e-ISSN: 2395-0056 p-ISSN: 2395-0072

# ON STRONG BLAST DOMINATION OF GRAPHS

# M. Lalitha kumari<sup>1</sup>, Dr.V. Anusuya<sup>2</sup>

<sup>1</sup>M.Phil Scholar, Department of Mathematics, S.T.Hindu college, Nagercoil, TamilNadu, India <sup>2</sup>Dr.V.Anusuya, Assistant Professor, Department of Mathematics, S.T.Hindu college, Nagercoil, TamilNadu, India \*\*\*

**Abstract** - In this paper, we introduce another new domination parameter called, the strong blast domination of a graph with real life applications. A subset S of V of a non-trivial graph G is said to be strong blast dominating set if S is a connected dominating set and the induced sub graph  $\langle V - S \rangle$  is triple connected and also for each vertex v in V-S there exist some vertex u in S such that  $d(u) \ge d(v)$ . Strong Blast Domination Number is denoted by  $\gamma_{sc}^{tc}(G)$ . In this paper, we find the Strong Blast Domination Number for Complete graph, Wheel graph, Peterson graph, Total graph of friendship graph, Total graph of Path, Total graph of Cycle, Total graph of wheel, Quasi-total graph of Complete graph and Friendship graph.

Key Words: Domination number, connected domination number, Triple connected, Strong domination, Total graph, Quasi-total graph.

## 1. INTRODUCTION

Throughout this paper, we consider only finite simple undirected connected graph. The order and the size are denoted by n and m respectively. Degree of a vertex v is denoted by d(v), the maximum degree of a graph G is denoted by  $\Delta(G)$ . A path on n vertices is denoted by  $P_n$ . A graph is complete if every pair of its vertices are adjacent. A complete graph on n vertices denoted by  $K_n$ . A cycle of length n is denoted by  $C_n$ .

The total graph T(G) of a graph G is defined with the vertex set  $V(G) \cup E(G)$ , in which two vertices are adjacent if and only if (i) Both are adjacent edges or vertices in G (or) (ii) One is a vertex and other is an edge incident to it in G. The Quasi-total graph P(G) of a graph G is a graph whose vertex set is  $V(G) \cup E(G)$  and two vertices are adjacent if and only if they correspond to two non adjacent vertices of G or to two adjacent edges of G or one is a vertex and other is an edge incident with it in G. The friendship graph  $F_n$  is an one-point union of n copies of cycle  $C_3$ . A Wheel graph  $W_n$  of order n is a graph that contains a cycle of order n-1 and for which every vertex in the cycle is connected to one other vertex. The Peterson graph is an undirected graph with 10 vertices and 15 edges. The Windmill graph  $D_n^m(G)$  is an undirected graph constructed for  $m \ge 2$ ,  $n \ge 2$  by taking m copies of the complete graph  $K_n$  with a vertex in common. The n -barbell graph is an undirected graph obtained by connecting two copies of a complete graph  $K_n$  by a bridge. The n -Sunlet graph is obtained by attaching a pendent edge to all the vertex of the cycle  $C_n$ .

In our literature survey, we are able to find many authors have introduced various new parameters by imposing conditions on the dominating sets. In that sequence, the concept of connectedness plays an important role in any network.

A subset S of V of a non-trivial graph G is called a dominating set of G if every vertex in V-S is adjacent to at least one vertex S. The domination number  $\gamma(G)$  is the minimum cardinality of a dominating set. The concept of triple connected graphs was introduced by Paulraj Joseph et.al [5]. A graph G is said to be triple connected if any three vertices lie on a path. In [2] the authors introduced the concept of complementary triple connected domination number of a graph. A subset S of V of a non-trivial graph G is said to be complementary triple connected dominating set, if S is a dominating set and the induced sub graph  $\langle V - S \rangle$  is triple connected. The minimum cardinality taken over all complementary triple connected dominating sets is called the complementary triple connected domination number of G and is denoted by  $\gamma_{ctc}(G)$ .

In [1], the authors introduced Blast domination number of a graph with real life applications. A subset S of V of a non-trivial graph G is said to be Blast dominating set, if S is a connected dominating set and the induced sub graph < V - S > is triple connected. The minimum cardinality taken over all Blast dominating sets is called the Blast domination number and is denoted by  $\gamma_c^{tc}(G)$ . A dominating set S is said to be a strong dominating set if for every vertex v in V-S dominated by some vertex u of S such that  $d(u) \ge d(v)$  and is denoted by  $\gamma_s(G)$ .

