International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 07 Issue: 07 | July 2020 www.irjet.net p-ISSN: 2395-0072

Application of Wood-Armer Moments in the Design of Concrete Skew Slab Bridges

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Abstract - *Skewed bridges are those bridges which are not orthogonal to the traffic direction. Bridges having smaller angle of skew (<20°) are usually considered as right bridges and thus the analysis and design becomes less complex. Due to the skew effect in bridges having skew angles greater than 20°, additional torsional effects are developed on the bridge deck which makes the analysis and design quite complex. Skew slab bridges are usually designed by grillage analogy method and thus torsional moments cannot be incorporated in the design. The current practice is to provide a mesh reinforcement at the obtuse corners of the slab bridge which is decided by the designer himself. But if the skewed slab bridges are modelled using finite element method, then the torsional moments can be incorporated in the design using Wood and Armer moments thus making the design much safer. Concrete skew slab bridges of varying skew angles are modelled by grillage analogy using STAAD Pro and finite element method using SAP 2000. The bridge modelled by grillage analogy method is designed by following the conventional method and the latter one is designed by using Wood-Armer Equations. The span of bridges considered are 8 m, 10 m, 12 m, 15 m for skew angles 0°,15°,30°,45° and 60°.*

Key Words: **Skew Bridge, Wood and Armer Moments, Skew Angle, Grillage Analogy, Finite Element method.**

1. INTRODUCTION

A slab bridge consists of a reinforced concrete slab supported on the abutments. They are supported at two edges and the remaining two ends are free. These supports are normally orthogonal to traffic direction. They are common for short spans up to 15 m, whereas voided slabs or other structural systems are more economical for longer spans. But sometimes, the bridges may not be at right angles to the supports due to many reasons. Such skewed bridges are commonly used to cross the roadways, waterways or railways that are not perpendicular to the bridge at the intersection. Skew bridges are characterized by their skew angle which is defined as the angle between normal to the centerline of the bridge and the centerline of abutment or pier cap (3). The behavior of skew bridges differs widely from that of normal bridges and therefore, the design of skew bridges needs special attention.

In small skew angle bridges say up-to 20°, the behavior is found out to be similar to that of normal bridges and are typically designed as normal right-angle bridges where the force flow is always straight towards the supports (fig 1a). There is no considerable variation in values of parameters like bending moment, shear force and torsional moment. However, if the skew angle increases beyond 20°, then there could be a considerable variation in values of shear force, bending moment, and torsional moment since the load always takes the shortest path to reach the supports (fig 1b).

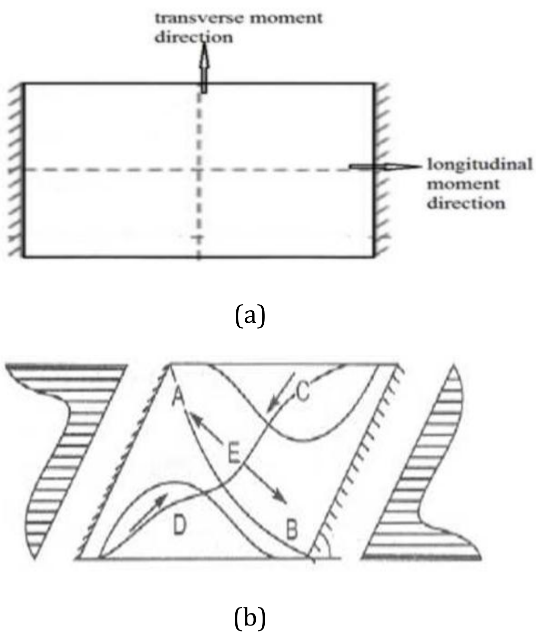


Fig 1: Force flow in Non-Skewed and Skewed Bridges

1.1 Scope of Work

For the analysis and design of skew slabs with higher skew angles, there is no generalized method. Due to the lack of standard methdology, design is very much individualized and the designs vary from designer to designer depending upon his own experience and concept of modelling of bridge decks. Skew slab bridges are usually designed by grillage analogy method and the resulting high torsional moments could not be incorporated in the design. The current practice is to provide a mesh reinforcement at the obtuse corners of the slab bridge where high torsion is acted upon. But if the skewed slab bridges are modelled using finite element method, then the torsional moments can be incorporated in the design by converting such moments to equivalent bending moments using Wood-Armer moments thus making the design much safer.

