

Creative Equiped Convex Topological Spaces for Distributional Transform

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Abstract - To satisfy the user's requirements of open management and visual query in theoretical forward models a novel scheme and study for spaces that makes the Gelfand Shilov technique to generalize the Laplace Stieltjes transform a simple objective function a combination of two different transforms in the Distributional Generalized sense appropriate domains for harmonic analysis is proposed technology to be taken into consideration during the planning modeling operating Cauchy problems and performing various operations due to wide spread applicability to solve the PDE involving distributional condition. In addition the primitive is used by giving convenient explanations for more general situations to achieve and enjoy a slightly faster decay in domain even in polynomial case by changing the scheme from one dimension to higher follows from the property of strong continuity at origin implies continuity at any point. However Cauchy problems with solutions which are not continuous at zero include important Differential problems that often arise in applications alongwith the well imposed

Key Words : Laplace Stieltjes Transform, Continuous Linear Functional, Transmission Image, Multinormed Space, Distributional Generalized Sense, The Cauchy Problems, Gelfand Shilov Spaces.

1. INTRODUCTION

The systematic theory of distributional integral transform that objects develops well established valuable techniques of generalized functions also known as distributions due to wide spread applicability in real life situations have its origin in the work of Schwartz[1], Zemanian[2], Brychkov[3], Snedonn[4]. The roots and mathematical approach of the methods are of great interest to gain appropriate flavor in several branches of engineering stress back to the work of Heaviside[1890], Todor [5], Hamed[6], Capiello[7] due to the concept of imposing conditions on the decay of the fundamental functions in Mar[8], Gabriella[9] at infinity with growth of the derivative to all the integrable functions used to formulate generalized solutions of partial differential equations as well as ordinary differential equations involving distributional boundary conditions for propogation of heat in cylindrical coordinates esppecially in the quantum field theory as the order of the derivative increases. The linear part of such equations in Dusan[10], Jaeyoung[11], Geetha[12] is connected to study the local regularity properties of analyzing functions as a motivation for formulating the generalized Laplace Stieltjes transform defind in Gulhane

[13] a widest one result on the connection between the transforms not satisfying admissibility conditions with both local and global behavior of the transform. Dmitrii in [14] designed a theoretical forward platform over integral representations of the generalized hypergeometric functions to establish new inequalities by collecting a number of consequences of properties for completely monotonic Stieltjes class.

We studied a crucible role in mathematical analysis, mathematical physics and engineering of generalized functions in the form of a continuous collection of six distinct volumes by Gelfand[15], Irina[16] as an introduction to generalized functions and presents various applications to analysis, partial differential equations, stochastic process, representation theory where many continuous non-continuous problems naturally lead to differential equations whose solution is a work by Paul Dirac[1920], Fisher[17] for Dirac delta distributions used in modeling quantum electronics as $\delta(t)$ equals to zero for nonzero functions and ∞ for t equals to zero. The major protection devices in a generalized distribution theory a class of Gelfand Shilov spaces [18, 19, 20] their closed subspaces consisting of analytic signals which are almost exponentially localized in time and frequency variables control the decay of the transforms independently in each variables in Cordero[21] since the appropriate support of transform in positive domain which do not contain explicit regularity conditions. The spaces have gained more attention in Feichtinger[22], Toft[23] connection with the modulation spaces localization operators the corresponding pseudo-differential calculus in Teofanov[24, 25] the projective descriptions of a general class of Gelfand Shilov spaces of Roumieu type are indispensable for achieving completed tensor product representations of different important classes of vector valued ultra-differentiable functions of Roumieu. The main interest comes historically from Quantum Mechanics, where the exponential decay of eigen functions have intensively studied. Gelfand Shilov type spaces Robertson [26] in which the topology of bounded convergence is assigned to the dual function study with the Symbol-Global operator's type in the context of time-frequency analysis.