Now, we introduce a new domination parameter called Strong Blast Domination number. A subset S of V of a nontrivial graph G is said to be strong blast dominating set, if S is a connected dominating set and the induced sub graph < V - S > is triple connected and also for each  $v \in V - S$  there exist  $u \in S$  such that  $d(u) \ge d(v)$  and SBDN is denoted by  $\gamma_{sc}^{tc}(G)$ .

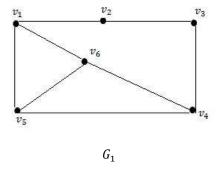
© 2020, IRJET **Impact Factor value: 7.34** ISO 9001:2008 Certified Journal Page 5484

## 2. RESULTS

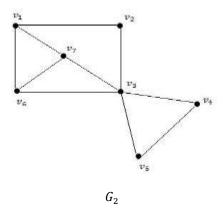
- For complete graph  $K_n$  with  $n \ge 4$ ,  $\gamma_{sc}^{tc}(K_n) = 1$ .
- For wheel graph  $W_n$   $(n \ge 4)$ ,  $\gamma_{sc}^{tc}(W_n) = 1$ .
- For Peterson graph  $\gamma_{sc}^{tc}(G) = 5$ .
- For the Windmill graph  $D_n^m$ ,  $n \ge 4$ ,  $m \ge 2$ ,  $\gamma_{sc}^{tc}(D_n^m) = 1$ .
- For the n-barbell graph G with  $n \ge 4$ ,  $\gamma_{sc}^{tc}(G) = 2$ .

# 3. EXAMPLES

**3.1** For the graph  $G_1$ ,  $S = \{v_3, v_4, v_6\}$  is the Strong blast dominating set and  $\gamma_{sc}^{tc}(G_1) = 3$ .



3.2 For the graph  $G_2$ ,  $S = \{v_3, v_7\}$  is the Strong blast dominating set and  $\gamma_{sc}^{tc}(G_2) = 2$ .



### 4. MAIN RESULTS

### Theorem 4.1

Let *G* be a connected graph of order n,  $n \ge 4$ . Then  $\gamma_{sc}^{tc}(G) = 1$  if and only if  $G = K_n$ .

### **Proof:**

Suppose  $\gamma_{sc}^{tc}(G) = 1$  and let  $S = \{v\}$  be a  $\gamma_{sc}^{tc} - set$  of G. Suppose  $G \neq K_n$ , then there exists  $x, y \in V(K_n)$  such that d(x,y) = 2. Then  $(S \setminus \{v\}) \cup \{x\} = \{x\}$ , which is not a dominating set of G, since  $xy \notin E(K_n)$ . Therefore  $G = K_n$ .

Conversely, suppose that  $G = K_n$ . Then  $S = \{v\} \subseteq V(G)$  is a  $\gamma_{sc}^{tc} - set$  of G. Since we have  $G = K_n$  with  $n \ge 4$  and  $|V - S| \ge 3$ , the complement  $\langle V - S \rangle$  is triple connected. And for all  $v \in V - S$  there exist  $u \in S$  such that  $d(u) \ge d(v)$ . Hence  $\gamma_{sc}^{tc}(G) = 1$ .

# Theorem 4.2

Let G be a connected graph of order at least 5 vertices with no isolated vertices. Then  $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \leq \gamma_{sc}^{tc}(G) \leq n - \Delta(G)$ .

e-ISSN: 2395-0056

e-ISSN: 2395-0056 Volume: 07 Issue: 03 | Mar 2020 www.irjet.net p-ISSN: 2395-0072

### **Proof:**

Let G be a connected graph and S be the  $\gamma_{sc}^{tc}$  – set of S. Each vertex can dominate at most itself and  $\Delta(G)$  other vertices. Hence  $\gamma_{sc}^{tc}(G) \ge \left[\frac{n}{1+\Delta(G)}\right]$ .

For the upper bound, let v be a vertex of maximum degree  $\Delta(G)$ . Then v dominates itself and each of its neighbors N(v). Also vertices in V - N[v] dominates themselves. Thus V - N[v] is a dominating set with cardinality  $n - \Delta(G)$ . Therefore  $\gamma_{sc}^{tc}(G) \leq n - \Delta(G)$ .

