	1	4/3
	Z _j	-4	-2	-3	5	0	0	1	-1	
	Z _j -C _j	-4	-2	-3	5	0	0	1	0	

Where $l = x_1 ; m = x_2 ; n = x_3$

We can observe in the above simplification $Z_j - C_j < 0$, hence current basic feasible solution is not optimum,

Introducing x_2 and delete S_1

		C _j	0						-1
C _B	Y _B	X _B	l=x ₁	m=x ₂	n=x ₃	S ₁	S ₂	S ₃	A ₁
0	x ₂	4/3	2/3	1	1/2	1/6	0	0	0
0	S ₂	0	7	0	-1	1	1	0	0
-1	A ₁	0	0	0	-13/2	-1/2	0	-1	1
	Z _j -C _j	0	0	0	13/2	1/2	0	1	0

Where $l = x_1, m = x_2, n = x_3$

In this Phase-1 process we can observe that there are no negative elements in the last row. Hence the Optimal solution of the new Objective function $Z_j - C_j \geq 0$ But Artificial variable may not be optimal hence continue phase-2.

Phase - 2

Consider Standard form $Max(z)$

$= 2l + 0m + n + 0S_1 + 0S_2 + 0S_3$

		C _j	2	0	1	0	0	0	0	R
C _B	Y _B	X _B	l	m	n	S ₁	S ₂	S ₃	A ₁	
1	m	4/3	2/3	1	1/2	1/6	0	0	0	4/2
0	S ₂	9	7	0	-1	1	1	0	0	9/7 →
0	A ₁	0	0	0	-13/2	-1/2	0	-1	1	==
	Z _j -C _j	4/3	-4/3	0	-1/2	1/6	0	0	0	

Still $Z_j - C_j < 0$, continuing the iteration

$R_2: R_2/7$ & $R_1: R_1 - (2/3) R_2$ New;

Introduce X_1 and delete S_2

		C_j	2	1	1	0			0
C_B	Y_B	X_B	l	m	n	S_1	S_2	S_3	A_1
1	m	$10/2$ 1	0	1	$25/4$ 2	$1/1$ 4	$-2/21$	0	0
2	l	$9/7$	1	0	$-1/7$	$1/7$	$1/7$	0	0
0	A_1	0	0	0	$-13/2$	$-1/2$	0	-1	1
	Z_j	$64/2$ 1	2	1	$13/4$ 2	$5/1$ 4	$4/21$	0	0
	$Z_j - C_j$	$64/2$ 1	0	0	$29/4$ 2	$5/1$ 4	$4/21$	0	0

Since all $(Z_j - C_j) \geq 0$ the given basic feasible solution is optimal.

$$\text{Max}(Z) = 64/21 ; l = 9/7 ; m = 10/21 ; n = 0$$

11. CONCLUSIONS

In this investigation article properties of LPP are discussed and graphical method of solving a LPP is applicable where two or more variables are involved. The methods applied for solving a linear programming problem are identified as simplex method approach. LPP provides technique practically applicable solutions since there might be other constraints operating outside the problem which must also be taken into consideration just because so many simplification methods are introduced for Linear Programming problem. Computational procedure of simplex method or Algorithm are the best optimal solution procedure in LPP.

In inequality cases surplus and slack, artificial variables are introduced and in simplex procedure where ever x_1, x_2, \dots enters corresponding surplus, slack variables are terminated. Two phase simplex method is used to solve a given problems in which some artificial variables are involved. Dual principles, Degeneracy conditions are well defined in LPP.

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