FUZZY SOFT HYPERIDEALS IN MEET HYPERLATTICES

Ramalakshmi.S[1], Senthamil Selvi.V[2]

[1,2] Students, Thassim Beevi Abdul Kader college for women, Kilakarai, Ramanathapuram, Tamilnadu, India.

---***---

Abstract: In this paper, we introduce the notion of meet hyperlattice and some properties related to it.

Keywords: Fuzzy soft hyperideal [1], Fuzzy soft homomorphism, meet hyperlattice, hyperideals.

Introduction:

We develop the theory of fuzzy soft hyperideals in meet hyperlattices by introducing the novel concept of fuzzy soft hyperideals. The theory introduced here is one of the initial ideas to be introduced in the development of the theory of fuzzy soft hyperideals. The properties and structural characteristics of these concepts are also investigated and discussed here.

Definition 1.1. Let (L, ∧, ∨) be a meet hyperlattice and (f, X) be a fuzzy soft set over L.

- (f, X) is called a fuzzy soft ∨-hyperideal over L if for all x Є X and a, b Є L
  (i) $\bigcap_{c \epsilon a \wedge b} f_x(c) \geq f_x(a) \cap f_x(b)$,
  (ii) $\bigcap_{c \epsilon a \vee b} f_x(c) \geq f_x(a) \cup f_x(b)$.

  That is, for each x Є X, f_x is a fuzzy ∨-hyperideals of L.

- (f, X) is called a fuzzy soft ∧-hyperideals over L if for all x Є X and a, b Є L,
  (i) $\bigcap_{c \epsilon a \vee b} f_x(c) \geq f_x(a) \cap f_x(b)$,
  (ii) $\bigcap_{c \epsilon a \wedge b} f_x(c) \geq f_x(a) \cup f_x(b)$.

  That is, for each x Є X, f_x is a fuzzy ∧-hyperideals of L.

  Next, let us illustrate this definition by the following examples.

Example 1.1. *A fuzzy soft ∧-hyperideals (f, X), for which X is a singleton, is a fuzzy ∧- hyperideal. Hence a fuzzy ∧-hyperideal is a particular type of fuzzy soft ∧-hyperideal. In a similar way, a fuzzy ∧-hyperideal is a particular type of fuzzy soft ∧-hyperideals.*

Example 1.2. Let (L, ∧, ∨) be the meet hyperlattice. Set X= {a, b}.

(1) Let (f, X) be a fuzzy soft set on L, where fuzzy sets f_α and f_β are as follows.

$$f_\alpha(a) = \begin{cases} 0.8, & a \in \{x, y\} \\ 0.4, & a \in \{z, s\} \end{cases}, \quad f_\beta(a) = \begin{cases} 0.6, & a \in \{x, y\} \\ 0.3, & a \in \{z, s\} \end{cases}$$

Then (f, X) is a fuzzy soft ∧-hyperideal over L.

(2) Let (f, X) be a fuzzy soft set on L, where fuzzy sets f_α and f_β are as follows.

$$f_\alpha(a) = \begin{cases} 0.5, & a \in \{x, y\} \\ 0.7, & a \in \{z, s\} \end{cases}, \quad f_\beta(a) = \begin{cases} 0.2, & a \in \{x, y\} \\ 0.4, & a \in \{z, s\} \end{cases}$$

Then (f, X) is a fuzzy soft ∨-hyperideal over L.

In what follows, we shall investigate some properties of fuzzy soft hyperideals.

Proposition 1.1. *Let (f, X) and (g, Y) be two fuzzy soft ⍰-hyperideals (⍰-hyperideals) over the meet hyperlattice (L, ⍰, ⍰). Then (f, X) ⊓ (g, Y) is a fuzzy soft ⍰-hyperideal (⍰-hyperideal) over L.*

Proof: Let (f, X) and (g, Y) be two fuzzy soft sets over A such that X ∪ Y ≠ ∅. The restricted intersection of (f, X) and (g, Y) is the fuzzy soft set (h, Z), where Z= X ∩ Y and $h_Z = f_Z \cap g_Z$, for all z Є Z. This is denoted by (h, Z) = (f, X) ⊓ (g, Y).

(f, X) ⊓ (g, Y) = (h, Z),

Where Z = X ∩ Y and $h_Z = f_Z \cap g_Z$,

That is, h_Z(a)= $f_Z(a) \cap g_Z(a)$ for all z Є Z and a Є L.