## Theorem 4.3

Let G be any connected graph with  $n \ge 5$  and  $\Delta(G) = n - 2$ . Then  $\gamma_{sc}^{tc}(G) = 2$ .

# **Proof:**

Let G be a connected graph with  $n \ge 5$  and  $\Delta(G) = n - 2$ . Let v be a vertex of degree  $\Delta(G) = n - 2$ . Let  $\{v_1, v_2, ..., v_{n-2}\}$  be the vertices which are adjacent to v and let  $v_{n-1}$  be the vertex which is not adjacent to v. Since G is connected,  $v_{n-1}$  is adjacent to some  $v_i$  for some i. Then  $S = \{v, v_i\}$  is a minimum connected dominating set. And the induced sub graph  $\langle V - S \rangle$  consists of at least 3 vertices which are lie on a path. So that  $\langle V - S \rangle$  is triple connected. Also for all  $v' \in V - S$  there exists a vertex  $u \in S$  such that  $d(u) \ge d(v')$ . Hence  $\gamma_{SC}^{tc}(G) = 2$ .

## Theorem 4.4

For any connected graph G with  $n \ge 5$  and  $\Delta(G) = n - 3$ , the strong blast domination number is either 2 or 3.

#### Proof:

Let G be the connected graph with  $n \ge 5$  and  $\Delta(G) = n - 3$ . Let v be the vertex of maximum degree n-3. Suppose  $N(v) = \{v_1, v_2, \dots, v_{n-3}\}$  and  $V - N(v) = \{v_{n-2}, v_{n-1}\}.$ 

Case i: If  $v_{n-1}$  and  $v_{n-2}$  are not adjacent in G.

Since G is connected, there are vertices  $v_i$  and  $v_j$  for some i,j  $(1 \le i \ne j \le n-3)$  which are adjacent to  $v_{n-1}$  and  $v_{n-2}$  respectively.

If i = j, then  $\{v, v_i\}$  is a  $\gamma_{sc}^{tc} - set$  and  $\gamma_{sc}^{tc}(G) = 2$ .

If  $i \neq j$  then  $\{v, v_i, v_i\}$  is a  $\gamma_{sc}^{tc}$  – set and  $\gamma_{sc}^{tc}(G) = 3$ .

Case ii: If  $v_{n-1}$  and  $v_{n-2}$  are adjacent in G. Then there is a vertex  $v_j$  for some j,  $(1 \le j \le n-3)$  which is adjacent to either  $v_{n-1}$  or  $v_{n-2}$  or both. In this case  $\{v, v_j, v_{n-1}\}$  or  $\{v, v_j, v_{n-2}\}$  or  $\{v, v_j\}$  is a  $\gamma_{sc}^{tc}$  – set of G. Hence the strong blast domination number is either 2 or 3.

# Theorem 4.5

A  $\gamma_{sc}^{tc}$  – set S of a graph G is a minimal dominating set if and only if  $\forall u \in S$ , there exist  $v \in V - S$  for which  $N[v] \cap S = \{u\}.$ 

# **Proof:**

Let S be a  $\gamma_{sc}^{tc}$  – set of G. Then for every vertex  $u \in S$ ,  $S - \{u\}$  is not a  $\gamma_{sc}^{tc}$  – set of G. Thus there is a vertex  $v \in (V - S) \cup \{u\}$  that is not dominated by S. This implies that  $N[v] \cap S = \{u\}$ .

Conversely let us presume that  $\forall u \in S$ , there exist  $v \in V - S$  for which  $N[v] \cap S = \{u\}$ . Suppose that S is not a  $\gamma_{sc}^{tc} - set$  of G. Then there is a vertex  $u \in S$  such that  $S - \{u\}$  is a  $\gamma_{sc}^{tc} - set$  of G. Hence u is adjacent to at least one vertex in  $S - \{u\}$ , also if  $S - \{u\}$  is a dominating set then every vertex in V - S is adjacent to at least one vertex in  $S - \{u\}$ . This is a contradiction to our assumption. Hence S is a  $\gamma_{sc}^{tc}-set$ .

Volume: 07 Issue: 03 | Mar 2020 www.irjet.net p-ISSN: 2395-0072

### Theorem 4.6

If G is a graph with no isolated vertices and S is a  $\gamma_{sc}^{tc}$  – set of G then V-S has a  $\gamma-set$ .

