An Investigation of Stresses induced in Curved Beams using MATLAB and Finite Element Analysis (FEA)

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Abstract - *Curved beams find many applications such as Crane hook, Portable hydraulic inverter, Offset bar, S-link etc. For the proper functioning of curved beam, it should have high material properties to withstand stresses induced in it. When it is subjected to load, the bending stress developed in it should be within safety limits. This report deals with stress analysis of crane hook using MATLAB SOFTWARE. The same is done in ANSYS WORKBENCH (FEA). Finally, the results of both are compared and the cross section of the crane hook which induces minimum stress for the given load, cross sectional area and radius of curvature of the curved beam is identified and recognized as a optimal cross section for the crane hook.*

Key Words: Curved beams, Bending stress, Crane Hook, MATLAB and Ansys workbench.

1. INTRODUCTION

If a beam is originally curved before applying any bending moment, then it is considered as curved beam. Its centroidal axis is not straight and is curved in the elevation hence it is said to be a curved beam. In Straight beam, the centroidal axis and the neutral axis coincide. But in curved beams, the neutral axis and the centroidal axis do not coincide and the neutral axis will be shifted towards the centre of curvature. Due to this shifting of the neutral axis towards the centre of curvature, the stress distribution in the curved beam will be non-linear.[3]

1.1 Classification of curved beams

Curved beams can be classified into 2 sections:

1. Beams with small initial curvature.
2. Beams with large initial curvature.

Though there is no clear distinct difference in the definition of both the sections; however, they are classified based on ratio of initial radius of curvature to the depth of the section. If this ratio exceeds number 10, then it belongs to classification of beams with small initial curvature, if it is the other way then it can be classified as beams with large initial curvature.[4]

The bending stress equation for curved beams is given by Design Data Handbook.[1]

$$\sigma_b = \frac{M}{Ae}\left(\frac{y}{R_n - y}\right)$$

M = Bending moment acting at the given section about the centroidal axis
A = Area of cross-section
e = Distance from the centroidal axis to the neutral axis $= R - R_n$
R = Radius of curvature of the centroidal axis
R_n = Radius of curvature of the neutral axis
y = Distance between the neutral axis to the considered fibre at which bending stress is needed to be calculated

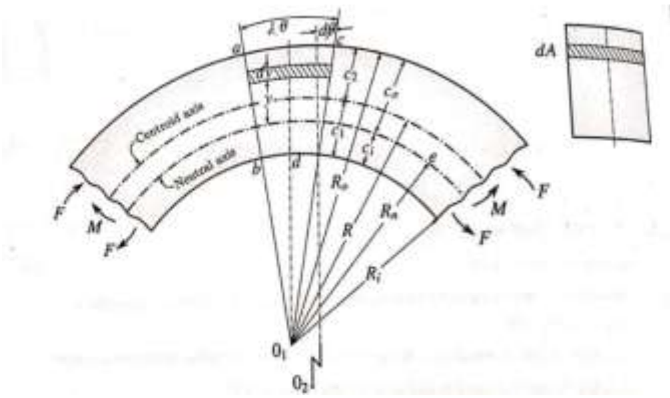

Fig 1: Parameters of a curved beam

1.2 Applications of curved beams

Curved beams find applications in many machine members, some of them are listed below:

- C-clamps
- Crane hook
- Frames of presses
- Chains, links & rings etc.

Curved beams are widely used in Civil Engineering applications. In RCC buildings they are normally seen around recreation purpose buildings (centre), convention centres, cement silos etc. where circular beams serve the purpose in the form of ring beams. Curved beams are also used to support curved glass applications in high-end housing and other structures.[3]

2. Stresses in Curved beams

There are some assumptions made while finding the bending stress for the curved beams. They are as follows:

• It is considered that the material throughout the beam is same (Homogeneous material)
• It should obey the Hooke's law (Stress is directly proportional to the strain in the beam)
• Each layer in the beam has to expand or contract freely and independently.
• The load should be applied in the plane (Plane of curvature) of bending
• The Young's modulus is to be same for both the tension and the compression.[8][3]

If the section is asymmetric then the maximum bending stress would be induced either at inside fiber or at outside fiber. By using these formulas, we can calculate the bending stress.[4]
The maximum Bending stress at inside fiber is given by Design Data Handbook.[1]

$$\sigma_{bi} = \frac{M}{Ae}(\frac{y_i}{R_i})$$

y_i = Distance between neutral axis to the inside fiber = $R_n - R_i$
R_i = Radius of curvature of inside fiber

The maximum Bending stress at outside fiber is given by Design Data Handbook.[1]

$$\sigma_{bo} = \frac{M}{Ae}(\frac{y_0}{R_0})$$

y_o = Distance between neutral axis to the inside fiber =$R_o - R_n$
R_o = Radius of curvature of outside fiber

In addition to the bending moment if there is axial load then the axial load can be alphabetically added to the bending stress to obtain the resultant stress on the given curved beam.[4]

$$\sigma = \sigma_d \pm \sigma_b$$

The above equations were derived by Winkler Bach to obtain the stresses induced in a curved beam.

