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COLORING FOR THE IDENTITY GRAPHS OF GROUPS

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Abstract - *Let $(Z_n, \oplus)$, addition modulo n be the group. The identity graph of $(Z_n, \oplus)$ is defined as: Let x, y be two elements in the graph are adjacent (or) can be joined by an edge if x.y=e. The vertices corresponds to the elements of the group, hence the order of the group $(Z_n, \oplus)$ corresponds to the number of vertices in the identity graph. In this paper we find the total chromatic number, edge chromatic number and vertex chromatic number for the identity graph of $(Z_n, \oplus)$.*

***Key Words*: vertex chromatic number, edge chromatic number, total chromatic number, identity graph of $(Z_n, \oplus)$.**

1. INTRODUCTION

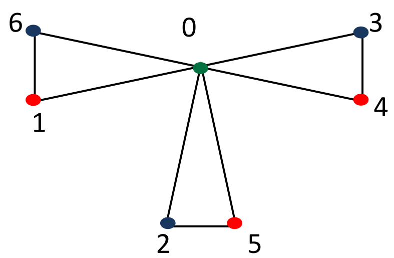
Throughout this paper, we consider only the identity graph of the group $(Z_n, \oplus)$. The order and the size are denoted by n and m respectively. Degree of a vertex v is denoted by $d(v)$, the maximum degree of a graph G is denoted by $\Delta(G)$. In Graph Theory, Graph coloring is a well-known area which is widely studied by many researchers various types of graph coloring and many open problems were discussed in the wonderful books. A vertex coloring is an assignment of labels or colors to each vertex of a graph such that no edge connects two identically colored vertices. The most common type of vertex coloring seeks to minimize the number of colors for a given graph. Such a coloring is known as a minimum vertex coloring, and the minimum number of colors which with the vertices of a graph G may be colored is called the chromatic number, denoted by vertex chromatic number $\chi(G)$. An edge coloring of a graph G is a coloring of the edges of G such that adjacent edges receive different colors. An edge coloring containing the smallest possible number of colors for a given graph is known as a minimum edge coloring and is denoted by edge chromatic number $\chi'(G)$. The total coloring of a graph G is an assignment of colors to the vertices and edges such that no two incident or adjacent elements receive the same color. The total chromatic number of G, denoted by χ'' (G), is the least number of colors required for a total coloring. In [2] the author W.B. Vasanth Kandaswamy Florentin Smarandache introduced Groups as graphs. Let $(Z_n, \oplus)$ be the group. The identity graph of $(Z_n, \oplus)$ is obtained as follows: Let x, y be two elements in the graph are adjacent (or) can be joined by an edge if x. y=e.

2. EXAMPLES

Example 2.1:

The identity graph of Z_7 = {0, 1, 2, 3, 4, 5, 6}, the group under addition modulo seven.

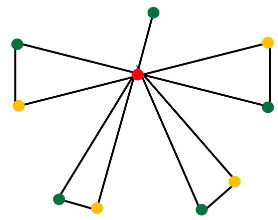
+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5



The chromatic number of the identity graph of $(Z_7, \oplus)$ is $\chi(G)$= 3.

Example 2.2:

Let Z_{10} = {0,1,2,3,4,5,6,7,8,9} be the group under addition modules ten . For the identify graph of the group Z_{10} $\chi''(G) = 4$



3. MAIN RESULTS

Theorem 3.1:

For the identify graph G of $(Z_n, \oplus)$, n ≥2

$$\chi(G) = \begin{cases} 2 \text{ if } n = 2 \\ 3 \text{ if } n \geq 3 \end{cases}$$

Proof:

Case (i): Suppose n is odd.

Then G contains n-1 vertices of degree 2 and one vertex of degree n-1, let it be v.

If n = 3, Then the identity graph of $(Z_3, \oplus)$ is a cycle C_3. Therefore it is clear that $\chi(G)$ = 3.

If n > 3, then the vertex v adjacent with n-3 triangles and let us color v by red. So we can color the remaining two vertices of each cycle with Blue and yellow.

$\therefore \chi(G)$ = 3

Case (ii): Suppose n is even.

If n = 2, then the result is obvious.

If n ≥4 then the identity graph of Z_n has one point union of $\frac{n-2}{2}$ copies of cycle c_3 with a vertex incident with v. Let us color v by Red. So this is possible to color the remaining two vertices of each cycle with Blue and yellow.

$\therefore \chi(G)$ = 3.

Note 3.2:

If n>3, the identity graph G of $(Z_n, \oplus)$ contained only one cut vertices which is the vertex's with the maximum degree

Theorem 3.3:

For identity graph G of Z_n with n≥2 $\chi'(G) = \begin{cases} n \text{ } if \text{ } n \text{ } is \text{ } odd \\ n-1 \text{ } if \text{ } n \text{ } is \text{ } even \end{cases}$

Proof:

If n=2, the result is obvious.

Hence let n >2. If n is odd. Now the edges of G can be n-colored as follows. Place the vertices of G in the form of a regular n-gon color the edge around the boundary using a different color for each edge.