Suppose that (f, X) and (g, Y) are two fuzzy soft ∨-hyperideals over the meet hyperlattice (L, ⍰, ∨).

If any a, b Є L and c Є a ∧ b, for all z Є Z, we have h_Z(c)= $f_Z(c) \cap g_Z(c) \geq (f_Z(a) \cap g_Z(b)) \cap (g_Z(a) \cap g_Z(b)) \geq (f_Z(a)$ $g_Z(a)) \cap (f_Z(b) \cap g_Z(b)) = h_Z(a) \cap h_Z(b)$.

Then we obtain $\bigcap_{c\epsilon a\wedge b} h_z$(c) ≥ $h_Z(a) \cap h_Z(b)$ for all z Є Z.

On the other hand, for all c Є a∨b and z Є Z, we have h_Z(c)= $f_Z(c) \cap g_Z(c) \geq (f_Z(a) \cup g_Z(b)) \cap$ $(g_Z(a) \cup g_Z(b))$ = $(f_Z(a) \cap g_Z(a)) \cup (f_Z(b) \cap g_Z(b)) = h_Z(a) \cup h_Z(b)$, which implies $\bigcap_{c\epsilon a\vee b} h_z$(c) ≥ $h_Z(a) \cup h_Z(b)$ for all z Є Z. Therefore, (f, X) ⊓ (g, Y) is a fuzzy soft ∧-hyperideals over L.

The case for ⍰-hyperideals can be similarly proved.

Proposition 1.2. *Let (f, X) and (g, Y) be two fuzzy soft ⍰-hyperideals (⍰-hyperideals) over the meet hyperlattice*

(L, ⍰, ⍰). Then (f, X) (g, Y) is a fuzzy soft ⍰-hyperideals (⍰-hyperideals) over L.

Proof: Suppose that (f, X) and (g, Y) are two fuzzy soft ⍰-hyperideals over the meet hyperlattice (L, ⍰, ⍰)

The extended intersection of two fuzzy soft sets (f, X) and (g, Y) over A is the fuzzy soft set (h, Z), where Z = X∪Y

$$h_Z = \begin{cases} f_Z, & if\ Z \in X - Y \\ g_Z, & if\ Z \in Y - Z \\ f_Z \cap g_Z, & if\ Z \in X \cap Y \end{cases}$$

For all z Є Z. This is denoted by (f, X) (g, Y) = (h, Z)

(f, X) (g, Y) = (h, Z), where Z = X ∪ Y and

$$\mathrm{h_Z} = \begin{cases} \mathrm{f_Z}, & \mathrm{if\ Z \in X - Y} \\ \mathrm{g_Z}, & \mathrm{if\ Z \in Y - Z} \\ \mathrm{f_Z \cap g_Z}, & \mathrm{if\ Z \in X \cap Y} \end{cases} \quad \text{for all z} \in \text{Z}$$

Now, for all z Є Z and a, b Є L, we consider the following cases.

Case 1: Z Є X-Y, then $h_Z = f_Z$. Since (f, X) is a fuzzy soft ∨-hyperideal over the meet hyperlattice (L, ⍰, ∨), h_Z is a fuzzy soft ∨-hyperideal over (L, ⍰, ∨).

Case 2: Z Є Y-X, then $h_Z = g_Z$. Analogous to the proof of case 1, we have h_Z is a fuzzy soft ∨-hyperideal over (L, ⍰, ∨).

Case 3: $Z \in X \cap Y$, then $h_Z = f_Z \cap g_Z$

Proposition 1.3: *Let (f, X) and (g, Y) be two fuzzy soft ⍰-hyperideals (⍰ -hyperideals) over the meet hyperlattice (L, ⍰, ⍰). Then (f, X) (g, Y) is a fuzzy soft ⍰-hyperideals (⍰-hyperideals) over L.*

Proof: Definition of (f, X) and (g, Y) are two fuzzy soft sets. Then (f, X) (g, y) is defined as (h, X × Y), where h (x, y) = $f_x \cap g_y$, for all (x, y) Є X × Y. we denote (f, X) (g, Y) = (h, X × Y).

We know that for all x Є X, y Є Y, f_x and g_y are fuzzy ∨-hyperideals of L and so is h (x, y) = f_x ∩ g_y, for (x, y) Є X × Y, because intersection of two fuzzy ∨-hyperideals is also a fuzzy ∨-hyperideal.

Therefore, (h, X × Y) = (f, X) (g, Y) is a fuzzy soft ∨-hyperideals over L.

