Some Results on Fuzzy Semi-Super Modular Lattices

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Abstract: *In this Paper, we introduce Fuzzy Semi-super modular Lattice , their definition and some theorems. The Definitions of Fuzzy Semi-Super modular Lattice, Fuzzy super modular lattice and their Characterization theorems are given.*

Keywords*:* Fuzzy Modular Lattice, Fuzzy Distributive Lattice, Fuzzy semi- Super modular Lattice. Commutative property, Associative law.

Introduction: The Concept of Fuzzy Lattice was already introduced by Ajmal, N [1], S. Nanda [3] and WilCox, L. R [6] explained modularity in the theory of Lattices, Iqbalunnisa andVasantha, W. B, [7] explained by Super modular Lattices, G. Gratzer [2], M. Mullaiand B. Chellappa [4] explained Fuzzy L-ideal and V. Vinoba and K. Nithya [5] Explained fuzzy modular pairs in Fuzzy Lattice and Fuzzy Modular Lattice. A few of definitions and results are listed that the fuzzy Semi-Super modular lattice using in this paper we explain fuzzy Semi-Super modular lattice, Definition of fuzzy semi-Super modular lattice, Characterization theorem of Fuzzy Semi-Super modular Lattice and some examples are given.

Definition: A lattice L is said to be fuzzy semi-super modular if it satisfies the following identity.

$\mu(a+b)\,\mu(a+c)\,\mu(a+d)\,\mu(a+e)=\mu(a)+\mu(b)\,\mu(c)\,\mu(a+d)\,\mu(a+e)+\mu(b)\,\mu(d)\,\mu(a+c)\,\mu(a+e)+\mu(b)\,\mu(e)\,\mu(a+c)\,\mu(a+d)+\mu(c)\,\mu(d)\,\mu(a+b)\,\mu(a+e)+\mu(c)\,\mu(e)\,\mu(a+b)\,\mu(a+d)+\mu(d)\,\mu(e)\,\mu(a+b)\,\mu(a+c)$

For all a, b, c, d, e in L.

Theorem: 1.1

If L is a Fuzzy lattice which is not Fuzzy Semi-super modular then L contains a Fuzzy set of five elements $\mu(a_1)$, $\mu(b_1)$,

$\mu(c_1)$, $\mu(d_1)$, $\mu(e_1)$ such that.

$\mu(a\vee a_1)\wedge\mu(a\vee b_1)\wedge\mu(a\vee d_1)\wedge\mu(a\vee e_1)>\mu(a)$ while

$\neq$

$\mu(a)>\mu(b_1\wedge c_1)\wedge\mu(a\vee d_1)\wedge\mu(a\vee e_1), \mu(b_1\wedge d_1)\wedge\mu(a\vee c_1)\wedge\mu(a\vee e_1), \mu(b_1\wedge e_1)\wedge\mu(a\vee c_1)\wedge$

$\mu(a\vee d_1)$, $\mu(c_1\wedge d_1)\wedge\mu(a\vee b_1)\wedge\mu(a\vee e_1)$, $\mu(c_1\wedge e_1)\wedge\mu(a\vee b_1)\wedge\mu(a\vee d_1)$, $\mu(d_1\wedge e_1)\wedge\mu(a\vee b_1)\wedge\mu(a\vee c_1)$holds.

[$\mu(b_1)$, $\mu(c_1)$, $\mu(d_1)$, $\mu(e_1)$ being distinct $\mu(b_1)\neq\mu(c_1)$,

Otherwise $\mu(b_1)=\mu(b_1\wedge c_1)$and $\mu(a\vee b_1)=\mu(a)\vee\mu(b_1\wedge c_1)$

A contradiction as it will imply equality of $\mu(a\vee b_1)\wedge\mu(a\vee c_1)\wedge\mu(a\vee d_1)\wedge\mu(a\vee e_1)=\mu(a)$]

Proof:

Let L be a Fuzzy modular lattice which is not Fuzzy Semi-super modular. As L is not Fuzzy Semi-super modular there exists elements $\mu(x)$,$\mu(P)$,$\mu(Q)$,$\mu(R)$,$\mu(S)$ such that.

