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Embedded Type-I and Type-II Censoring in Life-Testing Experiments to Determine the Reliability Sampling Plans for an Exponential Distribution

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Abstract - *Quality and Reliability Engineering has acquired its overwhelming application in industries as producer and consumer are aware of its vital importance in producing quality products. Reliability sampling is an algorithm driven tool of Statistical Quality Control which enables the Quality Control engineers to select appropriate sampling plans for testing the products and hence the decision of acceptance or rejection is made on the batches or lots. A new type of sampling plans called Mixed Censoring Reliability Sampling Plans are developed involving type I and type II censoring. In general, either type I censoring or type II censoring schemes are adopted in designing sampling plans. But in this study an attempt has been made to design sampling plans by blending the two censoring schemes. This pressurizes the producer to maintain the quality of the batches or lots. R-Language is used to determine the parameters of the reliability sampling plans. Necessary tables are constructed using the designing procedure and illustration is given for easy implementation in industries.*

***Key Words*: Reliability sampling plans, Censoring, mixed censoring, exponential distribution, life-testing experiments.**

1. INTRODUCTION

Censoring schemes are generally employed during the life test to make the inspection as a cost effective one. Time censoring (Type-I), Product censoring (Type-II) and hybrid censoring are some of the censoring schemes employed in the life test.

This paper proposes a general Bayesian framework for designing a variable acceptance sampling scheme with mixed censoring. A general loss function which includes the sampling cost, the time-consuming cost, the salvage value, and the decision loss is employed to determine the Bayes risk and the corresponding optimal sampling plan. An explicit expression of the Bayes risk is derived. The new model can easily be adapted to create life testing models for different distributions. Specifically, two commonly used distributions including the exponential distribution and the Weibull distribution are considered with a special decision loss function. We demonstrate that the proposed model is superior to models with Type I or Type II censoring

This paper develops and validates a reliability sampling methodology using simulation, and re-sampling; and which incorporates unit-to-unit variation in the determination of significant sample sizes for analytically intractable reliability cases. This sample size determination is very important because the reliability of the sampled vehicles should represent the reliability of the entire fleet. Smaller-than-required sample sizes may lead to an incorrect representation of the reliability of the fleet, which may mislead the Army to make poor decisions, such as deploying a fleet that may not be reliable. These type II errors can be minimized by incorporating a more realistic sampling methodology, as developed in this research. Prior to using this methodology, analytical formulas were used to compute reliability sample sizes with unit-to-unit variation assumed to be constant. This new methodology confirms the analytically derived solutions for fixed usage & true failure rate, as well as for fixed usage & varying vehicle true failure rate. Existing reliability data shows that unit-to-unit variation does exist. Vehicle variation in true failure rate is modeled with a Gamma prior distribution. Recent reliability data are used to validate the hypothesis that this is an adequate tool for reliability sampling when unit-to-unit variation exists. Results of this validation accept the hypothesis, validating that the methodology is an adequate tool. The Army is currently using this methodology for fleet assessment

The two censoring schemes have been studied by numerous authors who proposed a concept of grouping in which the experimenter might group the test units into several sets, each set as an assembly of test units, and then all the units are tested simultaneously until the first failures in each group are observed [11], [12], [10] and [8]. It was indicated that in a situation where the lifetime of a product is quite high and test facilities are scarce, but test material is relatively cheap, one can test 'mn' units by testing m sets, each containing n units [3]. The life test is then conducted by testing each of these sets of units separately until the occurrence of first failure in each set. Such a censoring scheme is called first-failure censoring. The exact likelihood Inference for the Exponential Distribution under Generalized Type-I and type-II hybrid censoring was discussed [4]. A reduction in testing effect and administrative convenience may be obtained by using intermittent inspection whereby items are inspected only at certain points of time using exponential distribution [9]. Already, mixed sampling plans by blending the process and product control measures was developed [6]. Also several mixed sampling plans have been proposed to suit the industrial requirements [16]. Mixed sampling product control plans for costly or destructive items were also developed [5].

In reliability studies the exponential distribution plays a role of importance analogous to that of the normal distribution in other areas of statistics. The desirability of the exponential distribution is due to its simplicity and its inherent association with the well-developed theory of Poisson process. Also, many times certain quantities computed from the exponential distribution serve as bounds for similar quantities that need to be computed from other, less tractable distributions. A reliability test plan for exponentiated log- logistic distribution was studied [14]. The Economic Reliability Group Acceptance Sampling Plans for Lifetimes using a Generalized Exponential Distribution was also developed [7]. A few authors proposed Group Acceptance Sampling Plans for Lifetimes using a Marshall-Olkin Extended Exponential Distribution [13]. An economic reliability test plan using the generalized exponential distribution was developed [1]. A new generalization of Lomax distribution called Exponential Lomax distribution was proposed [3].

