# Coefficient Bounds for Subclasses of Bi-univalent Functions Defined by (p,q)-Derivatives

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***Abstract*****—** This paper introduces two new subclasses $\boldsymbol{B_{\Sigma}^{p,q}(\lambda,\mu,\alpha)}$ and $\boldsymbol{B_{\Sigma}^{p,q}[\lambda,\mu,\gamma]}$ of bi-univalent functions by using (p, q)-derivatives and determine the bounds for first two coefficients for functions in these subclasses.

***Keywords- univalent function; bi-univalent function; coefficient bounds; (p,q)-derivative ; q-derivative***

## 1. INTRODUCTION

Let ✌ denote the class of functions $f$ given by,

$$f(z) = z + \sum_{m=2}^{\infty} a_m z^m \tag{1}$$

which are analytic in the open unit disk ♰ $= \{z : |z| < 1\}$. Furthermore, let ♠ represent the class of all functions $f \in$ ✌ in the form (1) which are univalent in ♰. The Koebe one-quarter theorem [5] ensures that the image of ♰ under every function $f \in$ ♠ contains a disk of radius ¼. Thus, every function $f \in$ ♠ has an inverse $f^{-1}$ satisfying $f^{-1}(f(z)) = z \quad (z \in \mathcal{U})$ and $(f^{-1}(w)) = w \quad (|w| < r_0(f),\ r_0(f) \geq 1/4)$.

A function $f \in$ ✌ is said to be bi-univalent in ♰ if both $f$ and its inverse $f^{-1}$ are univalent in ♰. Let Σ denote the class of bi-univalent functions defined in ♰. Since $f \in \Sigma$ has the Taylor-Maclaurin series expansion given by (1), its inverse $f^{-1}$ has the expansion

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3) + \cdots \tag{2}$$

Many authors introduced and investigated different subclasses of bi-univalent functions and obtained estimates for the initial coefficients for functions in these subclasses (see [1, 3, 6, 7, 11, 15, 14, 12, 17]).
In Geometric Function Theory, different subclasses of the normalized analytic function class ✌ have been analysed from various viewpoints. The $q$-calculus and the fractional $q$-calculus provide important tools that have been used for the investigation of different subclasses of ✌.
To begin with, we define the fractional *(p, q)*-derivative (see [4, 10]) for a complex function $f(z)$ as follows:

**Definition 1.** *For* $0 < q < p \leq 1$, *the (p, q)-derivative of a complex-valued function* $f(z)$ *is given by*

$$d_{p,q} f(z) = \begin{cases} \frac{f(pz) - f(qz)}{(p-q)z} & for\ z \neq 0 \\ f'(0) & for\ z = 0. \end{cases} \tag{3}$$

From the above definition, it is clear that

$$d_{p,q}(z^m) = [m]_{p,q} z^{m-1}, \tag{4}$$

where

$$[m]_{p,q} = \frac{p^m - q^m}{p - q} \tag{5}$$

Thus, for $f \in$ ✌ given by (1), we have

$$d_{p,q}f(z) = 1 + \sum_{m=2}^{\infty}[m]_{p,q}a_m z^{m-1} \quad (6)$$

For $p$=1, we obtain the $q$-derivative of $f(z)$ (see [9]) given by

$$d_q f(z) = \begin{cases} \frac{f(z)-f(qz)}{(1-q)z} & for\ z \neq 0 \\ f'(0) & for\ z = 0 \end{cases}$$

and thus, for $f \in \mathcal{A}$ given by (1), we have $d_q f(z) = 1 + \sum_{m=2}^{\infty}[m]_q a_m z^{m-1}$ where $[m]_q = \frac{1-q^m}{1-q}$.

Also, for $f \in \mathcal{A}$, we have $\lim_{q\to 1^-} d_q f(z) = f'(z)$.