$\mu(x\vee P)\wedge\mu(x\vee Q)\wedge\mu(x\vee R)\wedge\mu(x\vee S)>\mu(x)\vee[\mu(P\vee Q)\wedge\mu(x\vee R)\wedge\mu(x\vee S)]\vee[\mu(Q\vee R)\wedge\mu(x\vee P)\wedge\mu(x\vee S)]$

$\neq$

$[\mu(R\vee S)\wedge\mu(x\vee P)\wedge\mu(x\vee Q)]$ (1)

$\mu(a)=\mu(x)\vee[\mu(P\vee Q)\wedge\mu(x\vee R)\wedge\mu(x\vee S)]$

∨[μ (Q∧R)∧μ(x∨P)∧μ(x∨S)]

∨[μ(R∨S)∧μ(x∨P)∧μ(x∨Q)]

μ (b_1) =μ (P)

μ (c_1) =μ (Q)

μ (d_1) =μ (R)

μ (e_1) = μ(S)

thenμ (a∨ b_1) = μ (a) μ (b_1)

=μ(x)∨[μ(P)∧μ(Q)∧μ(x∨R)∧μ(x∨S)]

∨[μ (Q)∧μ(R)∧μ(x∨P)∧μ(x∨S)]

∨[μ(R) ∧μ(S)∧ μ(x∨P)∧μ(x∨Q)]∧ μ(P)

μ (a∨ b_1) =μ(x)∨μ(P)

=μ(x∨P)

Similarly

μ (a∨ c_1) =μ (x∨Q)

μ (a∨ d_1) =μ (x∨R) and μ (a∨ e_1) =μ (x∨S)

So

μ (a∨ b_1) ∧μ(a∨ c_1)∧μ(a∨ d_1)∧μ(a∨ e_1)

=μ(x∨P)∧μ(x∨Q)∧μ(x∨R)∧μ(x∨S)

Applying equation (1) in above equation

μ (a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)∧μ(a∨ e_1)

≥ μ(x)∨[μ(P∧Q)∧μ(x∧R)∧μ(x∧S)]∨[μ(Q∧R)∧μ(x∧P)∧μ(x∧S)]≠∨[μ(R∧S)∧μ(x∧P)∧μ(x∧Q)]

μ (a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)∧μ(a∨ e_1)>μ(a)

Hence proved

Theorem 1.2

If L is a fuzzy modular lattice which is not a fuzzy semi-super modular then L contains a set of five elements μ(a),μ(b),μ(c),μ(d),μ(e) such that

μ (a∨b) = μ (a∨c) =μ (a∨d) =μ (a∨e)>μ (a)

≠

Further μ(a)>μ(b∧c),μ(b∧d),μ(b∧e),μ(c∧d),μ(c∧e),μ(d∧e)

Proof:

As L is not Fuzzy semi-super modular, then by previous theorem, we can assert the existence of the set of five elements μ(a),μ(b_1),μ(c_1),μ(d_1),μ(e_1)in L such that

μ (a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)∧μ(a∨ e_1)>μ(a) (1)

≠

And>μ (b_1)∧μ(c_1)∧μ(a∨ d_1)∧μ(a∨ e_1),

μ (b_1)∧μ(d_1)∧μ(a∨ c_1)∧μ(a∨ e_1),

μ (b_1)∧μ(e_1)∧μ(a∨ c_1)∧μ(a∨ d_1),

μ (c_1)∧μ(d_1)∧μ(a∨ b_1)∧μ(a∨ e_1),

μ (c_1)∧μ(e_1)∧μ(a∨ b_1)∧μ(a∨ d_1),

μ (d_1)∧μ(e_1)∧μ(a∨ b_1)∧μ(a∨ c_1).