1.1 Formulation of the plan

In this Mixed Censoring Reliability Sampling Plans there are two stages, first stage being type I censoring and the second stage is dealt with type II censoring, where it is assumed that the random variable X follows an exponential distribution. The probability density function (pdf) is defined as:

$$f(x,\theta) = \theta e^{-\theta x}, \theta > 0 \qquad (1)$$

The cumulative distribution function (cdf) of exponential distribution is

$$F(x,\theta) = 1 - e^{-\theta}, \theta > 0 \qquad (2)$$

where θ is the parameter of the distribution.

1.2. Characteristics of Exponential Distribution:

Mean of the Exponential distribution

$$\bar{x}_E = \frac{1}{\theta}$$

Variance of the Exponential distribution

$$\sigma^2 = \frac{1}{\theta^2}$$

Skewness of the Exponential distribution

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

Kurtosis of the Exponential distribution

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

2. ALGORITHM FOR SENTENCING A LOT

Let the two stages be independent.

Step 1: Draw a random sample of size n_1 from the lot and put them into life test using type II censoring. Let it be $x_1, x_2, \ldots x_k$.

Step 2: Determine the skewness and kurtosis from the sample observations and hence the parameter θ is determined. Now determine the mean $\bar{x}_E$.

Step 3: If $\bar{x}_E \geq L$, accept the lot

Step 4: If $\bar{x}_E < L$, take a second sample of size n_2 and put them into life test using type I censoring

Step 5: In the second stage of life test, inspect and find the number of failures (d).

Step 6: Accept the batch or lot if d ≤ c, otherwise reject it.

3. MEASURES OF THE MIXED CENSORING RELIABILITY SAMPLING PLANS

Let $x_1, x_2 \ldots x_n$ be a random sample which follows a exponential distribution. Also let $d_1, d_2 \ldots d_n$ be the probability of failures. Hence the different measures are as follows

Operating Characteristic function:

By the addition theorem on probability, we get

$$P_a(p) = P_{Type-II,n_1}[\bar{x} \geq L] + P_{Type-I,n_2}[\bar{x} < L]P[d \leq c] \qquad (3)$$

Average Sample Number (ASN):

$$\text{ASN} = n_1 P[\bar{x} \geq L] + n_2 P[\bar{x} < L].P[d \leq c] \qquad (4)$$

Average Outgoing Quality (AOQ):

$$\text{AOQ} \approx p.P_a(p) \qquad (5)$$

4. DESIGNING THE MIXED CENSORING RELIABILITY SAMPLING PLANS INDEXED THROUGH AQL AND LQL

Step 1: Assume that the two stages are independent. Let β_1=0.95 and β_2= 0.10

Step 2: The first stage probability of acceptance is separated. Let it be $\beta_1^{'}$ and $\beta_2^{'}$ respectively. Also $\beta_1 \geq \beta_1^{"}$ and $\beta_2 \geq \beta_2^{"}$

Step 3: Using the procedure developed for non-normal distributions, the values of n_1 and k are determined using the following formula [17]:

$$n_1 = e_L \left(\frac{K_\alpha + K_\beta}{K^*_{1-p_1} - K^*_{1-p_2}}\right)^2 \quad (6)$$

$$k = -\frac{K_\alpha K^*_{1-p_2} + K_\beta K^*_{1-p_1}}{K_\alpha + K_\beta} \quad (7)$$

Step 4: The probability of acceptance associated with p_1 and p_2 is determined. Let them be denoted by $\beta_1^{"}$ and $\beta_2^{"}$

Where

$$\beta_1^{"} = \frac{\beta_1 - \beta_1'}{1 - \beta_1'} \quad (8)$$

$$\beta_2^{"} = \frac{\beta_2 - \beta_2'}{1 - \beta_2'} \quad (9)$$

Step 5: The values of n_2 and the corresponding acceptance number c are determined, such that

$$\sum_{x=0}^{c} \frac{e^{-n_2 p_1}(n_2 p_1)^x}{x!} \approx \beta_1^{"} \text{ and}$$

$$\sum_{x=0}^{c} \frac{e^{-n_2 p_2}(n_2 p_2)^x}{x!} \approx \beta_2^{"} \quad (10)$$

5. ILLUSTRATION

Let p_1 = 0.01 and p_2 = 0.1, the corresponding probability of acceptance be 0.95 and 0.10 respectively. Obtain the parameters of Mixed Censoring Reliability Sampling Plans using exponential distribution for the acceptance failure constant c = 3.