2.1 Geometrical properties of cross sections used in curved beams

There are different cross sections in curved beams like Solid rectangular section, Solid circular section, Hollow circular section, Solid Elliptical section, Triangular section, Trapezoidal section beam etc.

Following 3 major sections are considered for the study i.e., Circular, Elliptical and Trapezoidal.

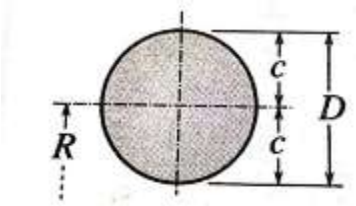

Fig 2: Circular section

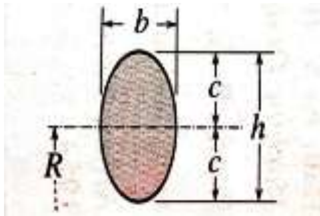

Fig 3: Elliptical section

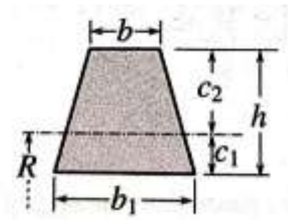

Fig 4: Trapezoidal section

Table -1: Geometric properties of cross section used in curved beams [1]

Cross sections	R_n
Circular	$\frac{0.5\,c^2}{R - \sqrt{R^2 - c^2}}$
Elliptical	$\frac{0.5\,c^2}{R - \sqrt{R^2 - c^2}}$
Trapezoidal	$\frac{A}{[\frac{b_1(R+c_2) - b(R-c_1)}{h} log_e \frac{R+c_2}{R-c_1}] - (b_1 - b)}$ (A-area of trapezium)

3. Case study – Crane hook

Due to characteristics of simple manufacturing and strong usability, crane hooks are widely used to handle materials. The hooks used in hoists and various types of cranes play a major role in lifting the heavy loads in many sectors, industries etc. Wrought iron and steel are the two widely used material for crane hook manufacturing. [5]

Fig 5: Crane hook

Here in our study 3 major cross sections of Crane hook i.e., circular, elliptical and trapezoidal sections and their behavior when loaded is studied. MATLAB codes for stress analysis of major cross sections has been obtained along with the stress values from ANSYS software. By comparing both the values cross section of crane hook which induce minimum stress is determined.

3.1 MATLAB Codes for Curved Beams

It is tedious to obtain the stresses induced in the curved beams using analytical method since it involves a lengthy time-consuming procedure. Hence the MATLAB codes for solving problems on curved beams given the load, section parameters and center of curvature is obtained. First it is coded to input the required parameters , then the codes are used to find R_o(radius of outer fiber), R_i(radius of inner fiber), R(radius of centroidal axis), R_n(radius of neutral axis) and finally the stresses i.e., the stresses in outer fiber, stresses in inner fiber and net stress.

PROBLEM

A crane hook has to handle a load of 100kN. Determine the stresses induced for the trapezoidal, circular and elliptical cross sections.

The section parameters have been selected such that overall cross-sectional area for all the three sections remain same for the comparison.

```
function [Si,So]=trap(C,b1,b,h,F);
Ri=C;
Ro=C+h;
A1=0.5*b1*h;
A2=0.5*b*h;
A=A1+A2;
R1=(h/3)+ C;
R2=(C+h)-(h/3);
R=((A1*R1)+(A2*R2))./(A1+A2);
c1=R-Ri;
c2=h-c1;
Rn=A./((((b1*(R+c2))-(b*(R-c1)))./h).*(log((R+c2)./(R-c1)))-(b1-b));
e=R-Rn;
Ci=c1-e;
Co=c2*e;
M=F*R;
Si=(M*Ci)./(A*e*Ri)
So=(M*Co)./(A*e*Ro)
```

Fig 6: MATLAB program for trapezoidal section

INPUT [Si, So] =trap (100,40,80,100,100000);
C-center of curvature of beam in mm = 100mm
b1- 40mm
b- 80mm
h- 100mm
F-load applied = 100kN