Put μ (b) =μ (b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)∧μ(a∨ e_1)

μ(c) =μ (c_1)∧μ(a∨ b_1)∧μ(a∨ d_1)∧μ(a∨ e_1)

μ (d) =μ (d_1)∧μ(a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ e_1)

μ (e) =μ (e_1)∧μ(a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)

μ (a∨b)≥min {μ(a),μ(b)}

≥min{μ(a), μ(b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)∧μ(a∨ e_1)}

≥Min {μ(a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)∧μ(a∨ e_1)}

Since L is fuzzy modular.

>μ(a) by (1)

≠

Similarly

μ (a∨c)≥min {μ(a),μ(c)}

≥min{μ(a), μ(c_1)∧μ(a∨ b_1)∧μ(a∨ d_1)∧μ(a∨ e_1)}

≥min {μ(a∨ c_1)∧μ (a∨ b_1)∧μ(a∨ d_1)∧μ(a∨ e_1)}

Since L is fuzzy modular.

>μ (a) by (1)

≠

μ (a∨d)≥min {μ(a),μ(d)}

≥min{μ(a), μ(d_1)∧μ(a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ e_1)}

≥Min {μ(a∨ d_1)∧μ(a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ e_1)}

Since L is fuzzy modular.

>μ (a) by (1)

≠

μ (a∨e)≥min {μ(a),μ(e)}

≥min{μ(a), μ(e_1)∧μ(a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)}

≥Min {μ(a∨ e_1)∧μ(a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)}

Since L is fuzzy modular.

>μ (a) by (1)

≠

Now μ (b∧c)≥min {μ(b),μ(c)}

≥min{ μ(b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)∧μ(a∨ e_1),

μ (c_1)∧μ(a∨ b_1)∧μ(a∨ d_1)∧μ(a∨ e_1)}

By commutative property

≥min{μ(b_1)∧μ($c_1 \vee d_1$)∧μ(a∨ e_1)∧μ(c_1)∧μ($b_1 \vee d_1$)∧μ(a∨ e_1)}

By associative law,

≥min{μ(b_1)∧μ($c_1 \vee d_1$),μ(a∨ e_1)∧μ(c_1),μ($b_1 \vee d_1$)∧μ(a∨ e_1)}

≥min{μ(b_1)∧μ(a∨ e_1),μ(c_1),μ($b_1 \vee d_1$)∧μ(a∨ e_1)}

≥Min { μ(a∨ e_1)∧ μ(a∨ e_1), μ(b_1)∧ μ($b_1 \vee d_1$),μ(c_1)}

By commutative and associative law

≥Min {μ(a∨ e_1),μ($b_1 \vee c_1$)}

By idempotent, absorption law

=μ (a∨ e_1)∧μ($b_1 \wedge c_1$)

=μ (a)>μ ($b_1 \wedge c_1$)

μ (b∧d)≥min {μ(b),μ(d)}

≥min{ μ(b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)∧μ(a∨ e_1),

μ (d_1)∧μ(a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ e_1)}

By commutative property

≥min {μ(b_1)∧μ($c_1 \vee d_1$)∧μ(a∨ e_1)∧μ(d_1)∧μ($b_1 \vee c_1$)∧μ(a∨ e_1)}

By associative law,

≥min {μ(b_1)∧μ($c_1 \vee d_1$),μ(a∨ e_1)∧μ(d_1),μ($b_1 \vee c_1$)∧μ(a∨ e_1)}

≥min{μ(b_1)∧μ(a∨ e_1), μ(d_1),μ($b_1 \vee c_1$)∧μ(a∨ e_1)}

≥Min {μ(a∨ e_1)∧ μ(a∨ e_1), μ(b_1)∧ μ($b_1 \vee c_1$) , μ(d_1)}

By commutative and associative law

≥Min {μ(a∨ e_1),μ($b_1 \wedge d_1$)}

By idempotent, absorption law

=μ (a∨ e_1)∧μ($b_1 \wedge d_1$)