Solution:

It is given that the probability of acceptance is 0.95 and 0.10 at AQL and LQL respectively

By assuming the first stage probability of acceptance β_1' = 0.65 and $\beta_2' = 0.05$

from the Table 1,

We get n_1 = 9; k = 4.7555; n_2 = 78 for C = 3.

Algorithm for sentencing a lot:

Step 1: Draw a sample of size 9.

Step 2: Find the skewness and kurtosis using the sample observations and hence determine the parameter θ.

Step 3: Find the mean $\bar{x}_E$

Step 4: If $\bar{x}_E \geq L$, accept the lot

Step 5: If $\bar{x}_E < L$, take a second sample of size 78 and put them into life test using type I censoring

Step 6: In the second stage of life test, inspect and find the number of failures (d).

Step 7: Accept the lot if d ≤ 3, otherwise reject it.

Table 1: Mixed Censoring Reliability Sampling plans indexed through AQL and LQL based on Exponential distribution (β_1=0.95; β_2=0.10; β_1'=0.65; β_2'=0.05; $\beta_1^{"}$=0.86; $\beta_1^{"}$=0.05)

AQL p_1	LQL p_2	n_1	k	Operating ratio Z	n_2										
					C=0	C=1	C=2	C=3	C=4	C=5	C=6	C=7	C=8	C=9	C=10
0.001	0.01	20	7.2404	2.854	300	474	630	775	915	1051	1184	1315	1443	1571	1696
0.002	0.02	16	6.5270	2.138	150	237	315	388	458	526	592	657	722	785	848
0.003	0.03	14	6.1013	1.881	100	158	210	258	305	350	395	438	481	524	565
0.004	0.04	13	5.7934	1.741	75	119	157	194	229	263	296	329	361	393	424
0.005	0.05	12	5.5500	1.650	60	95	126	155	183	210	237	263	289	314	339
0.006	0.06	11	5.3474	1.585	50	79	105	129	153	175	197	219	241	262	283
0.007	0.07	10	5.1730	1.536	43	68	90	111	131	150	169	188	206	224	242
0.008	0.08	10	5.0192	1.497	37	59	79	97	114	131	148	164	180	196	212
0.009	0.09	9	4.8812	1.465	33	53	70	86	102	117	132	146	160	175	188
0.01	0.1	9	4.7555	1.439	30	47	63	78	92	105	118	131	144	157	170
0.011	0.11	9	4.6400	1.417	27	43	57	70	83	96	108	120	131	143	154
0.012	0.12	8	4.5327	1.397	25	40	52	65	76	88	99	110	120	131	141
0.013	0.13	8	4.4324	1.380	23	36	48	60	70	81	91	101	111	121	130
0.014	0.14	8	4.3381	1.365	21	34	45	55	65	75	85	94	103	112	121
0.015	0.15	7	4.2488	1.352	20	32	42	52	61	70	79	88	96	105	113
0.016	0.16	7	4.1640	1.340	19	30	39	48	57	66	74	82	90	98	106
0.017	0.17	7	4.0831	1.329	18	28	37	46	54	62	70	77	85	92	100
0.018	0.18	7	4.0057	1.319	17	26	35	43	51	58	66	73	80	87	94
0.019	0.19	7	3.9314	1.310	16	25	33	41	48	55	62	69	76	83	89
0.02	0.2	6	3.8598	1.301	15	24	31	39	46	53	59	66	72	79	85

Table 2: Probability of Acceptance values and Average Sample Number values for Mixed Censoring Reliability Sampling Plans using Exponential distribution

p	P_{a1}	P_{a2}	P_{a3}	ASN_1	ASN_2	ASN_3
0.02	1.0000	1.0000	1.0000	10	10	10
0.04	1.0000	1.0000	1.0000	10	10	10
0.06	0.9999	1.0000	1.0000	10	10	10
0.08	0.9997	0.9999	1.0000	10	10	10
0.1	0.9991	0.9998	1.0000	10	10	10
0.2	0.9826	0.9923	0.9972	10	10	11
0.3	0.9341	0.9597	0.9789	10	11	11
0.4	0.8630	0.8982	0.9333	10	11	11