Similarly, we can prove that (h, X × Y) = (f, X) (g, Y) is a fuzzy soft ?-hyperideal over L.

Proposition 1.4:

Let (f, X) and (g, Y) be two fuzzy soft ?-hyperideals (?-hyperideals) over the meet hyperlattice (L, ?, ?). If for all x Є X and y Є Y, $f_x \subseteq g_y$ (or) $g_y \subseteq f_x$, then (f, X) (g, Y) is a fuzzy soft ?-hyperideals (?-hyperideals) over L.

Proof: If (f, X) and (g, Y) are two fuzzy soft sets. Then (f, X) Ũ (g, Y) is defined as (Õ, X × Y), where ? (x, y) = $f_x \cup g_y$, for all (x, y) Є X × Y. we can write (Õ, Z) = (f, X) Ũ (g, Y), where Z = X × Y, for all (x, y) Є Z, we have Õ (x, y) = $f_x \cup g_y$.

By hypothesis, for all (x, y) Є Z, $f_x \subseteq g_y$ *(or)* $g_y \subseteq f_x$.

Now, we assume that $f_x \subseteq g_y$ for any a , b Є L and c Є a ∧ b, we have ? (x, y) (c) = f_x(c) ∪ g_y(c) = g_y(c) ≥ g_y(a)∩ g_y(b)

= (f_x(a) ∪ g_y(a)) ∩ (f_x(b) ∪ g_y(b)) = ? (x, y) (a) ∩ ? (x, y) (b). Then we obtain $\bigcap_{c \in a \wedge b}$ Õ (x, y) (c) ≥ Õ (x, y) (a) ∩ ? (x, y) (b).

On the other hand, for all c Є a ∨ b, we have ? (x, y) (c) = f_x(c) ∪ g_y(c) = g_y(c) ≥ g_y(a)∪ g_y(b)

= (f_x(a) ∪ g_y(a)) ∪ (f_x(b) ∪ g_y(b)) = ? (x, y) (a) ∪ ? (x, y) (b). Hence, $\bigcap_{c \in a \wedge b}$ Õ (x, y) (c) ≥ Õ (x, y) (a) ∪ ? (x, y) (b). Therefore, (f, X) Ũ (g, Y) is a fuzzy soft ∨-hyperideal over L.

The case for ?-hyperideals can be similarly proved.

Definition 1.2. *Let (f, X) be a fuzzy soft set over L. The soft $(f, X)_t$ = {$(f_x)_{(t)}$: x Є X} for all t Є [0, 1), are called the t-level soft set and strong t- level soft set of the fuzzy soft set (f, X) respectively, where $(f_x)_t$ and $(f_x)_{(t)}$ are the t-level set and strong t-level set of the fuzzy set f_x, respectively.*

Theorem 1.1. *Let (f, X) be a fuzzy soft set over the meet hyperlattice (L, ?, ?). Then (f, X) is a fuzzy soft ?-hyperideals (?-hyperideals) over L if and only if for all x Є X and t Є [0,1] with $(f_x)_t \neq \phi$, the t-level soft set $(f, X)_t$ is a soft ?-hyperideals (?-hyperideals) over L.*

Proof: Let (f, X) be a fuzzy soft ∨-hyperideals over the meet hyperlattice (L, ?, ∨). Then for all x Є X, f_x is a fuzzy ∨-hyperideal of L. For t Є [0,1] with $(f_x)_t \neq \phi$,

Let a, b Є $(f_x)_t$, then f_x(a) ≥ t and f_x(b) ≥ t. Hence, for all c Є a ? b, we have f_x(c) ≥ f_x(a) ∩ f_x(b) ≥ t ∩ t = t, that is, c Є $(f_x)_t$, which implies a ∧ b ⊆ $(f_x)_t$.

On the other hand, let y Є L, for all c Є y ⋁ a, we have f_x(c) ≥ f_x(y) ∩ f_x(a) ≥ t, that is c Є $(f_x)_t$, which implies y ∨ a ⊆ $(f_x)_t$, then we obtain that $(f_x)_t$ is a ∨-hyperideal of L, for all x Є X. Therefore, $(f, X)_t$ is a soft ∨-hyperideal over L. For all x Є X. Let α = f_x(a) ∩ f_x(b), then we have f_x(a) ≥ α, f_x(b) ≥ α, which implies a , b Є $(f_x)_\alpha$. Since $(f_x)_\alpha$ is a ∨-hyperideal of L, then a ∧ b ⊆ $(f_x)_\alpha$. Hence, for all c Є a ? b, we have c Є $(f_x)_\alpha$. Thus, we can obtain f_x(c) ≥ α = f_x(a) ∩ f_x(b), which implies $\bigcap_{c \in a \wedge y} f_x$(c) ≥ f_x(a) ∩ f_x(b).