2. CREATION OF TOPOLOGICAL SPACES

In order to simplify the exposition we start by recalling some facts about one dimensional LS type spaces Gelfand Shilov involving both integral differentiation multiplication by function exponential concept under one umbrella having the approach to solve different types different order different degree ordinary differential

equations partial differential equations upto some desired order over some domain C^∞ the space $LS_{\alpha, A} = LS_{\alpha, A}(R^d)$ with constraints mainly on the decrease of the functions at infinity for $\alpha > 0$ consists of all infinitely differentiable functions $\varphi(t, x)$ for $0 < x < \infty, 0 < t < \infty$ satisfying the inequality for each nonnegative integer l, q

$$\gamma_{\alpha, k, l, q} \varphi = \sup_{\substack{0 < t < \infty \\ 0 < x < \infty}} \left| e^{at} (1+x)^k D_t^l (xD_x)^q \varphi(t, x) \right| \leq C_q A^k k^{k\alpha}$$

as the constants A and C_q depend on the everywhere differentiable testing function φ and $a \in R$. We get $k^{k\alpha} = 1$ for $k = 0$.

The topology of the multinormed space is generated by the countable multiform $\{\gamma_{\alpha, k, l, q}\}_{l, q=0}^\infty$

With this topology $LS_{\alpha, a}$ is a countably multiform complete, normed, real (or complex) strongest possible one with continuous induction map LS_{α, a_ν} to $LS_{\alpha, a}$ for every choice of $\nu > 0$.

Although some aspects were developed much earlier as if $\gamma_{\alpha, k, l, q} \varphi \leq C_k B^q q^{q\beta}$ where C_k is a function depend on l, q for the systematic study of exponential constructed space LS^β , which arise as a application of differentiable functions whose derivatives do or donot exist in the classical sense for the space having constraints mainly on the growth of the involved partial derivatives as l approaches to infinity for $\beta > 0$ as the origin.

The extensively used contribution for the development of the necessary facts related to the generalized functions theory by Schwartz hence the construction of Laplace Stieltjes transform theory of generalized distributional transform is based on the test function space LS consisting of all infinitely differentiable function $\varphi(t, x)$ defined for all positive values of t, x having continuous derivative over some domain $C^\infty(R^{d_1})$

$$\text{satisfying } \sup_{\substack{0 < t < \infty \\ 0 < x < \infty}} \left| e^{at} (1+x)^k D_t^l (xD_x)^q \varphi(t, x) \right| < \infty$$

Obviously the spaces $LS_{\alpha, a}, LS^\beta$, of all non negative numbers α, β are subspaces of the above testing function space for $0 < x < \infty, 0 < t < \infty$

Let there be given $\alpha_1, \beta_1 > 0, A_1, B_1 \in R$ be fixed, $\varphi(t, x)$ function defined for all positive values of t, x having continuous derivative over some domain $C^\infty(R^{d_1})$. Gelfand Shilov type space relative to Laplace transform $LS_{\alpha_1, A_1}^{\beta_1, B_1} = LS_{\alpha_1, A_1}^{\beta_1, B_1}(R^{d_1})$ is defined by

$$LS_{\alpha_1, A_1}^{\beta_1, B_1} = \left\{ \varphi \in C^\infty(R^{d_1}) / \exists C_{l, q_1} > 0, \sup_{\substack{0 < t < \infty \\ 0 < x < \infty}} \left| e^{at} (1+x)^k D_t^l (xD_x)^q \varphi(t, x) \right| \leq C_{l, q_1} A_1^k B_1^q a^{\alpha_1 k} l^{\beta_1 q} \right\}$$

where the constants C_{l, q_1}, A_1, B_1 depend on the everywhere differential testing function φ . From a topological point of view the spaces $LS_{\alpha_1}^{\beta_1}$ and $\sum_{\alpha_1}^{\beta_1}$ are given by the union and intersection for $A_1, B_1 \geq 0$ of $LS_{\alpha_1, A_1}^{\beta_1, B_1}$ respectively with their topologies having special paid attention on the inductive and projective limits:

$$LS_{\alpha_1}^{\beta_1} = \text{ind } \lim_{A_1, B_1 > 0} LS_{\alpha_1, A_1}^{\beta_1, B_1} \text{ and}$$

$$\sum_{\alpha_1}^{\beta_1} = \text{proj } \lim_{A_1, B_1 > 0} LS_{\alpha_1, A_1}^{\beta_1, B_1}$$

$LS_{\alpha_1}^{\beta_1}$ and $\sum_{\alpha_1}^{\beta_1}$ are nontrivial iff $\alpha_1 + \beta_1 \geq 0$ and $\alpha_1 \beta_1 > 0$. the union and intersection for $A_1, B_1 \geq 0$ of $LS_{\alpha_1, A_1}^{\beta_1, B_1}$.