=μ (a)>μ ($b_1 \wedge d_1$)

μ (b∧e)≥min{μ(b),μ(e)}

≥min{ μ(b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)∧μ(a∨ e_1),

μ (e_1)∧μ(a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)}

By commutative property,

≥min{ μ(b_1)∧μ(c_1 ∨ e_1)∧μ(a∨ d_1)∧ μ(e_1)∧μ(b_1 ∨ c_1)∧μ(a∨ d_1)}

By associative law

≥min{ μ(b_1)∧μ(c_1 ∨ e_1),μ(a∨ d_1)∧ μ(e_1),μ(b_1 ∨ c_1)∧μ(a∨ d_1)}

≥min{ μ(b_1)∧μ(a∨ d_1), μ(e_1),μ(b_1 ∨ c_1)∧μ(a∨ d_1)}

≥min{μ(a∨ d_1) ∧μ(a∨ d_1), μ(b_1)∧μ(b_1 ∨ c_1), μ(e_1)}

By commutative and associative law

≥min{μ(a∨ d_1),μ(b_1 ∧ e_1)}

By idempotent, absorption law

= μ (a∨ d_1)∧μ(b_1 ∧ e_1)

=μ (a)>μ(b_1 ∧ e_1)

μ(c∧d)≥min{μ(c),μ(d)}

≥min{ μ(c_1)∧μ(a∨ b_1)∧μ(a∨ d_1)∧μ(a∨ e_1),

μ (d_1)∧μ(a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ e_1)}

By commutative property,

≥min{ μ(c_1)∧μ(b_1 ∨ d_1)∧μ(a∨ e_1)∧ μ(d_1)∧μ(b_1 ∨ c_1)∧μ(a∨ e_1)}

By associative law

≥min{ μ(c_1)∧μ(b_1 ∨ d_1),μ(a∨ e_1)∧μ(d_1),μ(b_1 ∨ c_1)∧μ(a∨ e_1)}

≥min{ μ(c_1)∧μ(a∨ e_1),μ(d_1),μ(b_1 ∨ c_1)∧μ(a∨ e_1)}

≥min {μ(a∨ e_1)∧ μ(a∨ e_1), μ(d_1),μ(b_1 ∨ c_1)∧μ(c_1)}

By commutative and associative law

≥min{μ(a∨ e_1),μ(c_1 ∧ d_1)}

By idempotent, absorption law

= μ (a∨ e_1)∧μ(c_1 ∧ d_1)

=μ (a)> μ(c_1 ∧ d_1)

μ(c∧e)≥min{μ(c),μ(e)}

≥min{ μ(c_1)∧μ(a∨ b_1)∧μ(a∨ d_1)∧μ(a∨ e_1),

μ (e_1)∧μ(a∨ b_1)∧μ(a∨ c_1)∧μ(a∨ d_1)}

By commutative property

≥min{ $\mu(c_1)\wedge\mu(b_1 \vee e_1)\wedge\mu(a\vee d_1)\wedge\mu(e_1)\wedge\mu(b_1 \vee c_1)\wedge\mu(a\vee d_1)$}

By association law

≥min{ $\mu(c_1)\wedge\mu(b_1 \vee e_1)$,$\mu(a\vee d_1)\wedge\mu(e_1)$,$\mu(b_1 \vee c_1)\wedge\mu(a\vee d_1)$}

≥min{ $\mu(c_1)\wedge\mu(a\vee d_1)$,$\mu(e_1)$,$\mu(b_1 \vee c_1)\wedge\mu(a\vee d_1)$}

≥min{$\mu(a\vee d_1) \wedge\mu(a\vee d_1)$, $\mu(c_1)\wedge\mu(b_1 \vee c_1)$, $\mu(e_1)$}

