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Skolem Minkowski-4 Mean Labeling of Graphs

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Abstract - Let $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ be an simple and undirected graph with $\mathbf{p}$ vertices and $\mathbf{q}$ edges. Let us define a function $\phi: \mathbf{V(G)} \rightarrow \{\mathbf{1, 2, 3, \ldots, p}\}$ is called **Skolem Minkowski-4 Mean Labeling** of a graph G if we could able to label the vertices $\mathbf{x} \in \mathbf{V}$ with distinct elements from $\mathbf{1, 2, \ldots, p}$ such that it induces an edge labeling $\phi^*: \mathbf{E(G)} \rightarrow \{\mathbf{2, 3, \ldots, p}\}$ defined as,

$$\phi^*(\mathbf{e} = \mathbf{uv}) = \left\lceil \left(\frac{\phi(\mathbf{u})^4 + \phi(\mathbf{v})^4}{2}\right)^{\frac{1}{4}} \right\rceil,$$

is distinct for all edges $\mathbf{e} = \mathbf{uv} \in \mathbf{E}$. (i,e.) It indicates that, distinct vertex labeling induces a distinct edge labeling on the graph. The graph which admits Skolem Minkowski-4 Mean Labeling is called a **Skolem Minkowski-4 Mean Graph.** In this paper, we have investigated the Skolem Minkowski-4 Mean Labeling of some standard graphs like Path, Comb, Caterpillar, $\mathbf{P_n} \odot \mathbf{K_{1,2}}$, etc.

Key Words: Skolem Minkowski-4 Mean Labeling, Skolem Minkowski-4 Mean Graph, Path, Comb, Caterpillar, $P_n \odot K_{1,2}$.

1. INTRODUCTION

The graph G we used here are simple, finite and undirected graphs. V(G) and E(G) denotes the vertex set and edge set of a graph G. For graph theoretic terminology, we refer to Harary.F [3], Douglas B. West [1] and Gallian.J.A [2]. The concept of Mean Labeling of graphs was introduced by Somasundaram.S and Ponraj.R [5] in 2003. Sandhya.S.S, Somasundaram.S and Anusa.S [7] introduced the concept of Root Square Mean Labeling of graphs in 2014. V.Balaji, D.S.T.Ramesh and A.Subramanian [4] introduced the concept of Skolem Mean Labeling in 2007. On the same lines we define and study **Skolem Minkowski-4 Mean Labeling of graphs**.

2. BASIC DEFINITIONS

The following definitions are needed for the present study.

A. Definition

A walk in which all the vertices say $u_1, u_2, u_3, \ldots, u_n$ are distinct is called a **Path.** A Path is denoted by P_n . The Path P_n has n vertices and $n - 1$ edges.

B. Definition

The graph attained by attaching a single pendent edge to each vertex of a Path is called **Comb.** Generally, it has 2n vertices and $2n - 1$ edges.

C. Definition

A tree which yields a Path when its pendant vertices are removed is called a **Caterpillar.** It has 3n vertices and $3n - 1$ edges.

D. Definition

The $P_n \odot K_{1,2}$ is a graph attained by attaching the complete bipartite graph $K_{1,2}$ to each vertex of the path P_n . It has 3n vertices and $3n - 1$ edges.

3. MAIN RESULTS

Theorem: 1

Path P_n is a Skolem Minkowski-4 Mean graph for every n.

Proof:

Let P_n be a path $u_1, u_2, u_3, \ldots, u_n$ of length n. The path P_n has n vertices and $n - 1$ edges.
Let us define a function $\phi: V(G) \rightarrow \{1,2,3, \ldots, p\}$ by

$$\phi(u_i) = i \text{ ,} 1 \leq i \leq n$$

Then the induced edge labels are,

$$\phi^*(u_i u_{i+1}) = i + 1 \text{ ,} 1 \leq i \leq n - 1$$

Then we attain a distinct edge labels.
Therefore, P_n is a Skolem Minkowski-4 Mean graph.