## **Proof:**

Let S be a  $\gamma_{SC}^{tc}$  – set of G. Suppose V-S has no dominating set of G. Then for some vertex  $v \in S$  there is no edge from v to any vertex in V-S. Then  $S-\{v\}$  would be a dominating set, which contradict the minimality of S. Hence V-S has a dominating set.

#### Result 4.7

For complete graph  $K_n$ ,  $n \ge 5$  the complement of a  $\gamma_{sc}^{tc} - set$  is also a  $\gamma_{sc}^{tc} - set$ .

#### Theorem 4.8

If *G* is a n –Barbell graph with  $n \ge 4$ , then

- 1) G has a  $\gamma set$  S such that every vertex in S has at least three neighbors in V S.
- 2)  $\gamma(G) = \gamma_{sc}^{tc}(G)$ .

#### **Proof:**

Let G be a n -Barbell graph with  $n \ge 4$ , obtained by joining two complete graphs by a bridge. Let  $e = \{v_1u_1\}$  be the edge joining the two complete graphs.

Let the vertices of the complete graphs be  $\{v_i \mid 1 \le i \le n\}$  and  $\{u_i \mid 1 \le j \le m\}$ .

Let  $V(G) = \{v_1, v_2, ..., v_n, u_1, u_2, ..., u_m\}$ . Since  $v_1$  is adjacent to  $\{v_2, v_3, ..., v_n\}$  and  $u_1$  is adjacent to  $\{u_2, u_3, ..., u_m\}$ , where  $d(v_1) = n$  and  $d(u_1) = m$ , the vertices  $v_1$  and  $u_1$  dominates all the vertices of the Barbell graph. Let  $S = \{v_1, u_1\}$  be the dominating set of G. Therefore,  $\gamma(G) = \gamma(K_n) + \gamma(K_m) = 2$ .

Clearly, the vertices  $v_1$  and  $u_1$  are adjacent to at least three neighborhoods of V-S. Hence (1) follows.

Since  $e = \{v_1u_1\}$  is the bridge which connects the complete graphs, which is connected. Also since  $n \ge 4$ , the induced sub graph < V - S > must has at least three vertices which are lie on a path. Therefore < V - S > must be triple connected. And all the vertices  $v \in V - S$  strongly dominated by a vertex  $u \in S$ . Hence S is a  $\gamma_{sc}^{tc} - set$ .

Thus, 
$$\gamma_{sc}^{tc}(G) = 2$$
. Hence,  $\gamma(G) = \gamma_{sc}^{tc}(G)$ . This proves (2).

# Theorem 4.9

For any corona graph  $C_m \circ P_n$ ,  $\gamma_{sc}^{tc}(C_m \circ P_n) = m$  where  $n, m \ge 3$ .

# **Proof:**

Let  $V(C_m) = \{u_1, u_2, \dots u_m\}$ . Let  $P_{n_1}$  be the path having vertex set  $\{u_{11}, u_{12}, \dots u_{1n}\}$  and  $P_{n_2}$  be the path having vertex set  $\{u_{21}, u_{22}, \dots u_{2n}\}$ . Continuing like this we get  $P_{n_m}$  be the path having vertex set  $\{u_{m1}, u_{m2}, \dots u_{mn}\}$ .

Vertex set of the corona  $C_m \circ P_n$  is,  $\{u_1, u_{11}, u_{12}, \dots u_{1n}, u_2, u_{21}, u_{22}, \dots u_{2n}, \dots, u_m, u_{m1}, u_{m2}, \dots u_{mn}\}$ . Here  $u_1$  is adjacent to  $u_{1i}$ ,  $u_2$  is adjacent to  $u_{2i}$  ...and  $u_m$  is adjacent to  $u_{mi}$ ,  $i = 1, 2, \dots n$ .

Therefore  $S = \{u_1, u_2, ... u_m\}$  is a dominating set of  $C_m \circ P_n$ . Also each vertex of S has at least 3 neighbors in V - S. Thus V - S induces a triple connected subgraph. Also, clearly all the vertices in S has maximum degree as  $\Delta(G) \ge m+1$ . Thus for each vertex u in V - S there is a vertex v in S such that  $d(v) \ge d(u)$ . Hence S is a  $\gamma_{sc}^{tc} - set$ . ie,  $\gamma_{sc}^{tc}(C_m \circ P_n) = |S| = m$ .