By commutative and associative law

≥min{$\mu(a\vee d_1)$,$\mu(c_1 \wedge e_1)$}

By idempotent, absorption law

= $\mu(a\vee d_1)\wedge \mu(c_1 \wedge e_1)$

=$\mu(a) > \mu(c_1 \wedge e_1)$

μ (d∧e)≥min{μ(d),μ(e)}

≥min{ $\mu(d_1)\wedge\mu(a\vee b_1)\wedge\mu(a\vee c_1)\wedge\mu(a\vee e_1)$,

$\mu(e_1)\wedge\mu(a\vee b_1)\wedge\mu(a\vee c_1)\wedge\mu(a\vee d_1)$}

By commutative property

≥min{ $\mu(d_1)\wedge\mu(b_1 \vee e_1)\wedge\mu(a\vee c_1)\wedge\mu(e_1)\wedge\mu(b_1 \vee d_1)\wedge\mu(a\vee c_1)$}

By associative law

≥min{ $\mu(d_1)\wedge\mu(b_1 \vee e_1)$,$\mu(a\vee c_1)\wedge\mu(e_1)$,$\mu(b_1 \vee d_1)\wedge\mu(a\vee c_1)$}

≥min{ $\mu(d_1)\wedge\mu(a\vee c_1)\wedge\mu(e_1)$,$\mu(b_1 \vee d_1)\wedge\mu(a\vee c_1)$}

≥min{ $\mu(a\vee c_1) \wedge\mu(a\vee c_1)$, $\mu(d_1)\wedge \mu(b_1 \vee d_1)$, $\mu(e_1)$}

By commutative and associative law

≥min{ $\mu(a\vee c_1)$,$\mu(d_1 \wedge e_1)$}

By idempotent, absorption law

= $\mu(a\vee c_1)\wedge\mu(d_1 \wedge e_1)$

=$\mu(a) > \mu(d_1 \wedge e_1)$.

Theorem: 1.3

If a fuzzy lattice L is fuzzy semi-super modular then for μ(b)≥μ(c) ,μ(d)≥μ(e) and μ(a∨b)=μ(a∨c), μ(a∧b)=μ(a∧c) and μ(a∨d)=μ(a∨e),μ(a∧d)=μ(a∧e)for any μ(a) imply μ(c)=μ(d) and μ(d)=μ(e).

Proof:

Given L is a fuzzy semi-super modular lattice, and for μ(b)≥μ(c) ,μ(d)≥μ(e) and μ(a∨b)=μ(a∨c), μ(a∧b)=μ(a∧c) and μ(a∨d)=μ(a∨e),μ(a∧d)=μ(a∧e)for any μ(a)

To prove μ(c) =μ (d) and μ(d)=μ(e).

L is a fuzzy semi-super modular lattice

Then L is a fuzzy modular lattice, by the theorem

Every modular lattice is semi-super modular lattice.

Hence we have, L is a fuzzy modular lattice, and for μ(b)≥μ(c) ,μ(d)≥μ(e) and μ(a∨b)=μ(a∨c), μ(a∧b)=μ(a∧c) and μ(a∨d)=μ(a∨e),μ(a∧d)=μ(a∧e)for any μ(a)

⇒ μ(b) = μ (c)and μ(d)=μ(e).

[Fuzzy dual of the fuzzy modular lattice is fuzzy modular]

Every fuzzy modular lattice is fuzzy semi-super modular lattice.

Proof:

Follow from the theorem

In any fuzzy lattice L the following are equivalent.

(i)μ(a∨b) ∧μ (a∨c)=μ (a) ∨[μ (b) ∧μ (a∨c)]

(ii)μ (a∧b) ∨μ (a∧c)=μ (a) ∧[μ (b) ∨μ (a∧c)]

∀ μ (a), μ (b), μ (c) in L.