2. PARAMETRIC STUDY

The total span of the bridges considered for the analysis are 8m, 10m, 12m and 15m for skew angles 0°, 15°, 30° ,45° and 60° each. Among the 40 bridge models, 20 bridge models were analyzed by grillage analogy method (STAAD Pro) and 20 models by finite element method (SAP 2000). The thickness of slab considered is 1000mm throughout and the total carriageway width is 12 m (fig 2). The loads considered for analysis included the self-weight as dead load and vehicular loads as live loads. All the other loads were neglected for reducing the complexity in analysis. The vehicular loads considered are one lane of IRC Class 70R for every two lanes with one lane of IRC Class A on the remaining lane or 3 lanes of IRC Class A. The grade of concrete adopted is M30 and the steel is of Fe500 grade.

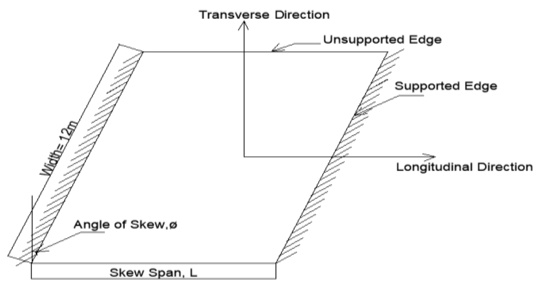


Fig 2: Model of Skew Bridge considered for analysis

2.1 Grillage Analogy method of bridge analysis

In grillage analogy method, the whole bridge structure is converted into appropriate number of longitudinal and transverse beams such that given prototype bridge deck and the equivalent grillage of beams are subjected to identical deformations under loading. The analysis was conducted using the software STAAD Pro. A few approximations were allowed in calculating the properties of all interconnected beams since exact modelling and calculation of properties of structure is tedious and time consuming. All the outer elements are considered as dummy elements and therefore all the properties are assumed to be minimum. The 3-lane slab bridge considered is simply supported over the abutments. Therefore, one end of bridge is assumed as pinned support and other end is fixed but support. The skew length of the bridge increases with an increase in skew angle.

2.1.1 Results and Discussions

All the 20 models were analyzed using the software STAAD Pro V8i SS5 and the following results for Shear Force, Torsional Moment and Longitudinal Bending Moment are taken. The load combination considered for tabulating the result is 1 Dead Load + 1 Live Load. Table 1 to 4 shows the results for all the models analyzed.

Table 1: Results for 8 m skew slab bridge

SKEW (in degree)	SHEAR FORCE (kN)	TORSIONAL MOMENT (kNm)	MOMENT (kNm)
0	430.3	74	723.8
15	476	147	685.4
30	586.6	211.8	605.1
45	837.5	666	555.1
60	944.1	716.3	504.2

Table 2: Results for 10 m skew slab bridge

SKEW (in degree)	SHEAR FORCE (kN)	TORSIONAL MOMENT (kNm)	MOMENT (kNm)
0	540.3	97.6	1113
15	577.6	220.3	1071.4
30	674.1	341.9	943.9
45	862	706.5	787.8
60	1027.6	753.8	730.7

Table 3: Results for 12 m skew slab bridge

SKEW (in degree)	SHEAR FORCE (kN)	TORSIONAL MOMENT (kNm)	MOMENT (kNm)
0	662.5	154.8	1651.1
15	712	299.3	1608.5
30	761.5	543.7	1407.7
45	960.8	874.5	927
60	1133.5	1097.4	777.9

Table 4: Results for 15 m skew slab bridge

SKEW (in degree)	SHEAR FORCE (kN)	TORSIONAL MOMENT (kNm)	MOMENT (kNm)
0	803	207.9	2568.1
15	881.8	547.6	2517
30	938.1	883.3	2235.6
45	1140.7	1109.5	1740.4
60	1244.5	1197.2	1355

Shear Force ratio v/s skew angle

The shear force obtained from analysis is presented in the form of F_x/F_0 (chart 1), where F_x is the maximum shear force in the bridge for a given skew angle of x (between 0° to 60°)

and F_0 is the maximum shear force for a non-skewed bridge (skew angle = 0°). The shear force due to dead load and live load increases with an increase in skew angle for all the models analyzed. From figure 1a, it can be seen that more force will get concentrated at the obtuse corners of the bridge slab.

