

Review of Entropy Generation Minimization in Heat Exchanger

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Abstract - There are various types of heat exchangers that are used for various applications. A helically coiled heat exchanger is one of the popularly used types of heat exchangers in industrial applications as it offers certain benefits, such as less size, larger heat transfer rate, efficient performance in high pressure and temperature differentials and more economical cost. This study intends to seek out the optimum pure mathematics and operational conditions of helically whorled heat exchangers for each laminar and turbulent flows supported the second law of physics. In order to accomplish the main goals of this work, first, a dimensionless function for entropy generation number comprising four dimensionless variables, i.e. Prandtl number (Pr , the ratio of helical pipe diameter to the tube diameter (δ), Dean number (De) and the duty parameter of heat exchanger, is derived. Next, the entropy generation number is minimized to increase analytical expressions for the optimal values of δ , De number (for laminar flow) and Reynolds number (Re , for turbulent flow) of the heat exchangers.

Key Words: Entropy generation; Helical coil; Irreversibility; Thermodynamic second law

1. INTRODUCTION

In spite of comparatively complex design and manufacturing process, helically coiled heat exchangers are widely used in manufacturing applications because they contribute some advantages that make them the most relevant heat exchanger type in many specific cases. This heat exchanger type is more suitable for the applications with limited space, lower flow rates, the high-pressure drop of one or both of the fluids along with the heat exchanger and the applications with multiphase fluids [1-6]. In a helical heat exchanger, the fluids are subjected to a centrifugal force as they flow through winding shape tubes. This centrifugal force produces counter-rotating vortices, so-called secondary flows, in the fluids and in this way, magnifies significantly the rate of heat transfer between the fluids [7]. Although the accidental motion of fluids and the behavior of secondary flows through the curvatures make it difficult to simulate the heat transfer procedure through a helical tube-in-tube [8], several studies have been conducted on these heat exchangers so far. For example, investigated hydro dynamically and heat transfer characteristics of this type of heat exchanger experimentally. Taken out an experimental study on a vertical helically coiled heat exchanger and found the coil surface area a very practical geometrical parameter on overall heat transfer coefficient while the effect of the diameters of the tubes was negligible. analyzed the review of

fluid to fluid helical coil heat exchangers under different boundary conditions, such as constant heat flux, constant wall temperature and constant heat transfer coefficient. In another work, Jayakumar et al. [12] studied numerically the variation of Nusselt number (Nu) along the tube length and compared their results with their results derived from experimental tests. Rahul et al. developed a novel correlation for calculating the overall heat transfer coefficient of helical tube-in-tube heat exchangers. Humic and Humic [4] assessed the effect of using nano-fluids on the performance of double tube helical heat exchangers, finding 14-19% enhancement in the overall heat transfer coefficient. Most recently, San et al. [3] Examined the heat transfer review of a helical heat exchanger including a tube with a rectangular cross-section and two cover plates for water as the helical fluid and air as radial flow material. Several more studies in this area could be addressed in the literature.

Therefore, the main goal of this work is minimizing the rate of entropy generation in tube-in-tube helical heat exchangers for both turbulent and laminar flows by finding the optimal geometry and operational conditions of this type of heat exchanger. Overall, four parameters, namely, the ratio of the helical pipe diameter to the diameter of both of the inner and outer tubes (δ_1 and δ_2), Re number (for turbulent flow) and De number (for laminar flow), are going to be optimized for obtaining the lowest rate of entropy generation in this work. It is worth mentioning that fixed geometry is considered for the heat exchanger for finding optimal operational conditions, and by the same token, fixed operational conditions are considered for the heat exchanger to optimize its geometry.

2. PROBLEM DESCRIPTION AND FORMULATION

In this section, detailed mathematical modeling of tube-in-tube helical heat exchangers based on the second law of thermodynamics is presented. Fig. 1-a and 1-b illustrate the schematic diagram and the geometry of a tube-in-tube helical heat exchanger, respectively. According to Fig. 1-a, in this type of heat exchanger, the fluid passing through the internal helical tube (called tube) exchanges thermal energy with a secondary fluid that passes through the space between the inner and outer tubes walls (called annulus). Fig. 1-b gives information about the details of the heat exchanger geometry.