Proof: (i) ⟹(ii)

Let μ (a), μ (b), μ (c) in L be arbitrary, then

μ (a ∧b) ∨μ (a ∧c)≥ min{ μ (a ∧b), μ (a ∧c)}

≥min{ μ (a ∧c), μ (a∧ b)}, by commutative law

≥min{ μ (a ∧c) ∨μ (a), μ (a∧ c) ∨μ (b)}, by (i)

≥min{ μ (a) ∨μ (a ∧c), μ (b) ∨μ (a ∧c)}, by commutative law

≥min{ μ (a), μ (b) ∨μ (a ∧c)}, by absorption law

=μ (a)∧ [μ (b) ∨μ (a ∧c)]

Hence μ (a ∧b) ∨μ (a ∧c)=μ (a) ∧[μ (b) ∨μ (a ∧c)],

for all μ (a), μ (b), μ (c) in L.

(ii) ⇒(i)

Let μ (a), μ (b), μ (c) in L be arbitrary

μ (a∨ b)∧μ (a ∨c) ≥min{ μ (a ∨b), μ (a ∨c)}

≥min{ μ (a∨ c), μ (a ∨b)}, by commutative law

≥min{ μ (a ∨c) ∧μ (a), μ (a∨ c) ∧μ (b)}, by (ii)

≥min{ μ (a)∧ μ (a ∨c), μ (b) ∧μ (a ∨c)},

By commutative law

≥min{ μ (a), μ (b) ∧μ (a ∨c)}, by absorption law

=μ (a) ∨[μ (b) ∧μ (a ∨c)]

Hence μ (a ∨b) ∧μ (a ∨c)=μ (a)∨ [μ (b) ∧μ (a ∨c)], for all μ (a), μ (b), μ (c) in L.

Proof:

Follow from the theorem

In any fuzzy lattice L the following are equivalent.

(iii)$\mu(a \vee d) \wedge \mu (a \vee e) = \mu (a) \vee [\mu (d) \wedge \mu (a \vee e)]$

(iv)$\mu (a \wedge d) \vee \mu (a \wedge e) = \mu (a) \wedge [\mu (d) \vee \mu (a \wedge e)]$

$\forall$ $\mu (a), \mu (d), \mu (e)$ in L.

Proof : (iii) $\Longrightarrow$(iv)

Let $\mu (a), \mu (d), \mu (e)$ in L be arbitrary, then

$\mu (a \wedge d) \vee \mu (a \wedge e) \geq \min\{ \mu (a \wedge d), \mu (a \wedge e)\}$

$\geq \min\{ \mu (a \wedge d), \mu (a \wedge e)\}$, by commutative law

$\geq \min\{ \mu (a \wedge d) \vee \mu (a), \mu (a \wedge e) \vee \mu (d)\}$, by (iii)

$\geq \min\{ \mu (a) \vee \mu (a \wedge e), \mu (d) \vee \mu (a \wedge e)\}$, by commutative law

$\geq \min\{ \mu (a), \mu (d) \vee \mu (a \wedge e)\}$, by absorption law

$= \mu (a) \wedge [\mu (d) \vee \mu (a \wedge e)]$

Hence $\mu (a \wedge d) \vee \mu (a \wedge e) = \mu (a) \wedge [\mu (d) \vee \mu (a \wedge e)]$,

for all $\mu (a), \mu (d), \mu (e)$ in L.

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(iv) $\Rightarrow$(iii)

Let $\mu (a), \mu (d), \mu (e)$ in L be arbitrary

$\mu (a \vee d) \wedge \mu (a \vee e) \geq \min\{ \mu (a \vee d), \mu (a \vee e)\}$

$\geq \min\{ \mu (a \vee e), \mu (a \vee d)\}$, by commutative law

$\geq \min\{ \mu (a \vee e) \wedge \mu (a), \mu (a \vee e) \wedge \mu (d)\}$, by (iv)

$\geq \min\{ \mu (a) \wedge \mu (a \vee e), \mu (d) \wedge \mu (a \vee e)\}$,

By commutative law

$\geq \min \{ \mu (a), \mu (d) \wedge \mu (a \vee e)\}$, by absorption law

$= \mu (a) \vee [\mu (d) \wedge \mu (a \vee e)]$

Hence $\mu (a \vee d) \wedge \mu (a \vee e) = \mu (a) \vee [\mu (d) \wedge \mu (a \vee e)]$, for all $\mu (a), \mu (d), \mu (e)$ in L.