Torsional Moment ratio v/s skew angle

The torsional moment obtained from analysis is presented in the form of T_x/T_0 (chart 2), where T_x is the maximum torsional moment in the bridge for a given skew angle of x (between 0° to 60°) and T_0 is the maximum torsional moment for a non-skewed bridge (skew angle = 0°). Due to the force concentration at the obtuse corners of the bridge slab, there is a chance for possible uplift at the acute corners. The torsional moment due to dead load and live load increases with an increase in the skew angle for all the models considered.

Bending Moment ratio v/s skew angle

The bending moment obtained from analysis is presented in the form of M_x/M_0 (chart 3), where M_x is the maximum torsional moment in the bridge for a given skew angle of x (between 0° to 60°) and M_0 is the maximum moment for a non-skewed bridge (skew angle = 0°). The trend of the curve obtained in this case is different from that obtained for shear force and torsional moment ratios. The longitudinal bending moment due to dead load and live load decreases with an increase in the skew angle for all the models considered. The reason for this trend can be attributed to the fact that the force flows through the strip connecting the obtuse corners of the bridge slab and as the skew angle increases, the length of the strip decreases and thus the longitudinal bending moment also decreases.

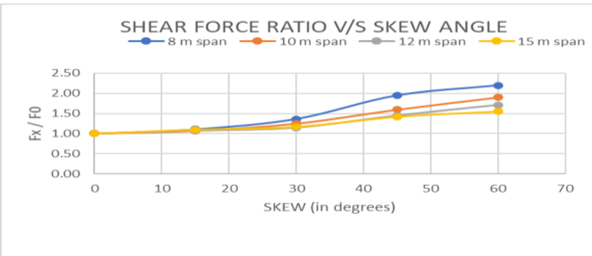


Chart 1: Shear Force Ratio v/s Skew Angle

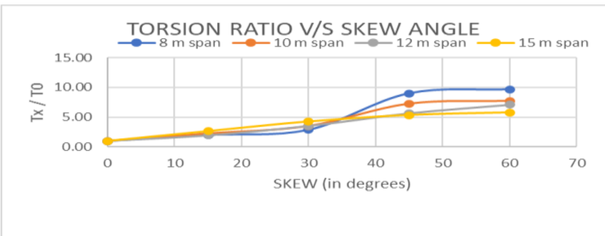


Chart 2: Torsion Ratio v/s Skew Angle

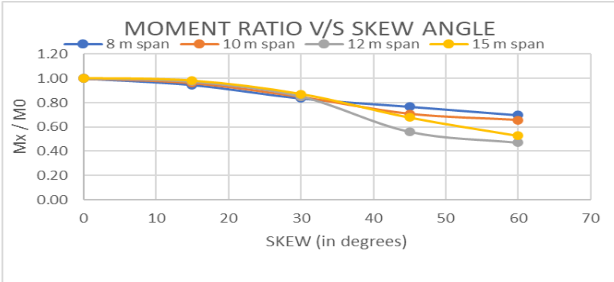
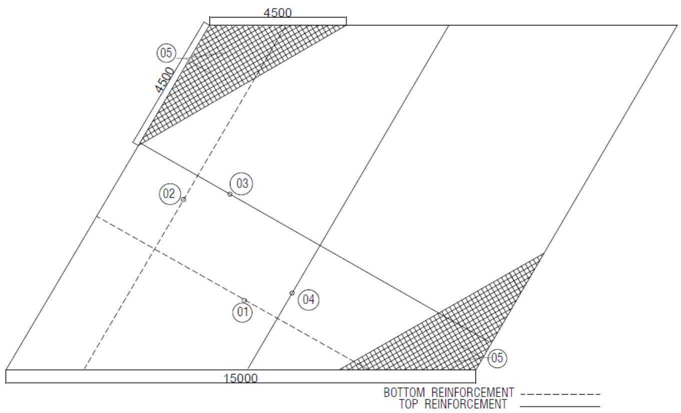