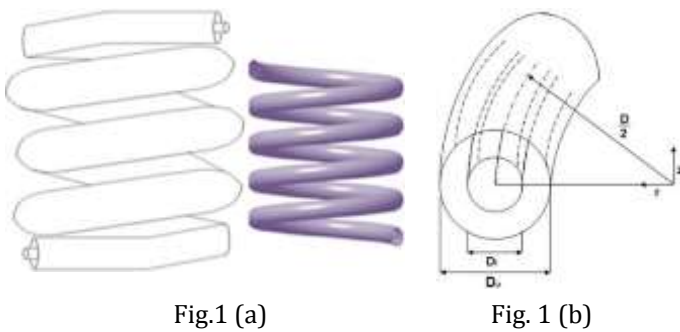


Fig.1 (a)

Fig. 1 (b)

The sensible heat transfer rate was determined either from the temperature difference on the tube-side or the shell-side of a heat exchanger as

A. Geometry and parameters of helical coils

Fig. 1 shows a typical helical coiled tube. The principal geometric dimensions include the inner radius of the tube (r_0), the curvature radius of the coil (a) and the coil pitch (increase of height per rotation, b). The curvature ratio, d , is defined as the coil-to-tube radius ratio, r_0/a and the nondimensional pitch, k , is defined as $b/2\pi a$. There are four important dimensionless parameters in the flow field of the helical coils. They are Reynolds number (Re), Nusselt number (Nu), Dean number (Dn) and the Helical number (He). Their definitions are as follows:

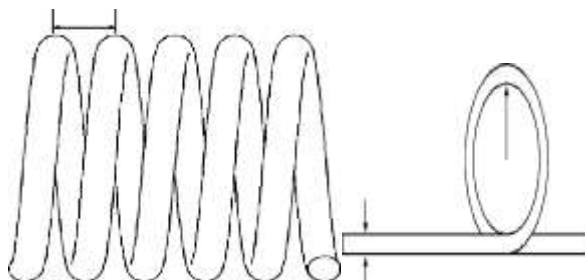


Fig. 2. Geometry of a helical coil tube.

$$Nu = \frac{\bar{h}(2r_0)}{k}, Dn = Re(r_0/a)^{1/2}$$

$$He = Dn / [1 + (b/2\pi a)^2]^{1/2}$$

3. ENTROPY GENERATION

In the present study we focus on the steady, fully developed and laminar convection in the helical coils with constant wall heat flux. As all the thermodynamic systems, the entropy in the helical coils is generated from the irreversibility due to the heat transfer with finite temperature differences and the friction of fluid flow. We denote \dot{m} and qV as the mass flow rate in the coils and the heat transfer rate per unit coil length, respectively. The heat transfer is to be transmitted to the fluid flow due to the temperature difference DT formed between the coil wall temperature ($T+DT$) and the bulk temperature of the stream (T). Taking the coil passage of length dx as the thermodynamic system, the first and second law can be expressed as:

$$\dot{m}dh = q'dx \tag{1}$$

$$\dot{S}'_{gen} = \dot{m} \frac{ds}{dx} - \frac{q'}{T + \Delta T} \tag{2}$$

where \dot{S}'_{gen} is the entropy generation rate per unit length. By using the thermodynamic relation

$$Tds = dh - vdp \tag{3}$$

The entropy generation rate per unit coil length

$$\therefore -\frac{dP}{dx} = \frac{\dot{m}^2 f}{\rho r_0^5 \pi^2}$$

$$\dot{S}'_{gen} = \frac{q'\Delta T}{T^2(1 + \Delta T/T)} + \frac{\dot{m}}{T\rho} \left(-\frac{dP}{dx} \right) \tag{4}$$

In Eq. (4) the pressure gradient can be replaced by

$$-\frac{dP}{dx} = \frac{f\rho V^2}{r_0} \tag{5}$$

Where, f is the friction factor. By considering

$$\dot{m} = \rho V \pi r_0^2 \tag{6}$$

the pressure gradient term becomes

$$-\frac{dP}{dx} = \frac{\dot{m}^2 f}{\rho r_0^5 \pi^2} \tag{7}$$

From the energy balance, the average heat transfer coefficient (\bar{h}) of the stream can be calculated by

$$\bar{h} = q' / \pi(2r_0)\Delta T \tag{8}$$

Through the definition of Nusselt number, the DT in the Eq. (8) can be expressed by

$$\Delta T = q' / \pi Nuk \tag{9}$$

Substituting Eqs. (7) and (9) into Eq. (4), the \dot{S}'_{gen} can be expressed as

$$\dot{S}'_{gen} = \frac{(q')^2}{T^2 \pi Nuk + Tq'} + \frac{\dot{m}^3 f}{T\rho^2 r_0^5 \pi^2}$$