Evidently the space $LS_{\alpha_1}^{\beta_1}$ of all non negative numbers α, β is contained in the intersection of the spaces $LS_{\alpha, a}, LS^\beta$, whereas space as a union of countably normed spaces were able to define sequential convergence in all metioned spaces such that these spaces became sequentially complete.

The Gelfand Shilov type distributional spaces $(LS_{\alpha_1}^{\beta_1})'$ and $(\sum_{\alpha_1}^{\beta_1})'$ are given by the intersection and union for $A_1, B_1 \geq 0$ of $(LS_{\alpha_1, A_1}^{\beta_1, B_1})'$ and its topological sence is given by the projective and inductive limits:

$$(LS_{\alpha_1}^{\beta_1})' = \bigcap_{A_1, B_1 > 0} (LS_{\alpha_1, A_1}^{\beta_1, B_1})' \text{ and } (\sum_{\alpha_1}^{\beta_1})' = \bigcup_{A_1, B_1 > 0} (LS_{\alpha_1, A_1}^{\beta_1, B_1})'$$

Here $(LS_{\alpha_1}^{\beta_1})'$ is the dual of $LS_{\alpha_1}^{\beta_1}$ and $(\sum_{\alpha_1}^{\beta_1})'$ is the dual of $\sum_{\alpha_1}^{\beta_1}$.

Gelfand Shilov type spaces as a exponential sence as well as polynomial approach relative to Stieltjes transform $LS_{\alpha_2, A_2}^{\beta_2, B_2} = LS_{\alpha_2, A_2}^{\beta_2, B_2}(R^{d_2})$ is defined for convenience under

proper coordination of the variables and parameters in a unified manner by

$$LS_{\alpha_2, A_2}^{\beta_2, B_2} = \left\{ \varphi \in C^\infty(\mathbb{R}^d) / \exists C_{l_2 q_2} > 0, \right. \\ \left. \sup_{\substack{0 < t < \infty \\ 0 < x < \infty}} \left| e^{at} (1+x)^k D_t^l (xD_x)^q \varphi(t, x) \right| \right. \\ \left. \leq C_{l_2 q_2} A_2^k B_2^q k^{k\alpha_2} q^{q\beta_2} \right\}$$

where the constants $C_{l_2 q_2}, A_2, B_2$ depend on the everywhere differential testing function φ . The spaces $LS_{\alpha_2}^{\beta_2}$ and $\sum_{\alpha_2}^{\beta_2}$ are given by the union and intersection for $A_2, B_2 \geq 0$ of $LS_{\alpha_2, A_2}^{\beta_2, B_2}$ and its topology is given by the inductive and projective limits:

$$LS_{\alpha_2}^{\beta_2} = \text{ind} \lim_{A_2, B_2 > 0} LS_{\alpha_2, A_2}^{\beta_2, B_2} \quad \text{and} \\ \sum_{\alpha_2}^{\beta_2} = \text{proj} \lim_{A_2, B_2 > 0} LS_{\alpha_2, A_2}^{\beta_2, B_2}$$

$LS_{\alpha_2}^{\beta_2}$ and $\sum_{\alpha_2}^{\beta_2}$ are nontrivial iff $\alpha_2 + \beta_2 \geq 0$ and $\alpha_2 \beta_2 > 0$. the union and intersection for $A_2, B_2 \geq 0$ of $LS_{\alpha_2, A_2}^{\beta_2, B_2}$.

The Gelfand Shilov type distributional spaces $(LS_{\alpha_2}^{\beta_2})'$ and $(\sum_{\alpha_2}^{\beta_2})'$ are given by the intersection and union for $A_2, B_2 \geq 0$ of $(LS_{\alpha_2, A_2}^{\beta_2, B_2})'$ and its topological sense is given by the projective and inductive limits:

$$(LS_{\alpha_2}^{\beta_2})' = \bigcap_{A_2, B_2 > 0} (LS_{\alpha_2, A_2}^{\beta_2, B_2})' \quad \text{and} \quad (\sum_{\alpha_2}^{\beta_2})' = \bigcup_{A_2, B_2 > 0} (LS_{\alpha_2, A_2}^{\beta_2, B_2})'$$

Here $(LS_{\alpha_2}^{\beta_2})'$ is the dual of $LS_{\alpha_2}^{\beta_2}$ and $(\sum_{\alpha_2}^{\beta_2})'$ is the dual of $\sum_{\alpha_2}^{\beta_2}$.