Chart 3: Moment Ratio v/s Skew Angle

2.1.2 Design of Skew Bridge by Grillage Analogy Method

Limit State Method which is a method is based on the concept of safety and serviceability has been adopted for the design of slab bridge. Excel sheets were prepared for the design and the reference code selected was IRC:112-2011. A 15 m span slab bridge with a skew angle of 30° is selected for simplicity.

For the main reinforcement, 25 mm diameter bars at 130 mm center to center was adopted and 12 mm diameter bars at 130 mm center to center was adopted as the distribution reinforcement. For the top reinforcement, 12 mm diameter bars at 130 mm center to center was adopted along the longitudinal direction and 10 mm diameter bars at 130 mm center to center was adopted as distribution reinforcement. Mesh reinforcement is also provided at the obtuse corners of slab where high torsion is acting (fig 3). Shear check was also done and the slab was found to be safe against shear. The total quantity of steel required is calculated to be around 11052 kg.



BAR NO	BAR DIA (mm)	SPACING (mm)
01	25	130
02	12	130
03	12	130
04	10	130
05	10	150 (MESH)

Fig 3: Reinforcement detailing of slab bridge modelled by grillage analysis

2.2 Finite Element Method of Bridge Analysis

The basic idea in finite element method is to find the solution of complicated problem by replacing it by their individual component or elements whose behavior is readily understood. These elements are interconnected at specified joints or nodal points which possess an appropriate number of degrees of freedom. The software used for finite element analysis is SAP2000. The dimension of models was same as that of grillage analysis. The slab bridge is simply supported over the abutments. Shell elements were considered for the analysis and the loads considered are dead load and live load as in case of grillage analysis. The bridge is analyzed as a shell element having a thickness of 1 m in bending and membrane is assigned as shown in the fig 8. The loads considered for the finite element analysis are self-weight of the slab and the live loads (IRC Class A and IRC 70R Wheeled).

2.2.1 Results and Discussions

Bridges of span 8 m, 10 m, 12 m and 15 m were analyzed for skew angles of 0°, 15°, 30°, 45° and 60° using the software SAP2000 and the following results for Shear Force, Torsional Moment, Longitudinal Bending Moment and Transverse Bending Moment are taken. The load combination considered for tabulating the result is 1 Dead Load + 1 Live Load.

In the tables show below from table 5 to 8, M11 (+) represents the longitudinal sagging moment, M11(-) represents the longitudinal hogging moment, M22(+) represents the transverse sagging moment and M22(-) represents the transverse hogging moment.

Table 5: Results for 8 m skew slab bridge

SKEW (°)	LONG. MOMENT kNm/m		TRANS. MOMENT kNm/m		TORSION kNm/m	SHEAR FORCE kN/m
	M11 (+)	M11 (-)	M22 (+)	M22 (-)		
0	390	10	175	158	47	168
15	386	39	184	182	97	175
30	357	69	196	174	138	224
45	320	103	207	116	298	313
60	238	130	221	86	318	369

Table 6: Results for 10 m skew slab bridge

SKEW (°)	LONG. MOMENT kNm/m		TRANS. MOMENT kNm/m		TORSION kNm/m	SHEAR FORCE kN/m
	M11 (+)	M11 (-)	M22 (+)	M22 (-)		
0	564	15	186	190	58	198
15	559	47	197	231	108	228
30	499	81	207	234	185	254
45	425	122	227	171	359	359
60	325	158	241	108	402	398

Table 7: Results for 12 m skew slab bridge

SKEW (°)	LONG. MOMENT kNm/m		TRANS. MOMENT kNm/m		TORSION kNm/m	SHEAR FORCE kN/m
	M11 (+)	M11 (-)	M22 (+)	M22 (-)		
0	770	21	199	220	89	247
15	761	56	218	281	158	284
30	671	102	244	297	261	301
45	543	133	259	233	424	398
60	427	182	272	124	517	471

