Meet-Semidistributive in Semilattice

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Abstract: *The main idea followed in this paper is fuzzy meet semi distributive semilattice. In this we worked some theorem related to the concept and give some definition related to the concept.*

Keywords: Lattice, Semilattice, Sublattice, Modular Lattice, Meet

I.INTRODUCTION

Some authors published many papers related to lattice concept. Many definitions given in preliminaries are based on the reference paper. Here we worked with the topic of Fuzzy meet-semidistributive semilattice.

II.PRELIMINARIES

2.1 Definition: let L be a non-empty set. Let ∨ and ∧ is said to be two binary operations defined on L. Then (L, ∨, ∧) is said to be a lattice, if the following condition are satisfied.

1. **Idempotent**

   $x \wedge x = x$ and $x \vee x = x$ for all $x \in L$

2. **Commutative**

   $x \wedge y = y \wedge x$ and $x \vee y = y \vee x$ for all $x, y \in L$

3. **Associative**

   $x \wedge (y \wedge z) = (x \wedge y) \wedge z$

   $x \vee (y \vee z) = (x \vee y) \vee z$ for all $x, y, z \in L$

4. **Absorbtion**

   $x \wedge (x \vee y) = x$ and $x \vee (x \wedge y) = x$ for all $x, y \in L$

2.2 Definition: let S be a non-empty set. Let ∨ and ∧ is said to be two binary operations defined on S. Then (S, ∨, ∧) is said to be a semilattice, if the following condition are satisfied.

1. **Idempotent**

   $x \wedge x=x$ and $x \vee x=x$ for all $x \in S$

2. **Commutative**

   $x \wedge y = y \wedge x$ and $x \vee y = y \vee x$ for all $x, y \in S$

3. **Associative**

   $x \wedge (y \wedge z) = (x \wedge y) \wedge z$

   $x \vee (y \vee z) = (x \vee y) \vee z$ for all $x, y, z \in S$

2.3 Definition: A sublattice of a lattice L is the nonempty subset of L that is a lattice with same meet and join operations as L.

2.4 Definition: let μ be a fuzzy set in L then μ is called a sublattice of L if

$i)$ μ(x+y) ≥ min(μ(x), μ(y))

$ii)$$\mu$(x.y) ≥ min($\mu$(x), μ(y))

2.5 Definition: A lattice (L, ∨, ∧) is called a modular lattice if

x ∨ (y ∧ z) = (x ∨ y) ∧ z whenever x≤z

2.6 Definition: A lattice (L, ∨, ∧) is called distributive if the following additional identity holds for all x, y and z in L

x ∧ (y ∨ z) = (x ∧ y) ∨ (x ∧ z) or

x ∨ (y ∧ z) = (x ∨ y) ∧ (x ∨ z).

III. MAIN RESULT

3.1 Definition: A fuzzy lattice L is called a Fuzzy meet-semidistributive if

μ(x ∧ y) = μ(x ∧ z) ⟹ μ(x ∧ y) = μ(x) ∧ μ(y ∨ z) for all μ(x) , μ(y) , μ(z) ∈ L.

Theorem 3.2: Every fuzzy meet-semidistributive semilattice is fuzzy semilattice but every fuzzy semilattice is not the fuzzy meet semidistributive semilattice.

Proof: Given S is a meet-semi distributive semilattice

⟹ μ(p ∧ q) = μ(p) ∧ μ(q ∨ r) for all μ(p) , μ(q) , μ(r) ∈ S.

μ(p ∧ q) = μ(p) ∧ μ(q ∨ r)

≥min{ μ(p) , μ(q ∨ r)}

≥min{ μ(p) , min{μ(q) , $\mu(r)$}}

≥min{ μ(p) , min{μ(r) , $\mu(q)$}} by commutative law

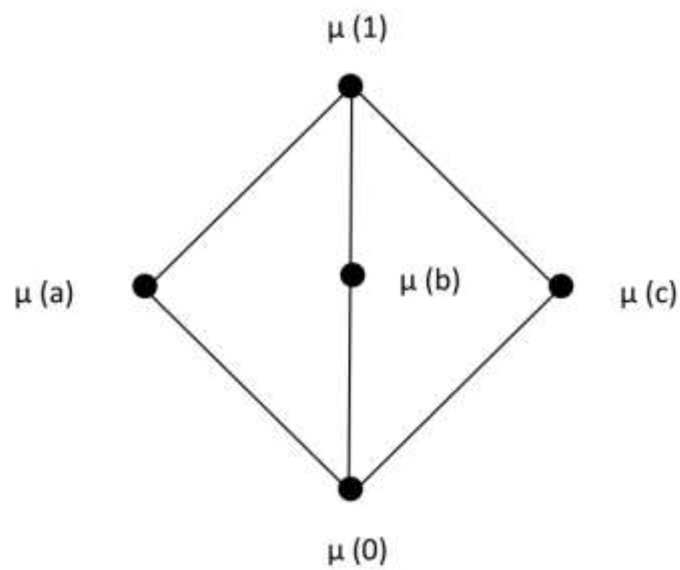
≥min{ μ(p) , μ(r ∨ q)}

= μ(p) ∧ μ(r ∨ q)

=μ(p ∧ r)

Hence S is a fuzzy semilattice. The converse is not true. That is every fuzzy semilattice need not be fuzzy meet-semi distributive.