Table 2: Results for stresses obtained for Trapezoidal section

Maximum stress at inner fiber Si	171.0421 MPa
Maximum net stress induced (Si+$\frac{F}{A}$)	187.7088 MPa

```
function [Si,So] = circl(d,Ri,f);
c=d/2;
A=pi*(c^2);
Ri;
Ro=Ri+d;
R=Ri+c;
Rn=.5*(c^2)/(R-((R^2)-(c^2))^(.5));
e= R-Rn;
cg=R;
M=f*cg;
Si=(M*c)/(A*e*Ri);
So=(M*c)/(A*e*Ro);

end
```

Fig 7: MATLAB Program for Circular Section

INPUT [Si, So] =Circl (100,100,500000);
d-diameter=80mm
Ri-radius of inner fiber=100mm
F-load applied=100kN

Table 3: Results for stresses obtained for circular section

Maximum stress at inner fiber Si	283.6048 MPa
Maximum net stress induced (Si+$\frac{F}{A}$)	300.2714 MPa

```
function [Si,So] = ellipt(C,F,c,b);
A=pi*2*c*b;
F=100000;
Ri=C;
Ro=C+(2*c);
h=c+c;
R=C+c;
Rn=(0.5*(c^2))./(R-(sqrt((R^2)-(c^2))));
e=R-Rn;
Ci=c-e;
Co=c+e;
M=F*R;
Si=(M*Ci)./(A*e*Ri)
So=(M*Co)./(A*e*Ro)
```

Fig 8: MATLAB program for elliptical section

INPUT [Si, So] =Ellipt (100,10000,40,20);
C=center of curvature=100mm

F=100kN
c=40mm
b=20mm

Table 4: Results for stresses obtained for elliptical section

Maximum stress at inner fiber Si	215.47 MPa
Maximum net stress induced (Si+$\frac{F}{A}$)	232.1366 MPa

4. FEA of Crane hook

Finite element method is a numerical method which gives approximate solutions for any engineering problems. Optimization using FEM is less time-consuming compare to experimental technique. In present we use ANSYS 16.0 to analyze the maximum and minimum stresses and its position. This curved structure is discretized into finite elements for the analysis. Stress at each point is determined.

Here in this case the required cross-sectional crane hook is designed with required dimensions. It is then meshed and given load is applied on the hook. It is then solved to get the required solution.

Von mises stress is a value used to determine if a given material will yield or fracture. It is mostly used for ductile material, such as metals. The Von mises yield criterion states that if the Von mises stress of a material under load is equal or greater than the yield limit of the same material under simple tension, which is easy to determine experimentally, then the material will yield. And it is the widely accepted and used failure theory.[6]

ANALYSIS OF CRANE HOOK WITH TRAPEZOIDAL SECTION

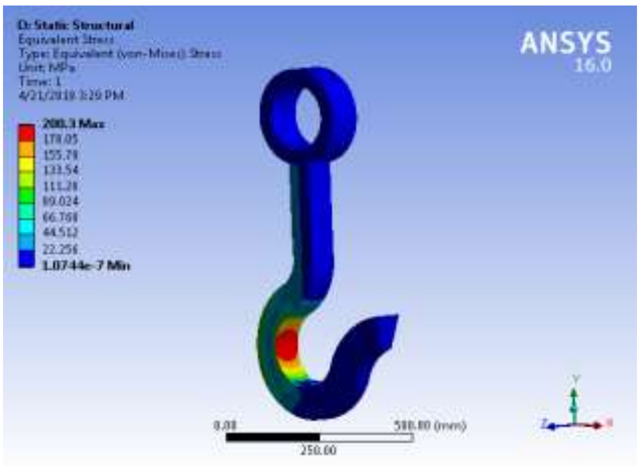

Fig 9: Von mises stress

Von mises stress thus obtained for trapezoidal section is 200.3 Mpa.

ANALYSIS OF CRANE HOOK WITH CIRCULAR SECTION

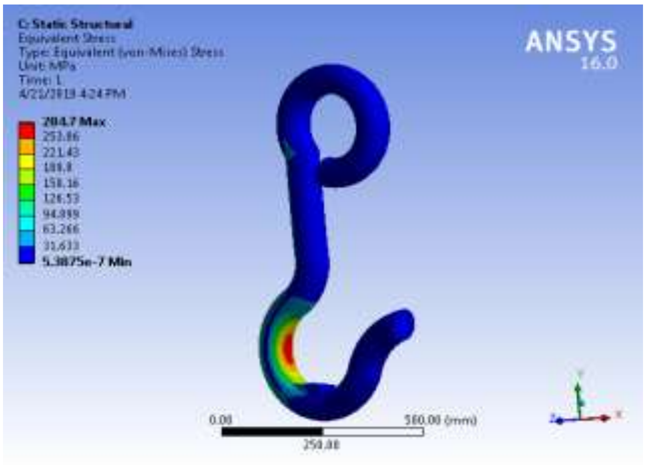

Fig 10: Von mises stress

Von mises stress thus obtained for cicular section is 284.7 Mpa.