On the other hand, let β = f_x(a), then we have f_x(a) ≥ β, that is, a Є $(f_x)_\beta$, and β Є [0,1]. Then for all y Є L, y ∨ a ⊆ $(f_x)_\beta$. Hence for all c Є y ∨ a, we have c Є $(f_x)_\beta$. Thus, we have f_x(c) ≥ β = f_x(a). Similarly, f_x(c) ≥ f_x(y), which implies $\bigcap_{c \in y \vee a} f_x$(c) ≥ f_x(y) ∪ f_x(a). Therefore, (f, X) is a fuzzy soft ∨-hyperideals over L.

The case for ?-hyperideals can be similarly proved.

Theorem 1.2.

Let (f, X) be a fuzzy soft set over the meet hyperlattice(L, ⃞, ∨).Then (f ,X) is a fuzzy soft ⃞-hyperideal(⃞-hyperideal) over L if and only if for all x ϵ X and t ϵ [0,1) with $(f_x)_{(t)}$ is a soft ⃞-hyperideal(⃞-hyperideal) over L.

Proof: ⇐Now, assume that (f, X) is not a fuzzy soft ∨-hyperideal over L. Then there exists x ϵ X such f_x is not a fuzzy ∨-hyperideal of L. That is, there exists a_0, b_0 ϵ L ,such that $\bigcap_{c\epsilon a_0 \wedge b_0} f_x(\mathrm{c}) < f_x(a_0) \cap f_x(b_0)$ or $\bigcap_{c\epsilon a_0 \vee b_0} f_x(\mathrm{c}) < f_x(a_0) \cup f_x(b_0)$

Now, we consider the following cases

(i) If $\bigcap_{c\epsilon a_0 \wedge b_0} f_x(\mathrm{c}) < f_x(a_0) \cap f_x(b_0)$,Let t=$\bigcap_{c\epsilon a_0 \wedge b_0} f_x(\mathrm{c})$.Then there exists $c_0 \ \epsilon\ a_0 \wedge b_0$ such that $f_x(c_0) = t$. Hence t=$f_x(c_0) < f_x(a_0) \cap f_x(b_0)$.Then we get $a_0, b_0\ \epsilon\ (f_x)_{(t)}$, but $c_0 \notin (f_x)_{(t)}$.Thus, we obtain $a_0 \wedge b_0 \nsubseteq (f_x)_{(t)}$.

(ii) If $\bigcap_{c\epsilon a_0 \vee b_0} f_x(\mathrm{c}) < f_x(a_0) \cup f_x(b_0)$,let t=$\bigcap_{c\epsilon a_0 \vee b_0} f_x(\mathrm{c})$.Then there exists $c_0\ \epsilon\ a_0 \vee b_0$ such that $f_x(c_0)$=t.Hence t=$f_x(c_0)$< $f_x(a_0) \cap f_x(b_0)$.Then we have t ϵ [0,1) and $f_x(a_0)$>t or $f_x(b_0)$>t ,that is, $a_0\ \epsilon\ (f_x)_{(t)}$ or $b_0\ \epsilon\ (f_x)_{(t)}$,but $b_0 \notin\ (f_x)_{(t)}$.Thus, we also obtain $a_0 \vee\ b_0 \nsubseteq (f_x)_{(t)}$.

Thus, results in case (i) and (ii) contradict the fact that $(f, X)_{(t)}$ is a soft ∨-hyperideal over L. Therefore, (f, X) is a fuzzy soft ∨-hyperideal over L.

The case for ⃞-hyperideals can be similarly proved.

Definition 1.3.

Let (f, X) and (g, Y) be two fuzzy soft sets over L_1 and L_2 respectively and Let $(\varphi,\ \psi)$ be a fuzzy soft function from $L_1$to L_2.