Now we are ready to extend and construct the systematic theory of straightforward extension of two dimensional some LS type spaces of Laplace Stieltjes transform $LS_{\alpha_i, A_i}^{\beta_i, B_i}$ using Gelfand Shilov technique for $\alpha_i = \alpha_1, \alpha_2; \beta_i = \beta_1, \beta_2$ defined by

$$LS_{\alpha_i, A_i}^{\beta_i, B_i} = \left\{ \varphi \in C^\infty(\mathbb{R}^{d_i}) / \exists C_{l_i q_i} > 0, \right. \\ \left. \sup_{\substack{0 < t < \infty \\ 0 < x < \infty}} \left| e^{at} (1+x)^k D_t^l (xD_x)^q \varphi(t, x) \right| \right. \\ \left. \leq C_{l_i q_i} A_i^a A_2^k B_1^l B_2^q a^{a\alpha_i} l^{l\beta_i} k^{k\alpha_2} q^{q\beta_2} \right\}$$

where the constants $C_{l_i q_i}; A_i = A_1, A_2; B_i = B_1, B_2$ depend on the everywhere differential testing function φ . The

spaces $LS_{\alpha_1, \alpha_2}^{\beta_1, \beta_2}$ and $\sum_{\alpha_1, \alpha_2}^{\beta_1, \beta_2}$ a pair of Laplace Stieltjes transform as a very powerful mathematical tool applied in various areas of engineering and science with the increasing complexity of engineering problems are given by the union and intersection for $A_i, B_i \geq 0$ of $LS_{\alpha_i, A_i}^{\beta_i, B_i}$ and its topology is given by the inductive and projective limits:

$$LS_{\alpha_1, \alpha_2}^{\beta_1, \beta_2} = \text{ind} \lim_{A_1, B_1 > 0} LS_{\alpha_i, A_i}^{\beta_i, B_i} \quad \text{and} \\ \sum_{\alpha_1, \alpha_2}^{\beta_1, \beta_2} = \text{proj} \lim_{A_1, B_1 > 0} LS_{\alpha_i, A_i}^{\beta_i, B_i}$$

$LS_{\alpha_1, \alpha_2}^{\beta_1, \beta_2}$ and $\sum_{\alpha_1, \alpha_2}^{\beta_1, \beta_2}$ are nontrivial iff $\alpha_i + \beta_i \geq 0$ and $\alpha_i \beta_i > 0$ the union and intersection for $A_i, B_i \geq 0$ of $LS_{\alpha_i, A_i}^{\beta_i, B_i}$.

The Gelfand Shilov type distributional spaces $(LS_{\alpha_1, \alpha_2}^{\beta_1, \beta_2})'$ and $(\sum_{\alpha_1, \alpha_2}^{\beta_1, \beta_2})'$ are given by the intersection and union for $A_i, B_i \geq 0$ of $(LS_{\alpha_i, A_i}^{\beta_i, B_i})'$ and its topological sense is given by the projective and inductive limits:

$$(LS_{\alpha_1, \alpha_2}^{\beta_1, \beta_2})' = \bigcap_{A_1, B_1 > 0} (LS_{\alpha_i, A_i}^{\beta_i, B_i})' \quad \text{and} \\ (\sum_{\alpha_1, \alpha_2}^{\beta_1, \beta_2})' = \bigcup_{A_1, B_1 > 0} (LS_{\alpha_i, A_i}^{\beta_i, B_i})'$$

Here $(LS_{\alpha_1, \alpha_2}^{\beta_1, \beta_2})'$ is the dual of $LS_{\alpha_1, \alpha_2}^{\beta_1, \beta_2}$ and $(\sum_{\alpha_1, \alpha_2}^{\beta_1, \beta_2})'$ is the dual of $\sum_{\alpha_1, \alpha_2}^{\beta_1, \beta_2}$. The corresponding dual spaces introduced for the study of Cauchy problems in partial differential equations created as a model in technical subjects are the spaces of ultradistributions of Roumieu and Beurling respectively. Unless specified otherwise all the spaces introduced throughout will henceforth be considered equipped with their naturally Hausdorff locally convex topologies on these spaces are generated by the family of seminorms $\{\mathcal{V}_{a, k, l, q}\}$.