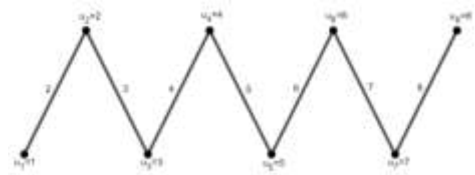

Figure 1: P_8

Theorem: 2

For every n, Comb $P_n \odot K_1$ is a Skolem Minkowski-4 Mean Graph .

Proof:

Let $P_n \odot K_1$ be a comb attained from a path $P_n = u_1, u_2, u_3, \ldots, u_n$ by attaching each vertex u_i to a

pendent vertex $v_i (1 \leq i \leq n)$ .The gragh G has 2n vertices and $2n - 1$ edges.
Let us define a function $\phi: V(G) \rightarrow \{1,2,3, \dots, p\}$ by

$$\phi(u_i) = \begin{cases} 2i \text{ , when } i = 1 \\ 2i - 1 \text{ ,} 2 \leq i \leq n \end{cases}$$

$$\phi(v_i) = \begin{cases} i \text{ , when } i = 1 \\ 2i \text{ ,} 2 \leq i \leq n \end{cases}$$

Then the induced edge labels are,

$$\phi^*(u_i u_{i+1}) = 2i + 1 \text{ ,} 1 \leq i \leq n - 1$$

$$\phi^*(u_i v_i) = 2i \text{ ,} 1 \leq i \leq n$$

Then we attain a distinct edge labels.
Therefore, $P_n \odot K_1$ is a Skolem Minkowski-4 Mean graph.

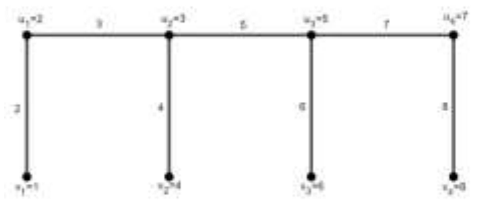

Figure 2: $P_4 \odot K_1$

Theorem: 3

Let G be a graph attained by joining a pendant edges to both sides of each vertex of a path P_n. Then G is a Skolem Minkowski-4 Mean graph.

Proof:
Let us consider a graph G which is attained by attaching a pendant edges to both sides of each vertex of a path P_n. Let P_n be a path $v_1, v_2, v_3, \dots, v_n$. Let u_i and w_i be the pendant vertices adjacent to $v_i (1 \leq i \leq n)$. The gragh G has $3n$ vertices and $3n - 1$ edges.
Let us define a function $\phi: V(G) \rightarrow \{1,2,3, \dots, p\}$ by

$$\phi(u_i) = \begin{cases} 3i - 2 \text{ , } if\ i\ is\ odd \\ 3i - 1 \text{ , } if\ i\ is\ even \end{cases}$$

$$\phi(v_i) = \begin{cases} 3i - 1 \text{ , } if\ i\ is\ odd \\ 3i - 2 \text{ , } if\ i\ is\ even \end{cases}$$

$$\phi(w_i) = \{3i \text{ ,} 1 \leq i \leq n$$

Then the induced edge labels are,

$$\phi^*(v_i v_{i+1}) = 3i + 1 \text{ ,} 1 \leq i \leq n - 1$$

$$\phi^*(v_i u_i) = 3i - 1 \text{ ,} 1 \leq i \leq n$$

$$\phi^*(v_i w_i) = 3i \text{ ,} 1 \leq i \leq n$$

Then we attain a distinct edge labels.
Therefore, G is a Skolem Minkowski-4 Mean graph.

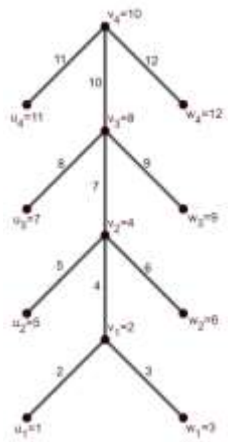

Figure 3: Caterpillar

Theorem: 4

$P_n \odot K_{1,2}$ is a Skolem Minkowski-4 Mean graph, for every n.