Table 8: Results for 15 m skew slab bridge

SKEW (°)	LONG. MOMENT kNm/m		TRANS. MOMENT kNm/m		TORSION kNm/m	SHEAR FORCE kN/m
	M11 (+)	M11 (-)	M22 (+)	M22 (-)		
0	1116	29	225	262	117	317
15	1098	81	258	354	269	346
30	952	158	309	390	394	399
45	743	144	307	329	496	478
60	555	201	319	161	564	527

Shear Force ratio v/s skew angle

The pattern for shear force ratio versus skew angle obtained from finite element analysis is similar to that obtained from grillage analysis (chart 4). Shear force increases with an increase in the skew angle and more force is found to concentrate at the obtuse corners of the bridge.

Torsional Moment ratio v/s skew angle

The torsional moment obtained from analysis is presented in the form of T_x/T_0 (chart 5). The pattern of the graph observed in finite element analysis is similar to pattern observed in grillage analysis. The maximum torsional moment increases with an increase in the skew angle. Up to a skew angle of 30°, the curve for all the models is almost following the same trend but after 30º there is noticeable variation in the pattern of the curves.

Bending Moment ratio v/s skew angle

The maximum longitudinal bending moment decreases with an increase in the skew angle and the pattern of curves obtained from both methods of analysis are the same (chart 6).

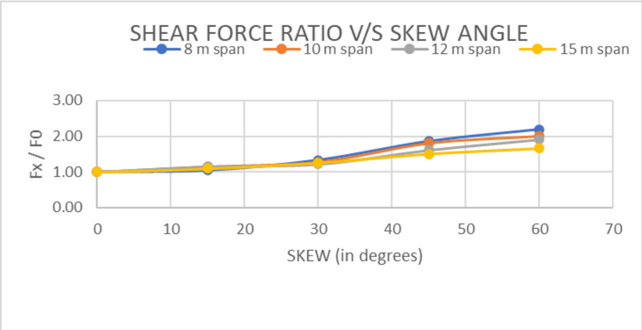


Chart 4: Shear Force Ratio v/s Skew Angle

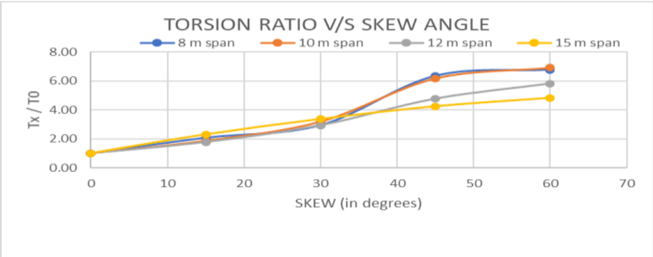


Chart 5: Torsion Ratio v/s Skew Angle

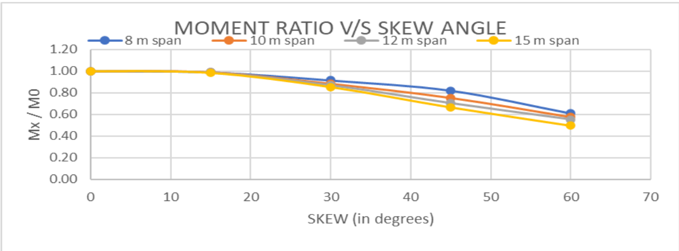


Chart 6: Moment Ratio v/s Skew Angle

2.2.2 Incorporation of twisting moments using WOOD-ARMER moments

The Wood-Armer method is a structural analysis method based on finite element analysis used to design the reinforcement for concrete slabs. This method provides simple equations to design a concrete slab based on the output from a finite element analysis software. This method explicitly incorporates twisting moment in slab design. In case of reinforced concrete slab which is reinforced by an orthogonal system of bars placed in the x and y directions, the problem is to determine the design moments M_{ud1} and M_{ud2}. The reinforcement should be designed for if adequate strength is to be available in all directions. Once M_{ud1} and M_{ud2} have been found, the reinforcement may be found out to resist these moments by normal analysis of a section in bending.