Let x be any one of the remaining edge. x divides the boundary into two segment, one say B_1 containing an odd number of edge and other containing an even number of edges. Color the edge x with the same color as the edge that occurs in the middle of B_1. Note that these two edges are parallel. The result is an n-edge coloring of G. Since any two edges having the same color are parallel and hence are not adjacent.

Hence $\chi'(G) \leq$ n (1)

Since G has n vertices and n is odd, it can have at most $\frac{1}{2}$ (n-1) mutually independent edges. Hence each color class can have at most $\frac{1}{2}$ (n-1) edges, so that the number of color classes is at least $\binom{n}{2}\frac{1}{2}(n-1) = n$ So that χ'(G) ≥ n (2)

(1) and (2) together implies that $\chi'(G) =$ n.

Let n be even and ≥ 4. Let G have vertices $v_1, v_2 \ldots\ldots\ldots v_n$. Color the edges of the sub graph of G' induced by the first n-1 points using the method described above. In this coloring, at each vertex, one color (the color assigned to the edges opposite to this vertex on the boundary) will be missing. Also, there missing colors are all different. This edge coloring of G' can be extended to an edge coloring of G by assigning the color that is missing at v_i to edge v_iv_n for every i, i <n

Hence χ'(G) ≤ n-i

Also χ'(G) ≥ $\Delta(B_n)$ = n-1.

Hence χ'(G) = n-1.

Theorem 3.4:

For the total coloring of identity graph G of Z_n

$$\chi''(G) = \begin{cases} 3 \text{ if } n = 2 \\ 3 \text{ if } n \text{ is odd} \\ 4 \text{ if } n \text{ is even and } n \geq 4 \end{cases}$$

Proof:

Case (i): Suppose n is odd

Then G has $\left\lfloor \frac{n}{2} \right\rfloor$ copies of cycle C_3 they have a single common point. Let it be v. Then $d\ (v) =\ n - 1$ and color v by Red. So color the remaining two vertices of each cycle with Blue and yellow

$\therefore \chi''(G)$= 3

Case (ii): Suppose n is even

For n = 2, by the definition of total coloring, it is clear that $\chi''(G)$= 3

Let v be a vertex of maximum degree n-1

I.e. d (v) = n-1

If n ≥ 4, then G contains $\left\lfloor \frac{n}{2} \right\rfloor$-1 copies of cycle c_3 and a vertex adjacent to v. Let us color v by red and color the remaining vertices of every cycles by blue and yellow. Since G has a pendant vertex which is adjacent with v, coloring that vertex by either blue or yellow.

Color the edges by a color which are not are adjacent coloring of its neighborhood.

Color the remaining edges by a new color orange. Therefore we need 4 colors as minimum possible

$\therefore \chi''(G) = 4$.

Theorem 3.5:
For the identity graph G of $(Z_n, \oplus)$. $K_2 \circ G$ is total colorable
Proof:
Let G be the identity graph of Z_n
The graphs $K_2 \circ G \cong G \vee G$

Denote the two copies of G by G_1 and G_2

The maximum degree of $G \vee G$ is $\Delta(G \vee G) = \Delta(G) + n$, where n is the order of G.

Color the elements (vertices and edges) of G_1 using coloring 1,2,………..Δ(G)+2

Assign n new colors to the vertices of G_2 and color the edges of G_2 as the edge coloring of G_1. Here the corresponding vertices G_1 & G_2 have common missing colors from {1,………Δ(G)+2}

Now assign a common missing color to the edges (all edges together give one 1-factor of $G \vee G$) between the corresponding vertices .At each vertex in G_2, there are n-1 available colors among the n vertex colors and using these available colos. we the remaining join edges between G_1 & G_2.

Theorem 3.6:

For identity graph G of Z_n, $\chi''(G) \geq \delta(G) + 1$.

Proof:

For a total coloring of graph G with the minimum color, the maximum degree vertex and the incident edges to this vertex must be colored. The maximum degree of G is n-1 and the minimum degree δ (G) =2 so the total coloring number of the graph G is at least one more than the minimum degree of G. $\therefore \chi''(G) \geq \delta(G)+1$.

4. CONCLUSION

In this paper we find the total chromatic number, edge chromatic number and vertex chromatic number for the identity graph of $(Z_n, \oplus)$.

REFERENCES

[1] Chartrand.G and Zhang,P., Chromatic graph theory, Chapman and Hall, CRC,2009

[2] GROUPS AS GRAPHS by W.B.Vasantha Kandasamy Florentin Smarandache.

[3] Introduction to Graph Theory by ROBIN J. WILSON, Fourth Edition.

[4] Invitation to Graph theory by S. Arumugam and S.Ramachandran.

[5] Jenson,T.R and Toft,B., Graph Coloring problems wiley- Interscience,2009

[6] Total Coloring of Central Graphs of a path, a Cycle and a Star by S.Sudha and K.Manikandan

[7] Total Coloring and Total Coloring of Thorny Graphs by Zeynep ORS YORGANCIOGLU and Pinar DUNDAR