Proof:
Let G be a graph attained by attaching each vertex of P_n to the central vertex of the complete bipartite graph $K_{1,2}$. Let P_n be a path $u_1, u_2, u_3, \dots, u_n$ and v_i , w_i be the vertices of $K_{1,2}$, which are attached to the vertex u_i of P_n.The gragh G contains $3n$ vertices and $3n - 1$ edges.
Let us define a function $\phi: V(G) \rightarrow \{1,2,3, \dots, p\}$ by

$$\phi(u_i) = \begin{cases} i \text{ , } when\ i = 1 \\ i + 2 \text{ , } when\ i = 2 \\ 3i - 1 \text{ ,} 3 \leq i \leq n \end{cases}$$

$$\phi(u_i) = \begin{cases} 2\, i \text{ , } when\ i = 1 \\ i + 3 \text{ , } when\ i = 2 \\ 3i - 2 \text{ ,} 3 \leq i \leq n \end{cases}$$

$$\phi(w_i) = \{3i \text{ ,} 1 \leq i \leq n$$

Then the induced edge labels are,

$$\phi^*(u_i u_{i+1}) = 3i + 1 \text{ ,} 1 \leq i \leq n - 1$$

$$\phi^*(u_i v_i) = 3i - 1 \text{ ,} 1 \leq i \leq n$$

$$\phi^*(u_i w_i) = 3i \text{ ,} 1 \leq i \leq n$$

Then we attain a distinct edge labels.
Therefore, $P_n \odot K_{1,2}$ is a Skolem Minkowski-4 Mean graph.

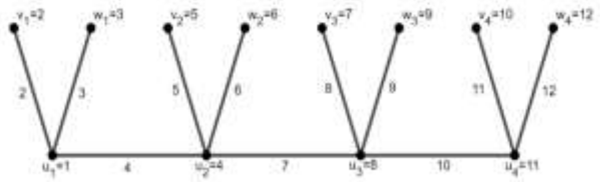

Figure 4: $P_4 \odot K_{1,2}$

Theorem: 5

Let G be a graph attained by attaching K_1 at each pendant vertex of a comb. Then G admits a Skolem Minkowski-4 Mean graph.

Proof:

Let G be a graph attained by attaching K_1 at each pendant vertex of a comb. The gragh G has $3n$ vertices and $3n-1$ edges.

Let us define a function $\phi: V(G) \to \{1,2,3,\dots,p\}$ by

$$\phi(u_i) = \begin{cases} i\,, when\ i = 1 \\ i+2\,, \text{when i} = 2 \\ 3i-1\,, 3 \le i \le n \end{cases}$$

$$\phi(u_i) = \begin{cases} 2\,i\,, \text{when i} = 1 \\ i+3\,, \text{when i} = 2 \\ 3i-2\,, 3 \le i \le n \end{cases}$$

$$\phi(w_i) = \{3i\,, 1 \le i \le n$$

Then the induced edge labels are,

$$\phi^*(u_i u_{i+1}) = 3i+1\,, 1 \le i \le n-1$$

$$\phi^*(u_i v_i) = 3i-1\,, 1 \le i \le n$$

$$\phi^*(v_i w_i) = 3i\,, 1 \le i \le n$$

Then we attain a distinct edge labels.

Therefore, G is a Skolem Minkowski-4 Mean graph.

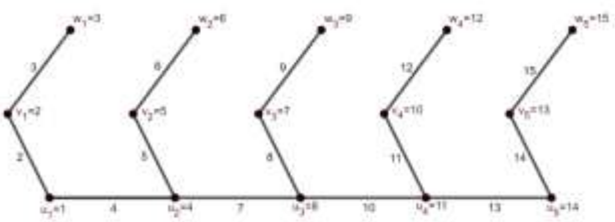

Figure 5: G

4. CONCLUSION

In this paper, we have introduced the notion of Skolem Minkowski-4 Mean Labeling and studied for some standard graphs. Illustrative examples are provided to support our investigation.

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