Conditions

$$M_a = \frac{(M_x+M_y)}{2} + \frac{(M_x-M_y)}{2}\cos(2\alpha) + M_{xy}\sin(2\alpha)$$

$$M_b = \frac{(M_x+M_y)}{2} - \frac{(M_x-M_y)}{2}\cos(2\alpha) - M_{xy}\sin(2\alpha)$$

$$M_{ab} = -\frac{(M_x-M_y)}{2}\sin(2a) + M_{xy}\cos(2\alpha)$$

Bottom Rebar

$$M_{ud1} = M_a - \frac{M_b\cos\phi}{(1+\cos\phi)} + \frac{M_{ab}\,(1-\cos 2\phi)}{\sin\phi}$$

$$M_{ud2} = \frac{M_a}{(1+\cos\phi)} + \frac{M_{ab}}{\sin\phi}$$

When $M_{ud1} > 0$ & $M_{ud2} > 0$

$$M_{ud1} = M_a - \frac{M_b\cos\phi}{(1+\cos\phi)} + \frac{M_{ab}\,(1-\cos 2\phi)}{\sin\phi}$$

$$M_{ud2} = \frac{M_a}{(1+\cos\phi)} + \frac{M_{ab}}{\sin\phi}$$

When $M_{ud1} < 0$ & $M_{ud2} > 0$

$M_{ud1} = 0$

$$M_{ud2} = \frac{(M_a * M_b - M_{ab}{}^2)}{(M_a(\sin\phi)^2 + M_b(\cos\phi)^2 - M_{ab}(\sin 2\phi))}$$

When $M_{ud1} > 0$ & $M_{ud2} < 0$

$$M_{ud1} = \frac{(M_a * M_b - M_{ab}{}^2)}{(M_b(\sin\phi)^2 + M_a(\cos\phi)^2 - M_{ab}(\sin 2\phi))}$$

$M_{ud2} = 0$

When $M_{ud1} < 0$ & $M_{ud2} < 0$

$M_{ud1} = 0$

$M_{ud2} = 0$

Top Rebar

$$M'_{ud1} = M_a + \frac{M_b\cos\phi}{(1-\cos\phi)} - \frac{M_{ab}\,(1+\cos 2\phi)}{\sin\phi}$$

$$M'_{ud2} = \frac{M_b}{(1-\cos\phi)} - \frac{M_{ab}}{\sin\phi}$$

When $M'_{ud1} < 0$ & $M'_{ud2} < 0$

$$M'_{ud1} = M_a + \frac{M_b\cos\phi}{(1-\cos\phi)} - \frac{M_{ab}\,(1+\cos 2\phi)}{\sin\phi}$$

$M'_{ud2} = \frac{M_b}{(1-\cos\phi)} - \frac{M_{ab}}{\sin\phi}$

When $M'_{ud1} > 0$ & $M'_{ud2} < 0$

$M'_{ud1} = 0$

$M'_{ud2} = \frac{M_b}{M_{ab}^2 / M_a}$

When $M'_{ud1} < 0$ & $M'_{ud2} > 0$

$M'_{ud1} = \frac{M_a}{M_{ab}^2 / M_b}$

$M'_{ud2} = 0$

When $M'_{ud1} > 0$ & $M'_{ud2} > 0$

$M'_{ud1} = 0$

$M'_{ud2} = 0$

The bending moment values in the longitudinal direction and transverse direction i.e., M_x and M_y and the torsional moments i.e., M_{xy} are obtained from SAP2000 for all the 20 models and it is substituted in the above equations using Microsoft Excel sheets. Thus, by using Wood-Armer equations, the torsional moments are converted to their equivalent bending moments in the longitudinal and transverse directions. From these moment values, the reinforcements in longitudinal and transverse direction can be easily calculated. The following tables from 9 to 12 represent the bending moment values for all the 20 models after the incorporation of torsional moments.