This paper introduces two new subclasses of bi-univalent functions defined by using *(p, q)*-derivatives and we determine bounds for the initial coefficients $|a_2|$ and $|a_3|$ for functions in these subclasses. For this purpose, we use the following lemma:

**Lemma 2.** [5] *If* $p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots$ *is analytic in* $Y$ *such that Re p(z)*$> 0$, *then* $|p_k| \leq 2$, *for each k.*

## 2. COEFFICIENT BOUNDS FOR THE FUNCTION CLASS $B_{\Sigma}^{p,q}(\lambda,\mu,\alpha)$.

In this section, we introduce the subclass $B_{\Sigma}^{p,q}(\lambda,\mu,\alpha)$ of the bi-univalent function class $\Sigma$ and obtain the bounds for $|a_2|$ and $|a_3|$ for the functions in this subclass.

**Definition 3.** *A function* $f \in \Sigma$ *given by (1) is said to be in the class* $B_{\Sigma}^{p,q}(\lambda,\mu,\alpha)$ *where* $0 < q < p \leq 1$,

$\lambda \geq 1,\ \mu \geq 0,\ 0 < \alpha \leq 1$ *if the following conditions are satisfied:*

$$\left|\arg\left\{(1-\lambda)\frac{f(z)}{z} + \lambda\, d_{p,q}f(z) + \mu z\left(d_{p,q}f(z)\right)'\right\}\right| < \frac{\pi\alpha}{2} \quad (z \in \mathcal{U}) \quad (7)$$

*and*

$$\left|\arg\left\{(1-\lambda)\frac{g(w)}{w} + \lambda\, d_{p,q}g(w) + \mu w\left(d_{p,q}g(w)\right)'\right\}\right| < \frac{\pi\alpha}{2} \quad (w \in \mathcal{U}) \quad (8)$$

*where* $2(1-\alpha)\sum_{m=1}^{\infty}\frac{(-1)^{m-1}}{\mu m+1} \leq 1$ *and g is the extension of* $f^{-1}$ *to* $Y$..

Now, we obtain the bounds for $|a_2|$ and $|a_3|$ for the function class $B_{\Sigma}^{p,q}(\lambda,\mu,\alpha)$.

**Theorem 4.** *Let f(z) given by (1) be in the class* $B_{\Sigma}^{p,q}(\lambda,\mu,\alpha)$. *Then*

$$|a_2| \leq min\left\{\frac{2\alpha}{|1-\lambda+(\lambda+\mu)[2]_{p,q}|}, \frac{2\alpha}{\sqrt{|2\alpha\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}+(1-\alpha)\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2|}}\right\} \quad (9)$$

*and*

$$|a_3| \leq \frac{4\alpha^2}{\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2} + \frac{2\alpha}{|1-\lambda+(\lambda+2\mu)[3]_{p,q}|}. \quad (10)$$

*Proof.* Let $f \in B_{\Sigma}^{p,q}(\lambda,\mu,\alpha)$. Then, from (7) and (8), we have

$$(1-\lambda)\frac{f(z)}{z} + \lambda\, d_{p,q}f(z) + \mu z\left(d_{p,q}f(z)\right)' = [k(z)]^{\alpha} \quad (z \in \mathcal{U}) \tag{11}$$

and

$$(1-\lambda)\frac{g(w)}{w} + \lambda\, d_{p,q}g(w) + \mu w\left(d_{p,q}g(w)\right)' = [h(w)]^{\alpha} \quad (w \in \mathcal{U}) \tag{12}$$

where

$$k(z) = 1 + k_1 z + k_2 z^2 + k_3 z^3 + \cdots \quad (z \in \mathcal{U}) \tag{13}$$

and

$$h(w) = 1 + h_1 w + h_2 w^2 + h_3 w^3 + \cdots \quad (w \in \mathcal{U}) \tag{14}$$

satisfying the conditions $Re\, k(z) > 0$ and $Re\, h(w) > 0$.