The expression of \dot{S}'_{gen} is similar to that derived by Bejan [9] for a straight pipe with circular cross section. But the final form is different. In the derivation of Bejan [9], the DT/T term has been ignored, whilst the term has been retained in the present derivation for accuracy. The entropy generation rate \dot{S}'_{gen} is non dimensionalized by qV/T and the non dimensional entropy generation rate is usually called entropy generation number NS [9]. NS is defined by $\dot{S}'_{gen} / (qV/T)$ and can be obtained from Eq. (10) as

$$N_s = \frac{1}{\pi N u k T / q' + 1} + \frac{f}{\pi^2 \rho^2 r_0^5 q' / m^3} \quad (11)$$

By denoting the dimensionless parameters η_1 and η_2 as

$$\eta_1 = \pi k T / q' \quad (13)$$

and

$$\eta_2 = 32 \dot{m}^2 \rho^2 q' / \mu^5 \pi^3 \quad (14)$$

N_s can be written as

$$N_s = \frac{1}{N_u \eta_1 + 1} + \frac{f \text{Re}^5}{\eta_2} \quad (15)$$

For the fully developed laminar convection in the helical coils with the constant heat flux, two reliable correlations for the friction factor and the Nusselt number were proposed by Manlapaz et al. [6] after their detailed survey of previous experimental and theoretical investigations. The correlations are as follows:

$$f = \frac{16}{\text{Re}} \left[\left(1 - \frac{0.18}{\left[1 + \left(\frac{35}{\text{He}} \right)^2 \right]^{1/2}} \right)^m + \left(1 + \frac{r_0}{3a} \right)^2 \frac{\text{He}}{88.33} \right]^{1/2} \quad (16)$$

And

$$\text{Nu} = \left[\left(\frac{48}{11} + \frac{51/11}{\left(1 + \frac{1342}{\text{PrHe}^2} \right)^2} \right)^3 + 1.816 \left(\frac{\text{He}}{1 + \frac{1.15}{\text{Pr}}} \right)^{3/2} \right]^{1/3} \quad (17)$$

where values of m are 2, 1 and 0 for $\text{Dn} < 20$, $20 < \text{Dn} < 40$ and $\text{Dn} > 40$, respectively. Invoking the expressions of f and Nu into Eq. (14), the entropy generation number NS becomes a function of Re , k , d , Pr , g_1 and g_2 . In the expression of NS , the first and the second terms of the right-hand side of Eq. (14) are the entropy generations caused by the irreversibility from the heat transfer and the fluid friction, respectively. The challenge of the design work comes from an important feature of the NS expression that a design parameter change will induce changes of the two terms with opposite signs, which makes the optimal trade-off become the chief task for the heat exchanger designers. The optimum design could be determined through the analysis of the entropy generation rate based on the entropy generation minimization principal.

3. DISCUSSION

Is a very complicated function of Re , k , d , Pr , g_1 and g_2 . To find its extreme through the mathematical operation based on calculus theorem is not easy. In the present study, we

directly calculate NS of different cases with various combinations of the parameters and try to find out the optimum case with the minimum. The following results are based on the physical properties for air ($\text{Pr}=0.711$). Five groups with different combination of g_1 and g_2 are investigated. In this section, the results of numerical simulation accomplished for entropy generation minimization of helical tube-in-tube heat exchangers with laminar and turbulent flows are presented. Fig. 2 illustrates the effect of changing the ratio of inner tube and annulus diameters for laminar flows, in specific De numbers, on the rate of entropy generation.

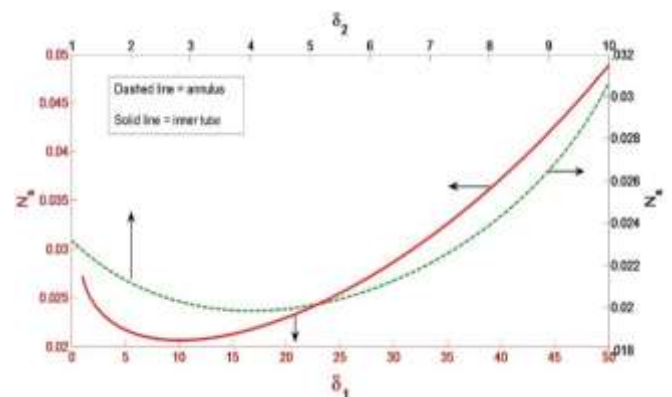


Fig. 3. Variation of N_s value with curvature ratios changes in laminar flow for both the inner tube and the annulus in specific De number