# Theorem 4.10

For the corona of  $W_n$  and  $K_m$  with  $n \ge 4$ ,  $m \ge 3$ ,

e-ISSN: 2395-0056

$$\gamma(W_n \circ K_m) = \gamma_s(W_n \circ K_m) = \gamma_c^{tc}(W_n \circ K_m) = \gamma_{sc}^{tc}(W_n \circ K_m) = n.$$

## **Proof:**

Let the vertices of  $W_n$  and  $K_m$  be  $\{v_1, v_2, ..., v_n\}$  and  $\{u_1, u_2, ..., u_m\}$  respectively. And for the corona  $W_n \circ K_m$ ,

$$\gamma(W_n \circ K_m) = n,$$

$$\gamma_s(W_n \circ K_m) = n$$
,

$$\gamma_c^{tc}(W_n \circ K_m) = n,$$

$$\gamma_{sc}^{tc}(W_n \circ K_m) = n.$$

Comparing the above equations we get,

$$\gamma(W_n \circ K_m) = \gamma_s(W_n \circ K_m) = \gamma_c^{tc}(W_n \circ K_m) = \gamma_{sc}^{tc}(W_n \circ K_m).$$

## **Observation 4.11**

For the corona of  $K_m$  and  $W_n$  with  $m \ge 3$ ,  $n \ge 4$ ,

$$\gamma(K_m \circ W_n) = \gamma_S(K_m \circ W_n) = \gamma_S^{tc}(K_m \circ W_n) = \gamma_{Sc}^{tc}(K_m \circ W_n) = m.$$

## Theorem 4.12

$$\gamma_{sc}^{tc}[T(K_{1,n})] = 1.$$

### **Proof:**

Let  $T(K_{1,n})$  be the total graph of the star graph having the vertex set

 $Vig[Tig(K_{1,n}ig)ig] = Vig(K_{1,n}ig) \cup Eig(K_{1,n}ig) = \{v_0\} \cup \{e_i/1 \le i \le n\} \cup \{v_i/1 \le i \le n\}$ , in which the vertices  $\{v_0\} \cup \{e_i/1 \le i \le n\}$  induces a clique of order n+1 and the vertex  $v_0$  is adjacent to  $\{v_i/1 \le i \le n\}$ . Since  $\{v_0\}$  dominates all the vertices of  $Tig(K_{1,n}ig)$  and is also connected, we get  $S = \{v_0\}$  forms the minimum Blast dominating set of  $Tig(K_{1,n}ig)$ . Since  $d(v_0) = 2n$  in  $Tig(K_{1,n}ig)$  and which is the maximum degree of a vertex in  $Tig(K_{1,n}ig)$ , for every vertex  $v \in Vig[Tig(K_{1,n}ig)ig] - \{v_0\}$ , there is a vertex  $v_0$  such that  $d(v_0) \ge d(v)$ . Hence S is a Strong blast dominating set. Therefore, the Strong Blast Domination Number of the total graph of star graph is,  $\gamma_{SC}^{tc}ig[Tig(K_{1,n}ig)ig] = 1$ .

## Theorem 4.13

For any 
$$n$$
 – Sunlet graph  $S_n$  ( $n \ge 3$ ),  $\gamma_{sc}^{tc}[T(S_n)] = n$ .

### **Proof:**

Let  $V(C_n) = \{v_1, v_2, ..., v_n\}$  and let  $u_i$  be the pendant vertex adjacent to  $v_i$ ,  $1 \le i \le n$ . Let  $x_i$  be a vertex of  $T(S_n)$  corresponding to the edge  $u_iv_i$  in  $S_n$ . Then  $S = \{v_1, v_2, ..., v_n\}$  is a  $\gamma_{sc}^{tc} - set$  of  $T(S_n)$ . Thus  $\gamma_{sc}^{tc}[T(S_n)] \le n$ . Further, any dominating set of  $T(S_n)$  must contains at least one of  $v_i, u_i, x_i \ \forall i$  and hence  $|S| \ge n$ . So that  $\gamma_{sc}^{tc}[T(S_n)] \ge n$ . Hence  $\gamma_{sc}^{tc}[T(S_n)] = n$ .

#### Theorem 4.14

For any wheel 
$$W_n$$
 ( $n \ge 4$ ),  $\gamma_{sc}^{tc}[T(W_n)] = \left\lfloor \frac{n}{2} \right\rfloor + 1$ .