Now, equating the coefficients of like terms in (11) and (12), we get

$$\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}a_2 = \alpha k_1 \tag{15}$$

$$\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}a_3 = \alpha k_2 + \frac{\alpha(\alpha-1)}{2}k_1^2 \tag{16}$$

$$-\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}a_2 = \alpha h_1 \tag{17}$$

and

$$\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}(2a_2^2 - a_3) = \alpha h_2 + \frac{\alpha(\alpha-1)}{2}h_1^2 \tag{18}$$

From (15) and (17), we get

$$h_1 = -k_1 \tag{19}$$

And

$$2\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2 a_2^2 = \alpha^2(k_1^2 + h_1^2) \tag{20}$$

Now, from (16), (18) and (20), we obtain

Thus, we have

$$2\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}a_2^2 = \alpha(k_2+h_2) + \frac{\alpha(\alpha-1)}{2}(k_1^2+h_1^2)$$

$$= \alpha(k_2+h_2) + \frac{\alpha(\alpha-1)}{2}\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2 a_2^2$$

$$a_2^2 = \frac{\alpha^2(k_2+h_2)}{2\alpha\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}+(1-\alpha)\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2} \tag{21}$$

Now, calculating the absolute values on both sides of (20) and (21) and by using Lemma 2, we get

$$|a_2|^2 \le \frac{\alpha^2(|k_1|^2+|h_1|^2)}{2\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2} \le \frac{4\alpha^2}{2\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2}$$

and

$$|a_2|^2 \le \frac{\alpha^2(|k_2|+|h_2|)}{|2\alpha\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}+(1-\alpha)\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2|} \le \frac{4\alpha^2}{|2\alpha\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}+(1-\alpha)\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2|}$$

from which we obtain (9).
Further, to find the bound on the coefficient $|a_3|$, we substract (18) from (16) and use (19) to obtain

$$a_3 = a_2^2 + \frac{\alpha(k_2-h_2)}{2\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}} \tag{22}$$

Substituting for $a_2^2$ from (20) in (22), we have

$$a_3 = \frac{\alpha^2(k_1^2+h_1^2)}{2\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2} + \frac{\alpha(k_2-h_2)}{2\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}} \tag{23}$$

Now, by finding the absolute values on both sides of (23) and using Lemma 2, we obtain

$$|a_3| \le \frac{\alpha^2(|k_1|^2+|h_1|^2)}{2\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2} + \frac{\alpha(|k_2|+|h_2|)}{2|\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}|} \le \frac{4\alpha^2}{\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2} + \frac{2\alpha}{|\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}|}$$

which is precisely (10).

Hence the Theorem 4 is proved.

**Remark 5.** *If we put $\lambda = 1$ in Definition 3, then the class $B_\Sigma^{p,q}(\lambda,\mu,\alpha)$ reduces to the class $H_{\sigma_B}^{p,q,\mu,\alpha}$ which was defined and studied by Motamednezhad and Salehian [10].*

Thus, from Theorem 4., we obtain the results as follows:

**Corollary 6.** *Let f(z) given by (1) be in the class $H_{\sigma_B}^{p,q,\mu,\alpha}$. Then*

$$|a_2| \le min\left\{\frac{2\alpha}{(1+\mu)[2]_{p,q}}, \frac{2\alpha}{\sqrt{2\alpha(1+2\mu)[3]_{p,q}+(1-\alpha)(1+\mu)^2[2]_{p,q}^2}}\right\}$$

and

$$|a_3| \le \frac{4\alpha^2}{(1+\mu)^2[2]_{p,q}^2} + \frac{2\alpha}{(1+2\mu)[3]_{p,q}}.$$

**Remark 7.** *If we put $\lambda = 1, p = 1$ and let $q \to 1^-$ in Definition 3, then the class $B_\Sigma^{p,q}(\lambda,\mu,\alpha)$ reduces to the class $H_\Sigma(\mu,\alpha)$ which was defined and studied by Frasin [6].*

Thus, from Theorem 4, we obtain the results as follows:

**Corollary 8.** *Let f(z) given by (1) be in the class $H_\Sigma(\mu,\alpha)$. Then,*

$$|a_2| \le min\left\{\frac{\alpha}{1+\mu}, \frac{2\alpha}{\sqrt{2(\alpha+2)+4\mu(\alpha+\mu-\alpha\mu+2)}}\right\}$$

and

$|a_3| \le \frac{\alpha^2}{(1+\mu)^2} + \frac{2\alpha}{(1+2\mu)}.$

### 3. COEFFICIENT BOUNDS FOR THE FUNCTION CLASS $B_{\Sigma}^{p,q}[\lambda,\mu,\gamma]$

Here we introduce the function class $B_{\Sigma}^{p,q}[\lambda,\mu,\gamma]$ using the definition as follows:

**Definition 9.** *A function $f \in \Sigma$ given by (1) is said to be in the class $B_{\Sigma}^{p,q}[\lambda,\mu,\gamma]$ where $0 < q < p \le 1$, $\lambda \ge 1,\ \mu \ge 0,\ 0 \le \gamma < 1$ if the following conditions are satisfied:*

$$Re\left\{(1-\lambda)\frac{f(z)}{z} + \lambda\, d_{p,q}f(z) + \mu z\left(d_{p,q}f(z)\right)'\right\} > \gamma \quad (z \in \mathcal{U}) \tag{24}$$

*and*

$$Re\left\{(1-\lambda)\frac{g(w)}{w} + \lambda\, d_{p,q}g(w) + \mu w\left(d_{p,q}g(w)\right)'\right\} > \gamma \quad (w \in \mathcal{U}) \tag{25}$$

*where* $2(1-\gamma)\sum_{m=1}^{\infty}\frac{(-1)^{m-1}}{\mu m+1} \le 1$ *and g is the extension of* $f^{-1}$ *to* $\mathcal{Y}$..

Now, we obtain the bounds for $|a_2|$ and $|a_3|$ for the function class. $B_{\Sigma}^{p,q}[\lambda,\mu,\gamma]$.

**Theorem 10.** *Let f(z) given by (1) be in the class* $B_{\Sigma}^{p,q}[\lambda,\mu,\gamma]$. *Then*

$$|a_2| \le min\left\{\frac{2(1-\gamma)}{|1-\lambda+(\lambda+\mu)[2]_{p,q}|}, \sqrt{\frac{2(1-\gamma)}{|1-\lambda+(\lambda+2\mu)[3]_{p,q}|}}\right\} \tag{26}$$

*and*

$$|a_3| \le \frac{2(1-\gamma)}{|1-\lambda+(\lambda+2\mu)[3]_{p,q}|} \tag{27}$$

*Proof.* Let $f \in B_{\Sigma}^{p,q}[\lambda,\mu,\gamma]$. Then, from (24) and (25), we have

$$(1-\lambda)\frac{f(z)}{z} + \lambda\, d_{p,q}f(z) + \mu z\left(d_{p,q}f(z)\right)' = \gamma + (1-\gamma)k(z) \quad (z \in \mathcal{U}) \tag{28}$$

and

$$(1-\lambda)\frac{g(w)}{w} + \lambda\, d_{p,q}g(w) + \mu w\left(d_{p,q}g(w)\right)' = \gamma + (1-\gamma)g(w) \quad (w \in \mathcal{U}) \tag{29}$$

where $k(z)$ and $h(w)$ are given by (13) and (14), respectively, with *Re k(z)* > 0 and *Re h(w)* > 0.

Now, equating the coefficients of like terms in (28) and (29), we get

$$\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}a_2 = (1-\gamma)k_1 \tag{30}$$

$$\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}a_3 = (1-\gamma)k_2 \tag{31}$$

$$-\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}a_2 = (1-\gamma)h_1 \tag{32}$$

and

$$\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}(2a_2^2-a_3) = (1-\gamma)h_2 \tag{33}$$

From (30) and (32), we get

$$h_1 = -k_1 \tag{34}$$

and

$$2\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2 a_2^2 = (1-\gamma)^2(k_1^2+h_1^2) \tag{35}$$