ANALYSIS OF CRANE HOOK WITH ELLIPTICAL SECTION

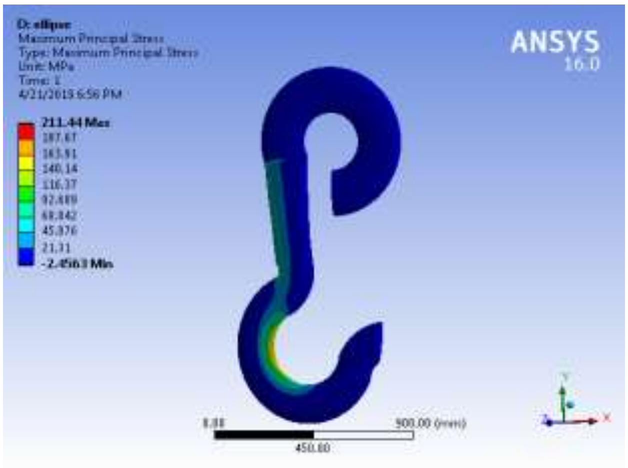

Fig 11: Von mises stress

Von mises stress thus obtained for elliptical section is 211.44 Mpa.

5. Comparison

The three major cross sections of crane hook are circular, elliptical and trapezoidal. In this section the stress values for crane hook using MATLAB and ANSYS is compared. It is found that ANSYS solution which happens to be an approximate solution deviates from the actual solution i.e., solution obtained using MATLAB. It is clearly observed that the deviation of values for all the three sections are within 10 percent.

Trapezoidal section validation

MATLAB Solution
Stress at inner fiber=171.15 MPa
Direct stress ($\frac{F}{A}$)=16.66 MPa
Net stress=187.81 MPa

ANSYS Solution
Von mises max stress=200.3 MPa
Percentage deviation= (200.3-187.81)/187.81
=6.64 %

Table 5: Comparison of stress values obtained for trapezoidal section

MATLAB solution for stress	187.81 MPa
ANSYS solution for stress	200.30 MPa
Percentage deviation	6.64 %

Circular section validation

MATLAB Solution
Stress at inner fiber=283.6048 MPa
Direct stress ($\frac{F}{A}$)=16.667 MPa
Net stress=300.2714 MPa

ANSYS Solution
Von mises max stress=284.7 MPa
Maximum principal stress=293.38 MPa

Percentage deviation= (300.2714-284.7)/300.2714
= 5.185 %

Table 6: Comparison of stress values obtained for circular cross section

MATLAB solution for stress	300.2714 MPa
ANSYS solution for stress	284.7 MPa
Percentage deviation	5.185 %

Elliptical section validation

MATLAB Solution
Stress at inner fiber=215.47 MPa
Direct stress ($\frac{F}{A}$)=16.667 MPa
Net stress=232.1366 MPa

ANSYS Solution
Von mises max stress=188.91 MPa
Maximum principal stress=211.44 MPa

Percentage deviation= (232.1366-211.47)/232.1366
= 8.90 %

Table 7: Comparison of stress values obtained for elliptical section

MATLAB solution for stress	232.1366 MPa
ANSYS solution for stress	211.44 MPa
Percentage deviation	8.90 %

Table 8: Comparison of all three cross sections

Cross-section of crane hook	MATLAB solution stress	ANSYS solution stress
Trapezoidal	187.7088 MPa	200.30 MPa
Circular	300.2714 MPa	284.7 MPa
Elliptical	232.1366 MPa	211.44 MPa

6. Conclusion

The MATLAB codes for different cross sections of curved beams were obtained and for the same i.e., circular, elliptical and trapezoidal sections FEA analysis was done. From the above table we can conclude that for the same cross-sectional area, R_i and same loading conditions trapezoidal section gives minimum stress compared to the circular and elliptical sections. Hence trapezoidal section crane hooks can we widely used as it is less stressed. However, stress obtained in elliptical section is less than the circular section and can be used widely rather than circular section crane hook.

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