Consider the fuzzy semilattice S of following figure:



This Fuzzy semilattice is not Fuzzy meet-semidistributive.

Here,

$\mu(p) \wedge \mu(q) = \mu(0)$

$\mu(p) \wedge \mu(r) = \mu(0)$

$\mu(q) \vee \mu(r) = \mu(1)$

$\mu(p) \wedge \mu(q \vee r) \geq \min\{ \mu(p), \mu(q \vee r)\}$

$\geq \min\{ \mu(p), \mu(q) \vee \mu(r)\}$

$\geq \min\{ \mu(p), \mu(1)\}$

$\geq \{ \mu(1) \wedge \mu(p)$

$= \mu(p)$

Thus $\mu(p) \wedge \mu(q) = \mu(p) \wedge \mu(r)$. But $\mu(p) \wedge \mu(q) \neq \mu(p) \wedge \mu(q \vee r) \Longrightarrow$ S is not fuzzy meet-semidistributive.

Theorem 3.3: Every Fuzzy distributive is Fuzzy meet-semi distributive but the fuzzy meet semi distributive is not fuzzy distributive.

Proof: Given S is a Fuzzy distributive semilattice

$\Longrightarrow \mu(p) \vee \mu(q \wedge r) = \mu(p \vee q) \wedge \mu(p \vee r)$ for all $\mu(p), \mu(q), \mu(r) \in S$ (1)

To prove: S is Fuzzy meet-semidistributive semilattice. That is to prove if

$\mu(p) \wedge \mu(q) = \mu(p) \wedge \mu(r)$ implies $\mu(p) \wedge \mu(q) = \mu(p) \wedge \mu(q \vee r)$, for all

$\mu(p), \mu(q), \mu(r) \in S$. First to claim that $\mu(p) \wedge \mu(q \vee r) = \mu(p \wedge q) \vee \mu(p \wedge r)$ for all $\mu(p), \mu(q), \mu(r) \in S$ For let $\mu(p), \mu(q), \mu(r) \in S$ be an arbitrary. Then

$\mu(p) \wedge \mu(q \vee r) = \mu(p \wedge q) \vee \mu(p \wedge r)$ for all $\mu(p), \mu(q), \mu(r) \in S$ (2)

By theorem, In a Fuzzy distributive semilattice, the duality of the Fuzzy distributive condition is holds.

Given S is a Fuzzy distributive semilattice $\Longrightarrow$ $\mu(p) \vee \mu(q \wedge r) = \mu(p \vee q) \wedge \mu(p \vee r)$ for all $\mu(p), \mu(q), \mu(r) \in S$.

To prove: $\mu(p) \wedge \mu(q \vee r) = \mu(p \wedge q) \vee \mu(p \wedge r)$ for all $\mu(p), \mu(q), \mu(r) \in S$.

Let $\mu(p), \mu(q), \mu(r) \in S$ be arbitrary. Then

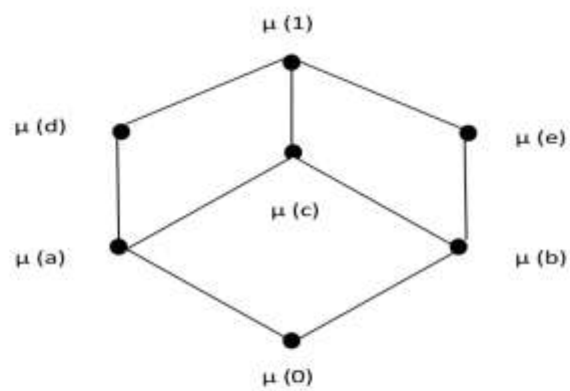
$$\begin{aligned}
\mu(p \wedge q) \vee \mu(p \wedge r) &\geq \min\{\mu(p \wedge q), \mu(p \wedge r)\} \\
&\geq \min\{\mu(p \wedge q) \vee \mu(p), \mu(p \wedge q) \vee \mu(r)\} \\
&\geq \min\{\mu(p) \vee \mu(p \wedge q), \mu(r) \vee \mu(p \wedge q)\} \\
&\geq \min\{\mu(p), \mu(r \vee p) \wedge \mu(r \vee q)\} \\
&\geq \min\{\mu(p) \wedge \mu(r \vee p), \mu(r \vee q)\} \\
&\geq \min\{\mu(p) \wedge \mu(p \vee r), \mu(q \vee r)\} \\
&= \mu(p) \wedge \mu(q \vee r), \text{ for all } \mu(p), \mu(q), \mu(r) \in S.
\end{aligned}$$