Also, from (31) and (33), we get

$$2\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}a_2^2 = (1-\gamma)^2(k_2+h_2) \tag{36}$$

Now, by finding the absolute values on both sides of (35) and (36) and using Lemma 2, we get

$$|a_2|^2 \le \frac{(1-\gamma)^2(|k_1|^2+|h_1|^2)}{2\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2} \le \frac{4(1-\gamma)^2}{\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2}$$

and

$$|a_2|^2 \le \frac{(1-\gamma)(|k_2|+|h_2|)}{|2\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}|} \le \frac{2(1-\gamma)}{|\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}|}$$

from which we obtain (26).

Further, to find the bound on the coefficient $|a_3|$, we substract (33) from (31) to get

$$a_3 = a_2^2 + \frac{(1-\gamma)(k_2-h_2)}{2\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}} \tag{37}$$

Substituting for $a_2^2$ from (35) in (37), we have

$$a_3 = \frac{(1-\gamma)^2(k_1^2+h_1^2)}{2\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2} + \frac{(1-\gamma)(k_2-h_2)}{2\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}} \tag{38}$$

Also, by substituting the value of $a_2^2$ from (36) in (37), we have

$$a_3 = \frac{(1-\gamma)k_2}{1-\lambda+(\lambda+2\mu)[3]_{p,q}} \tag{39}$$

Now, by finding the absolute values on both sides of (38) and (39) and using Lemma 2, we obtain

$$|a_3| \le \frac{(1-\gamma)^2(|k_1|^2+|h_1|^2)}{2\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2} + \frac{(1-\gamma)(|k_2|+|h_2|)}{2|\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}|} \le \frac{4(1-\gamma)^2}{\{1-\lambda+(\lambda+\mu)[2]_{p,q}\}^2} + \frac{2(1-\gamma)}{|\{1-\lambda+(\lambda+2\mu)[3]_{p,q}\}|}$$

and

$$|a_3| \le \frac{(1-\gamma)|k_2|}{|1-\lambda+(\lambda+2\mu)[3]_{p,q}|} \le \frac{2(1-\gamma)}{|1-\lambda+(\lambda+2\mu)[3]_{p,q}|}$$

from which we obtain (27).

Hence the Theorem is proved. ∎

**Remark 11.** *If we put* $\lambda = 1$ *in Definition 9, then the class* $B_{\Sigma}^{p,q}[\lambda,\mu,\gamma]$ *reduces to the class* $H_{\sigma_B}^{p,q,\mu}(\gamma)$ *which was defined and studied by Motamednezhad and Salehian [10].*

Thus, from Theorem 10, we obtain the result as follows:

**Corollary 12.** *Let f(z) given by (1) be in the class* $H_{\sigma_B}^{p,q,\mu}(\gamma)$ *. Then*

$$|a_2| \leq min\left\{\frac{2(1-\gamma)}{(1+\mu)[2]_{p,q}}, \sqrt{\frac{2(1-\gamma)}{(1+2\mu)[3]_{p,q}}}\right\}$$

and

$$|a_3| \leq \frac{2(1-\gamma)}{(1+2\mu)[3]_{p,q}}.$$

**Remark 13.** *If we put* $\lambda = 1, p = 1$ *and let* $q \to 1^-$ *in Definition 9, then the class* $B_{\Sigma}^{p,q}[\lambda,\mu,\gamma]$ *reduces to the class* $H_{\Sigma}^{\mu}(\gamma)$ *which was defined and studied by Frasin [6].*

Thus, from Theorem 10, we obtain the result as follows:

**Corollary 14.** *Let f(z) given by (1.1) be in the class* $H_{\Sigma}^{\mu}(\gamma)$. *Then,*

$$|a_2| \leq min\left\{\frac{1-\gamma}{1+\mu}, \sqrt{\frac{2(1-\gamma)}{3(1+2\mu)}}\right\}$$

and

$$|a_3| \leq \frac{2(1-\gamma)}{3(1+2\mu)}.$$

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