5.1 Construction of Tables

Table 1:

i) The values of p_1 and p_2 are assumed to known.

ii) The probabilities of acceptance values such as β_1=0.95, β_2=0.10, β_1'= 0.65 and β_2' =0.05 are assumed.

iii) For the second stage, the probabilities of acceptance are obtained using equations (8 and 9)

iv) The sample size n_1 and the corresponding values of k of the first stage for the given values of p_1 and p_2 are obtained.

v) Here, variable sampling plans using exponential distribution is used for obtaining the first stage parameters. And hence n_1 and k are determined using equations (6 and 7).

vi) The operating ratio Z (= p_2/p_1) is obtained for this sampling plan using chi-square variates.

vii) In the second stage c is fixed and hence $n_2$is obtained using equation (10). The values of n_2 are obtained through AQL and LQL.

viii) The values of n_1 and n_2 so obtained are the minimum sample sizes and they are presented in Table 1.

Table 2:

i) The values of p are assumed to be known.

ii) The corresponding probabilities of acceptance namely P_{a1}, P_{a2} and P_{a3} for varying values of n_1, n_2, k and C are obtained using equation (3)

iii) Then the ASN values are obtained using equation (4) and are presented

iv) Using the Table 2, the OC and ASN curves are constructed and presented in Figure 1 and Figure 2.

Figure 1: OC Curves for the Mixed Censoring Reliability Sampling plans using Exponential Distribution

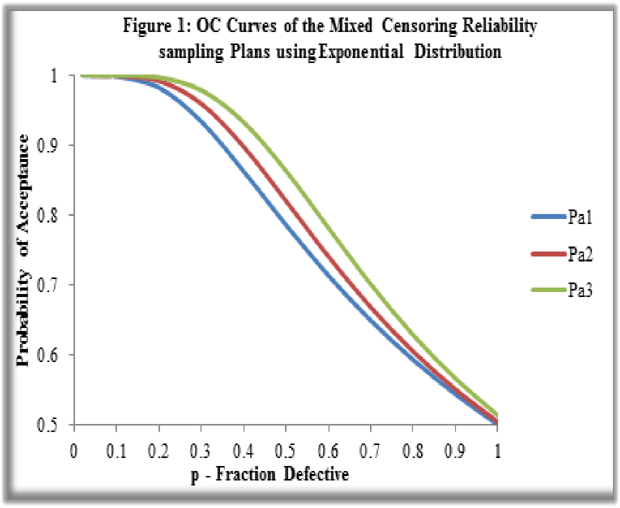
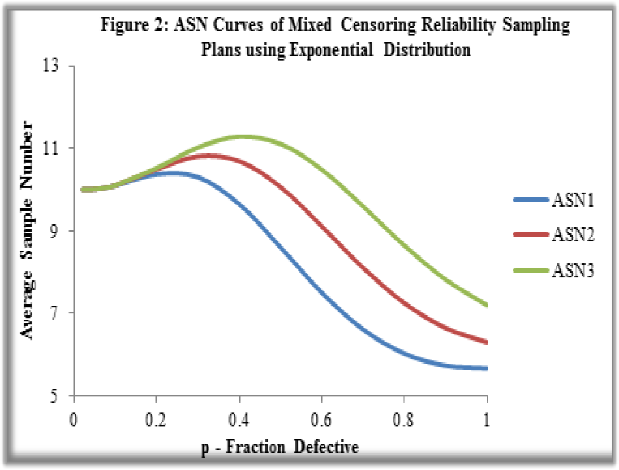


Figure 2: ASN Curves for Mixed Censoring Reliability Sampling Plans using Exponential Distribution



5.2 Construction of OC curves

The operating characteristic curves for the Mixed Censoring Reliability Sampling Plans for Exponential distribution are constructed and presented in Figure 1. The probability of acceptance values are obtained using equation (3) for n_1=10, n_2=10 and C= 2, 3, 4 and are presented in Table 2. From the Figure 1, P_{a1} is the probability of acceptance when n_1 = 10, n_2 = 10 and C = 2, P_{a2} is the probability of acceptance when for n_1 = 10, n_2 = 10 and C = 3 and P_{a3} is the probability of acceptance when for n_1 = 10, n_2 = 10 and C = 4.