Suppose $\mu(p) \wedge \mu(q) = \mu(p) \wedge \mu(r)$. Then

$$\begin{aligned}
\mu(p) \wedge \mu(q \vee r) &= \mu(p \wedge q) \vee \mu(p \wedge r) \quad \text{by (2)} \\
&= \mu(p \wedge q) \vee \mu(p \wedge q) \\
&= \mu(p \wedge q)
\end{aligned}$$

Thus $\mu(p \wedge q) = \mu(p \wedge r) \Longrightarrow \mu(p \wedge q) = \mu(p) \wedge \mu(q \vee r)$ for all $\mu(p), \mu(q), \mu(r) \in S$

$\Longrightarrow$ S is Fuzzy meet-semidistributive semilattice. The converse need not be true. That is every fuzzy meet-semidistributive semilattice need not be a Fuzzy distributive semilattice. We shall verify it by the following example. consider the Fuzzy semilattice K_7 of following figure.



This Fuzzy semilattice is Fuzzy meet-semidistributive but not Fuzzy distributive. Here

$$\begin{aligned}
\mu(p) \vee \mu(s \wedge t) &\geq \min\{\mu(p), \mu(s \wedge t)\} \\
&\geq \min\{\mu(p), \mu(0)\}
\end{aligned}$$

$= \mu(\mathrm{p}) \vee \mu(0)$

$= \mu(\mathrm{p})$

$\mu(\mathrm{p} \vee \mathrm{s}) \wedge \mu(\mathrm{p} \vee \mathrm{t}) \geq \min \{ \mu(\mathrm{p} \vee \mathrm{s}) \wedge \mu(\mathrm{p} \vee \mathrm{t})\}$

$\geq \min \{\mu(\mathrm{s}), \mu(1)\}$

$= \mu(\mathrm{s}) \wedge \mu(1)$

$= \mu(\mathrm{s})$

Therefore $\mu(\mathrm{p}) \vee \mu(\mathrm{s} \wedge \mathrm{t}) \neq \mu(\mathrm{p} \vee \mathrm{s}) \wedge \mu(\mathrm{p} \vee \mathrm{t}) \Longrightarrow K_7$ is not Fuzzy distributive.

Theorem 3.4: A Fuzzy meet-semidistributive semilattice S is Fuzzy distributive if and only if S does not contain a Fuzzy sublattice isomorphic to K_7

Proof: Assume that a Fuzzy meet-semidistributive semilattice S is Fuzzy distributive.

To prove: S does not contain a Fuzzy sublattice isomorphic to K_7. Suppose S contain a Fuzzy sublattice isomorphic to K_7. Thus, S is not Fuzzy distributive. This is a contradiction. Hence S does not contain a Fuzzy sublattice isomorphic to K_7. Conversely, assume that a Fuzzy meet-semi distributive semilattice S does not contain a Fuzzy sublattice isomorphic to K_7.

To prove: S is Fuzzy distributive.

Suppose S is not Fuzzy distributive. Then S contains a Fuzzy sublattice isomorphic to K_7. This is contradiction. Hence S is a Fuzzy distributive semilattice.

Theorem 3.5: Every Fuzzy modular semilattice need not be Fuzzy meet-semi distributive semilattice.

Proof: Given S is Fuzzy modular semilattice $\Longrightarrow$ S contain a Fuzzy sublattice isomorphic to M_3. A Fuzzy semilattice S is Fuzzy modular if and only if does not contain a Fuzzy sublattice isomorphic to N_5. Assume that a Fuzzy semilattice S is Fuzzy modular.

To prove S does not contain a Fuzzy sublattice isomorphic to N_5.

Suppose S contain a Fuzzy sublattice isomorphic to $N_5 \Longrightarrow$ S is not Fuzzy modular. This is Contradiction. Hence S does not contain a Fuzzy sublattice isomorphic to N_5.

Conversely Assume that a Fuzzy semilattice S does not contain a Fuzzy sublattice isomorphic to N_5.

To prove: S is Fuzzy modular.

Suppose S is not Fuzzy modular. S contain a Fuzzy sublattice isomorphic to N_5. This is a contradiction to our assumption $\Longrightarrow$ S is Fuzzy modular $\Longrightarrow$ S is not Fuzzy meet-semi distributive by theorem 3.1

V. REFERENCEs

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