## **Proof:**

Let  $V(W_n) = \{v, v_1, v_2, \dots, v_{n-1}\}$  and  $V[T(W_n)] = \{v \cup \{v_i/1 \le i \le n-1\} \cup \{u_i/1 \le i \le n-1\} \cup \{e_i/1 \le i \le n-1\}\}$  where  $u_i$  is the vertex of  $T(W_n)$  corresponding to the edge  $v_iv_{i+1}$  of  $W_n$  ( $1 \le i \le n-1$ ) and  $e_i$  ( $1 \le i \le n-1$ ) is the vertex of  $T(W_n)$  corresponding to the edge  $vv_i$  ( $1 \le i \le n-1$ ).

Case i: Suppose n is odd.

IRIET Volume: 07 Issue: 03 | Mar 2020

www.irjet.net

e-ISSN: 2395-0056 p-ISSN: 2395-0072

Let S' be the independent set of the cycle  $C_{n-1}$  with respect to the wheel  $W_n$ . Therefore  $S' = \{v_1, v_3, ..., v_{n-2}\}$  and  $|S'| = \left\lfloor \frac{n}{2} \right\rfloor$ . Then  $S = S' \cup \{v\}$  where v is the apex vertex of  $W_n$  is the  $\gamma_{sc}^{tc} - set$  of  $T(W_n)$  and  $|S| = \left\lfloor \frac{n}{2} \right\rfloor + 1$ . Hence  $\gamma_{sc}^{tc}[T(W_n)] = \left\lfloor \frac{n}{2} \right\rfloor + 1$ .

*case ii*: Suppose *n* is even.

As before let  $S' = \{v_1, v_3, ..., v_{n-2}\}$  be the independent set of the cycle  $C_{n-1}$  with respect to the wheel  $W_n$  and  $|S'| = \left|\frac{n}{2}\right|$ . Then  $S = S' \cup \{v\} \cup \{v_{n-1}\}$  is the  $\gamma_{sc}^{tc} - set$  of  $T(W_n)$  and  $|S| = \left|\frac{n}{2}\right| + 1$ . Hence  $\gamma_{sc}^{tc}[T(W_n)] = \left|\frac{n}{2}\right| + 1$ .

## Theorem 4.15

$$\gamma_{sc}^{tc}[P(K_n)] = n - 1, n \ge 3.$$

## **Proof:**

Let  $v_1, v_2, \ldots, v_n$  be the vertices of  $K_n$  and  $e_1, e_2, \ldots e_{\binom{n}{2}}$  be the edges of  $K_n$ . Then  $\{v_1, v_2, \ldots, v_n, e_1, e_2, \ldots e_{\binom{n}{2}}\}$  is the vertex set of  $P(K_n)$ . Therefore  $|V[P(K_n)]| = n + \binom{n}{2}$ . Each edge  $e_i$   $(1 \le i \le \binom{n}{2})$  is adjacent to 2(n-2) edges and incident with 2 vertices. Let  $e_i$  be a vertex of  $P(K_n)$  corresponding to  $v_a$  and  $v_b$ . Then  $e_i$  adjacent to 2(n-1) vertices of  $P(K_n)$ . Therefore  $e_i$  dominates 2n-1 vertices including itself in  $P(K_n)$ .

Thus there are  $\binom{n-1}{2}$  vertices in  $P(K_n)$  which are need to be dominated. Let such  $v_j's$   $(1 \le j \ne a \ne b \le n)$  and  $e_k's$   $(1 \le k \ne i \le n)$  belongs to the set M.  $\therefore M = \{v_1, v_2, ..., v_{n-2}, e_1, e_2, ... e_{\binom{n-2}{2}}\}$ .

Let  $e_1 \in M$  be a vertex of  $P(K_n)$  corresponding to  $v_b$  and  $v_c$ . So that  $e_1$  dominates at least three vertices in M. Choose  $e_2 \in M$  adjacent to  $e_1$  and which is also dominate at least 3 vertices of M. Proceeding like this until all the vertices of  $P(K_n)$  dominated by minimum number of vertices. Thus we get S as a minimum connected dominating set of cardinality n-1.

Also every vertex of S has maximum degree  $\Delta(G)$ . Therefore S is a  $\gamma_{sc} - set$ . And  $\langle V - S \rangle$  is triple connected. Hence S is a  $\gamma_{sc}^{tc} - set$  and  $\gamma_{sc}^{tc}[P(K_n)] = n-1$ .