(1) The image of (f, X) under (φ, ψ) , denoted by (φ,ψ) (f,X),is a fuzzy soft over L_2 defined by (φ, ψ) (f, X)=(φ(f), ψ(X)),
(2) where

$$\varphi(f)_k(\mathrm{b}) = \begin{cases} \bigcup\bigcup f_x(a) & , \quad if\ a \in \varphi^{-1}(b) \\ \varphi(a) = b \quad \psi(x) = k & , \quad for\ all\ k\epsilon\psi(x), b\epsilon L_2 \\ 0 & , \quad otherwise. \end{cases}$$

(3) The pre-image of (g ,Y) under (ϕ, ψ), denoted by $(\phi, \psi)^{-1}(g, Y)$, is a fuzzy soft set over L_1,defined by $(\phi, \psi)^{-1}(g, Y)$=($\varphi^{-1}(g), \psi^{-1}(Y)$), where $\varphi^{-1}(g)_x(a) = g_{\psi(x)}\big(\psi(a)\big)$, for all x$\epsilon\psi^{-1}(Y)$,a$\epsilon L_1$.

Definition 1.4

Let (f, X) and (g, Y) be two fuzzy soft sets over the meet hyperlattice L_1 and the meet hyperlattice L_2, respectively. Let (φ, ψ) be a fuzzy soft function from $L_1$to L_2. If φ is a homomorphism from $L_1$to L_2,then (φ, ψ) is said to be a fuzzy soft homomorphism from $L_1$to L_2.

Theorem 1.3.

Let (φ, ψ) be a fuzzy soft homomorphism from the meet hyperlattice $(L_1,\ \wedge_1,\ \vee_1)$ to the meet hyperlattice $(L_2,\wedge,\vee_2)$.If (g, Y) is a fuzzy soft $\vee_2$-hyperideal($\wedge_2$-hyperideal) over L_2,then $(\varphi, \psi)^{-1}(g, Y)$ is a fuzzy soft $\vee_1$-hyperideal($\wedge_1$-hyperideal) over L_1.

Proof: Let x ϵ $\psi^{-1}(Y)$ and $a_1, a_2\ \epsilon\ L_1$, c ϵ $a_1 \wedge a_2$.Suppose that $\varphi(a_1) = b_1$ and $\varphi(a_2) = b_2$.Since (g ,Y) is a fuzzy soft $\vee_2$-hyperideal over L_2,we have $\varphi^{-1}(g)_x(a_1) \cap \varphi^{-1}(g)_x(a_2) = g_{\psi(x)}(\varphi(a_1)) \cap g_{\psi(x)}(\varphi(a_2)) = g_{\psi((x)}(b_1) \cap g_{\psi(x)}(b_2) \le g_{\psi(x)}(t)$, for all t ϵ $b_1 \wedge_2 b_2 = \varphi(a_1 \wedge_1 a_2)$.Hence, for all c ϵ $a_1 \wedge_1 a_2$, $\varphi^{-1}(g)_x(a_1)\cap\ \varphi^{-1}(g)_x(a_2) \le g_{\psi(x)}(\varphi(c))=\varphi^{-1}(g)_x(c)$,that is,

$$\bigcap_{c\ \epsilon\ a_1 \wedge_1 a_2} \varphi^{-1}(g)_x(c) \ge \varphi^{-1}(g)_x(a_1) \cap \varphi^{-1}(g)_x(a_2)$$

Similarly, we obtain

$$\bigcap_{c\ \epsilon\ a_1 \vee_1 a_2} \varphi^{-1}(g)_x(c) \ge \varphi^{-1}(g)_x(a_1) \cup \varphi^{-1}(g)_x(a_2)$$

Therefore, $(\varphi, \psi)^{-1}(g, Y)$ is a fuzzy soft $\vee_1$-hyperideal over L_1.

Similarly, we can prove that $(\varphi, \psi)^{-1}(g, Y)$ is a fuzzy soft $\wedge_1$-hyperideal over L_1.

Conclusion

Hence, we have successfully introduced the fuzzy meet hyperlattice. And we investigated some of their properties.

REFERENCES

[1] https://www.researchgate.net/publication/286912054_Fuzzy_Isomorphism_Theorems_of_Soft_G-Hyperrings
[2] https://www.researchgate.net/publication/254221895_Fuzzy_soft_hypergroups
[3] P. F. He, X. L. Xin, J. M. Zhan, On hyperideals in hyperlattices, Journal of Mathematics, Volume 2013, Article ID 915217, 10 pages.
[4] V. Leoreanu - Fotea, F. Feng and J. M. Zhan, Fuzzy soft hypergroups, International Journal of Computer Mathematics, 89(2012) 963-974.