Table 9: Bending Moments after the incorporation of twisting moment (span = 8 m)

SKEW (°)	BOTTOM REBAR (kNm/m)		TOP REBAR (kNm/m)	
	M_{ud1}	M_{ud2}	M'_{ud1}	M'_{ud2}
0	484	437	104	205
15	580	483	233	279
30	633	495	345	312
45	916	618	699	414
60	874	556	766	404

Table 10: Bending Moments after the incorporation of twisting moment (span = 10 m)

SKEW (°)	BOTTOM REBAR (kNm/m)		TOP REBAR (kNm/m)	
	M_{ud1}	M_{ud2}	M'_{ud1}	M'_{ud2}
0	680	622	131	248
15	775	667	263	339
30	869	684	451	419
45	1143	784	840	530
60	1129	727	962	510

Table 11: Bending Moments after the incorporation of twisting moment (span = 12 m)

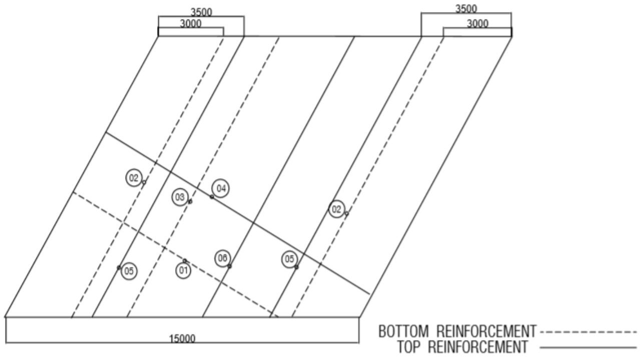
SKEW (°)	BOTTOM REBAR (kNm/m)		TOP REBAR (kNm/m)	
	M_{ud1}	M_{ud2}	M'_{ud1}	M'_{ud2}
0	948	859	199	309
15	1077	919	372	439
30	1193	932	624	558
45	1391	967	981	657
60	1461	944	1216	641

Table 12: Bending Moments after the incorporation of twisting moment (span = 15 m)

SKEW (°)	BOTTOM REBAR (kNm/m)		TOP REBAR (kNm/m)	
	M_{ud1}	M_{ud2}	M'_{ud1}	M'_{ud2}
0	1350	1233	263	379
15	1636	1367	619	623
30	1740	1346	946	784
45	1735	1239	1136	825
60	1683	1119	1329	725

2.2.3 Design of Skew Bridge analyzed by Finite Element Method

For the bottom main reinforcement, 25 mm diameter bars at 100 mm center to center spacing was adopted along the longitudinal direction and 25 mm diameter bars at 130 mm center to center was adopted as the transverse reinforcement up to a length of 3000 mm from the support where maximum hogging moment is acting. For the remaining 9000 mm 12 mm diameter bars at 130 mm center to center was adopted. For the top main reinforcement, 25 mm diameter bars at 200 mm center to center spacing was adopted as the longitudinal reinforcement while 25 mm diameter bars at 240 mm center to center was adopted for a length of 3500 mm from the support where maximum hogging moment is acting. For the remaining 8000 mm 10 mm diameter bars at 130 mm center to center was adopted. Total quantity of steel required is calculated to be around 16988kg.



BAR NO	BAR DIA (mm)	SPACING (mm)
01	25	100
02	25	130
03	12	130
04	25	200
05	25	240
06	10	130

Fig 4: Reinforcement detailing of slab modelled by Finite Element Analysis

3. CONCLUSIONS

After the analysis of various concrete skew bridges by grillage analogy method and finite element method, the following conclusions can be made from the results obtained from the two software:

- From both the analysis methods, it was observed that the shear force and torsional moment increases with an increasing skew angle. Up to a skew angle of 30º, the increase in these values are less and at higher skew, a sharp increase is observed.
- Longitudinal Bending Moment was found to be decrease with an increase in the skew value.
- The pattern of the curves obtained from grillage analysis and finite element analysis is same.
- The results for bending moment obtained from finite element analysis is lesser in magnitude as compared to grillage analogy method. But Grillage Analogy method is widely accepted by the designers due to its easiness to use.
- The quantity of reinforcement calculated from finite element analysis is found to be more as compared to grillage analysis but the former one is safer approach as it includes the effects of torsion in the design.

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