5.3 Construction of ASN Curves

The ASN curves for the Mixed Censoring Reliability Sampling Plans are constructed and presented in Figure 2. The Average Sample Number (ASN) values are obtained using equation (4) for n_1=10, n_2=10 and C= 2, 3, 4 and are presented in Table 2. From the Figure 2, ASN_1 curve is obtained for n_1=10, n_2=10 and C=2, ASN_2 curve is obtained for n_1=10, n_2=10 and C=3 and ASN_3 curve is obtained for n_1=10, n_2=10 and C=4.

6. CONCLUSION

In this paper, the Mixed Censoring Reliability Sampling Plans are developed for one parameter exponential distribution. The operating procedure for these plans is very simple and can be used effectively in the quality control section. It is found that the new reliability sampling plans give more pressure on the producer to maintain the quality of the lots or batches. It is observed from the tables that the first stage requires minimum sample sizes when compared to the second stage. Also as the acceptance constant increases the sample size increases. And it is found that the sample size increases in case of type I censoring compared to type II censoring. Whenever the probabilities of acceptance are known, various parameters like n_1, n_2, k and c can be easily obtained. Tables are given for easy selection and implementation in industries.

References

[1] Balasooriya, U, "Failure-censored Reliability Sampling Plans for the Exponential Distribution", Journal of Statistical Computation and Simulation, Vol.52, 1995, pp.337-349.

[2] Chandrasekar, B., Childs, A. and Balakrishnan, N, "Exact likelihood Inference for the Exponential Distribution under Generalized Type-I and type-II hybrid censoring", Naval research logistics, Vol.51 (7), 2004, pp. 994-1004.

[3] Jianwei Chen, Winlin Chou, Hulin Wu and Haibo Zhou, "Designing Acceptance Sampling Schemes for Life Testing with Mixed Censoring" Naval Research Logistics, Vol. 51(4), 2004, pp. 597 – 612.

[4] J.Nierwinski, "Reliability Sampling Methodology Using Simulation and Re-Sampling," in IEEE Transactions on Reliability, vol. 56, no. 1, 2007, pp. 125-131.

[5] Devaarul, S, "Certain Studies Relating to Mixed Sampling Plans and Reliability based Sampling Plans", Ph.D., Thesis, Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu, India, 2002.

[6] Kim, S.H. and Yum, B.J, "Comparisons of Exponential Life Test Plans with Intermittent Inspection", Journal of Quality Technology, Vol.32, 2000, pp.217-230.

[7] Rosaiah, K., Kantam, R. R. L. and Santhosh Kumar, C, "Reliability Test Plans for Exponentiated Log-Logistic Distribution", Economic Quality Control, Vol. 21(2), 2006, pp.165-175.

[8] Schilling, E. G, Acceptance Sampling in Quality thank you very much Control. New York: Marcel Dekker Inc , 1982.

[9] Kim, S.H. and Yum, B.J, "Comparisons of Exponential Life Test Plans with Intermittent Inspection", Journal of Quality Technology, Vol.32, 2000, pp.217-230.

[10] Lawless, J.F, Statistical Models and Methods for Lifetime Data, 2nd Edition., New York, 2003.

[11] Mann, N.R., Schafer, R.E. and Singpurwalla, N.D, Methods for Statistical Analysis of Reliability and Life Data, New York, 1974.

[12] Meeker, W.Q. and Escobar, L.A, Statistical Methods for Reliability Data, New York, 1998.

[13] Rao, G.S, "A Group Acceptance Sampling Plans for Lifetimes Following a Marshall-Olkin Extended Exponential Distribution" Vol. 6 (2), 2011, pp.592–601.

[14] Rosaiah, K., Kantam, R. R. L. and Santhosh Kumar, C, "Reliability Test Plans for Exponentiated Log-Logistic Distribution", Economic Quality Control, Vol. 21(2), 2006 pp.165-175.

[15] Schilling, E. G, Acceptance Sampling in Quality Control. New York: Marcel Dekker Inc, 1982.

[16] Suresh, K.K. and Devaarul, S, "Combining Process and Product Control for Reducing Sampling Costs", Economic Quality Control, Journal and Newsletter for Quality and Reliability, Vol. 17, No.2, 2002, pp. 187-194.

[17] Takagi, k, "On Designing Unknown-sigma Sampling Plans based on a Wide Class of Non-normal Distributions", Technometrics, Vol. 14, No.3, 1972, pp. 669-678.