Note that Eqs. 10 and 15 are in connection with this figure and the constant values of $\text{De}=500$, $B=109$ and $\text{Pr}=2.5$, adapted from Ref. [38], have been considered for depicting this figure. According to the figure, the lowest rate of entropy generation for the tube occurs in $\delta_1=10$. As can be seen, the value of N_s associated with this optimal point is almost 0.0204 and this value increases as δ_1 scats from 10 for either higher or lower values. For the annulus, on the other hand, there is no specific δ_2 associated with the minimum value of N_s . In other words, N_s remains at its optimal value of 0.02 when δ_2 value is in range of 3.5-5. Equations

Similarly, for the same values of B and Pr number and employing Eqs. 19 and 22, Fig. 3 illustrates the effect of changing the values of curvature ratios (δ_1 and δ_2) on the rate of entropy generation in the inner tube and the annulus of the helical heat exchanger in case of having turbulent flows, respectively. According to the given information in previous section, in contrast with the laminar flow analysis in which De number was considered in flow modelling, Re number is applied here.

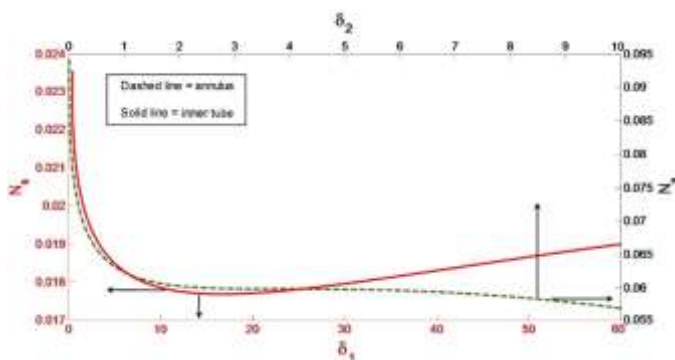


Fig. 4. Variation of N_s value with curvature ratio changes in turbulent flow for both the inner tube and the annulus in specific Re number

Obviously from the figure, in contrast with the inner tube for which there is a specific curvature ratio corresponding with the minimum rate of entropy generation ($\delta_1 \approx 16.5$), for the annulus, after sharp collapse in entropy generation rate by increasing the value of δ_2 from 0 to 1.5, entropy generation rate decreases with an almost constant mild pace as the value of δ_2 increases further. It is noteworthy that the optimal N_s value for the inner tube is 0.0177 while, for the annulus, the value of N_s decreases from 0.06 down to 0.057 for $1.5 < \delta_2 < 10$. This implies to this fact that although increasing the annulus curvature ratio could enhance the entropy generation performance of the device in a turbulent flow, its effect is negligible and $\delta_2 \approx 2$ may be a very good choice

4. CONCLUSION

This work aimed at optimizing tube-in-tube helical heat exchangers geometry and flow characteristics with either laminar flow or turbulent flow based on entropy generation minimization approach. For this objective, a helical tube-in-tube heat exchanger was theoretically investigated, it was mathematically modelled and dimensionless expressions for various parameters of the problem were developed. Defining the dimensionless expression of curvature ratio for both of the inner tube (δ_1) and the annulus (δ_2), it was found out that the geometry of the heat exchanger affects its performance significantly and one could find optimal curvature ratios for both of the inner tube and the annulus to minimize the demotion of thermal energy and viscous dispersion of mechanical energy in the heat exchanger. Then, the rate of entropy generation through the annulus and the inner tube with either laminar flow or turbulent flow was calculated. Finally, taking the results of the numerical simulation implemented into account, the optimal geometry (δ_1 and δ_2) and flow characteristics (Re number, De number) that optimize the performance of this type of heat exchanger based on the lowest rate of entropy generation were determined. Combining economic analysis with an entropy generation minimization analysis seems to be an

interesting investigation for future works in this context to promote application of double-pipe helical heat exchangers.

NOMENCLATURE

A	cross-sectional area of the tube, m^2
B	heat exchanger duty parameter
c_p	Specific heat of tube-fluid, J/kg.K
D	Diameter of tube, m
d	Coil diameter, m
D_i	Diameter of inner tube, m
D_o	Diameter of annulus, m
D_e	Dean number
f	Friction factor
h	heat transfer coefficient, W/m ² .K
k	thermal conductivity of the tube-fluid, W/m.K
\dot{m}	mass flow rate, kg/s
N_s	scaled dimensionless entropy generation rate
N_u	Nusselt number
q'	heat transfer rate per unit tube length, W/m
Q	dimensionless heat flux
Re	Reynolds number
\dot{S}_{gen}	entropy generation rate, W/K
S_t	Stanton number
T	Temperature(K)

Greek Symbols

δ	curvature ratio
ρ	Density(kg/m ³)
μ	viscosity coefficient(N s/m ²)

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