We consider the domain $-\infty < t < 0, 0 < x < \infty$ is in $\overline{LS}_{\alpha_1, \alpha_2}^{\beta_1, \beta_2}$ if φ smooth function $\overline{\varphi}(t, x) = \varphi(-t, x)$ is in $LS_{\alpha_1, \alpha_2}^{\beta_1, \beta_2}$ for which

$$\leq C_{l_i q_i} A_1^a A_2^k B_1^l B_2^q a^{a\alpha_1} l^{l\beta_1} k^{k\alpha_2} q^{q\beta_2}$$

satisfying all above mentioned properties. Corresponding to all defined spaces in above sections the spaces.

$$\overline{LS}_{\alpha_1}, \overline{LS}_{\alpha_2}^{\beta_1}, \overline{LS}_{\alpha_2}^{\beta_2}, \overline{LS}_{\alpha_1}^{\beta_1}, \overline{LS}_{\alpha_2}^{\beta_2}, \overline{LS}_{\alpha_1, \alpha_2}^{\beta_1, \beta_2}$$

which have domain $-\infty < t < 0, 0 < x < \infty$ can also be defined.

Depending on various choices of distributional spaces defined above defined in Gulhane [13] nondefined equipped with their naturally Hausdrof locally convex topologies generated by their respective corresponding total families of seminorms are as usual denoted by $T_{\alpha_1}, T^{\beta_1}, T_{\alpha_2}, T^{\beta_2}, T_{\alpha_1}, T^{\beta_1}, T_{\alpha_2}, T^{\beta_2}, T_{\alpha_1, \alpha_2}, T^{\beta_1, \beta_2}$. Moreover all the spaces having domain $-\infty < t < 0, 0 < x < \infty$ are equipped with their naturally Hausdrof locally convex topologies $\bar{T}_{\alpha_1}, \bar{T}^{\beta_1}, \bar{T}_{\alpha_2}, \bar{T}^{\beta_2}, \bar{T}_{\alpha_1}, \bar{T}^{\beta_1}, \bar{T}_{\alpha_2}, \bar{T}^{\beta_2}, \bar{T}_{\alpha_1, \alpha_2}, \bar{T}^{\beta_1, \beta_2}$.

Further we extend the space $LS_{\alpha_1, \alpha_2, m_1, m_2}^{\beta_1, \beta_2, n_1, n_2}$ defined by

$$LS_{\alpha_1, \alpha_2, m_1, m_2}^{\beta_1, \beta_2, n_1, n_2} = \left\{ \varphi \in C^\infty(\mathbb{R}^{d_1+d_2}) / \exists C_{l,q_i} > 0, \right. \\ \left. \sup_{\substack{0 < t < \infty \\ 0 < x < \infty}} \left| e^{at} (1+x)^k D_t^l (xD_x)^q \varphi(t, x) \right| \right. \\ \left. \leq C_{l,q_i} (m_1 + \delta_1)^a (m_2 + \delta_2)^k (n_1 + \eta_1)^l \right. \\ \left. (n_2 + \eta_2)^q a^{\alpha \alpha_1} l^{\beta_1} k^{\alpha \alpha_2} q^{\beta_2} \right\}$$

where $\delta_1, \delta_2, \eta_1, \eta_2$ are any numbers greater than zero lose the property of strongly continuity at $x = 0, t = 0$ being strongly continuous at $0 < x < \infty, 0 < t < \infty$ equipped with their naturally Hausdrof locally convex topologies generated by their respective corresponding total families of seminorms as usual denoted by $T_{\alpha_1, \alpha_2, m_1, m_2}^{\beta_1, \beta_2, n_1, n_2}$ for the domain $0 < x < \infty, 0 < t < \infty$ and $\bar{T}_{\alpha_1, \alpha_2, m_1, m_2}^{\beta_1, \beta_2, n_1, n_2}$ for the domain $-\infty < t < 0, 0 < x < \infty$.

3. CONCLUSION

For the functional analyst engineers from a topological point of view described spaces as a union of countability normed spaces able to define sequential convergence in all above mentioned spaces so become sequentially complete are interesting because of rich structure used to solve the equation of propagation of heat in cylindrical coordinates imposing the generalized boundary conditions.

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