# Theorem 4.16

$$\gamma_{sc}^{tc}[P(F_n)] = 2n, n \ge 1.$$

# **Proof:**

Let  $\{v_1, v_2, v_3, \dots, v_{2n+1}\}$  be the vertex set of  $F_n$  and  $\{e_1, e_2, \dots, e_{3n}\}$  be the edge set of  $F_n$ .

∴ The vertex set of  $P(F_n)$  is,  $\{v_i/1 \le i \le 2n+1\} \cup \{e_i/1 \le j \le 3n\}$  and  $|V[P(F_n)]| = 5n+1$ .

We prove this result by induction on *n*.

# Suppose n = 1:

Then  $F_1$  has 3 vertices and 3 edges. Therefore  $P(F_1)$  has 6 vertices namely  $\{v_1, v_2, v_3, e_1, e_2, e_3\}$ . Let  $v_1$  be the apex vertex of  $F_1$ . Let  $e_1$  be a vertex of  $P(F_1)$  which is incident with  $v_1$  in  $F_1$ . Therefore  $e_1$  dominates 5 vertices including itself. Since  $P(F_1)$  is connected, the remaining one vertex is adjacent to some  $e_i$  ( $2 \le i \le 3$ ). Choose  $e_2$  adjacent with  $e_1$  so that  $e_2$  dominates the remaining one vertex of  $P(F_1)$ .

∴  $S = \{e_1, e_2\}$  is a minimum connected dominating set and  $\deg(e_1) = \deg(e_2) = \Delta(G) = 4$ . Thus for every vertex  $v \in V - S$  there exist some vertices  $e_i \in S$  such that  $\deg(e_i) \ge \deg(v)$ . Therefore S is a strong dominating set, also |V - S| = 4 and for triple connected we need at least 3 vertices. So  $\langle V - S \rangle$  is triple connected. Thus S is a  $\gamma_{sc}^{tc} - set$  and  $\gamma_{sc}^{tc}[P(F_1)] = 2$ . Hence the result is true for n = 1.

Let us assume that the result is true for n-1. i.e. There exist a  $\gamma_{sc}^{tc}$  – set S' such that |S'|=2(n-1).

$$\therefore \ \gamma^{tc}_{sc}[P(F_{n-1})] = 2(n-1).$$

Volume: 07 Issue: 03 | Mar 2020 www.irjet.net p-ISSN: 2395-0072

Now, we prove the result for n.

 $F_n = F_{n-1} + C_3$  such that any one of the vertex of  $C_3$  incident with the apex vertex of  $F_{n-1}$ . For  $F_{n-1}$ , we have,  $\gamma_{sc}^{tc}[P(F_{n-1})] = 2(n-1)$ . Now for  $F_n$ ,  $\{S'\} \cup \{u\} \cup \{v\}$  where  $\{u\}$ ,  $\{v\}$  are the edges belongs to  $C_3$  is the  $\gamma_{sc}^{tc} - set$  of  $P(F_n)$ . Let  $S'' = \{S'\} \cup \{u\}$ ,  $\{v\}$ .

$$|S''| = |S'| + 1 + 1$$
  
=  $2(n-1) + 2$   
=  $2n$ 

 $\therefore \gamma_{sc}^{tc}[P(F_n)] = 2n$ . Thus the result is true for n. Hence the result is true for all n.

#### 3. CONCLUSION

In this paper, we computed the exact values of strong blast domination for some total graph, quasi total graph, corona product of some graphs. Also we derive several general results on this domination parameter.

### REFERENCES

- [1] G. Mahadevan, A. Ahila and Selvam Avadayappan, Blast Domination Number of a Graph Preprint [Communicated in Global Journal Of Pure and Applied Mathematics].
- [2] G. Mahadevan, A. Selvam, J. Paulraj Joseph, B. Ayisha and T. Subramanian, Complementary triple connected domination number of a graph, Advances and Applications in Discrete Mathematics, Vol. 12 (I) (2013), 39 54.
- [3] G. Mahadevan, A. Selvam, J. Paulraj Joseph and T. Subramanian, Triple connected domination number of a graph, International Journal of Mathematical Combinatorics, Vol.3 (2012), 93 104.
- [4] Harary.F Graoh Theory, Addison Wesley Reading Mass(1972).
- [5] J. Paulraj Joseph, M. K. Angel Jebitha, P. Chithra Devi and G. Sudhana, Triple connected graphs, Indian Journal of Mathematics and Mathematical Sciences, Vol.8, No.I(2012),61–75.
- [6] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, Inc., New York, 1998.

e-ISSN: 2395-0056