Theorem: 1.4

If L is a Fuzzy lattice, for $\mu (a) \geq \mu (b)$, $\mu (a \vee c) = \mu (b \vee c)$ and $\mu (a \wedge c) = \mu(b \wedge c)$ forany $\mu (c)$ imply $\mu (a) = \mu (b)$. Then L is Fuzzy modular but not a Fuzzy semi-super modular lattice.

Proof:

First we shall prove fuzzy super modular lattice

Given L is a Fuzzy lattice, for μ (a) ≥μ (b), μ (a ∨c)=μ (b ∨c) and μ (a ∧c)= μ(b ∧c) forany μ (c) imply μ (a)=μ (b).

Then by the equivalent theorem 1.3

(i)μ(a∨b) ∧μ (a∨c)=μ (a) ∨[μ (b) ∧μ (a∨c)]

(ii)μ (a∧b) ∨μ (a∧c)=μ (a) ∧[μ (b) ∨μ (a∧c)]

L is a Fuzzy modular lattice.

⇒L has a Fuzzy sublattice isomorphic to M4 or M3, 3.

⇒L is not a Fuzzy supermodular lattice, by theorem

(A Fuzzy modular lattice L is Fuzzy super modular if and only if it does not contain a fuzzy sub lattice isomorphic to either M4 or M3, 3).

By the lemma,

Any fuzzy super modular lattice is fuzzy semi-super modular lattice.

Conclusion: This paper is proved that If L is a Fuzzy lattice which is not Fuzzy Semi-super modular then L contains a Fuzzy set of five elements μ (a_1), μ (b_1), μ (c_1), μ (d_1), μ(e_1) such that.

μ (a∨ a_1)∧μ(a∨ b_1) ∧ μ$(a \vee d_1)$ ∧ μ$(a \vee e_1) >$ μ(a) while

≠

μ (a)>μ$(b_1 \wedge c_1)$∧ μ(a∨ d_1) ∧ μ$(a \vee e_1)$, μ$(b_1 \wedge d_1)$

∧ μ$(a \vee c_1)$ ∧ μ$(a \vee e_1)$, μ$(b_1 \wedge e_1)$ ∧ μ$(a \vee c_1)$ ∧

μ (a∨ d_1), μ $(c_1 \wedge d_1)$ ∧μ (a∨b_1) ∧μ $(a \vee e_1)$, μ $(c_1 \wedge e_1)$∧

μ$(a \vee b_1)$∧μ(a∨ d_1),μ$(d_1 \wedge e_1)$∧μ$(a \vee b_1)$∧μ(a∨ c_1)holds.

[μ (b_1), μ (c_1), μ (d_1), μ (e_1) being distinct μ(b_1)≠ μ(c_1),

Otherwise μ (b_1)= μ$(b_1 \wedge c_1)$ and μ(a∨ b_1)=μ(a)∨ μ$(b_1 \wedge c_1)$ A contradiction as it will imply equality of

μ(a∨ b_1)∧ μ(a∨ c_1)∧μ(a∨ d_1)∧μ(a∨ e_1)=μ(a)], If L is a fuzzy modular lattice which is not a fuzzy semi-super modular then L contains a set of five elements μ(a),μ(b),μ(c),μ(d),μ(e) such that

μ (a∨b) = μ (a∨c) = μ (a∨d) = μ (a∨e)>μ (a)

≠

Further μ(a)>μ(b∧c),μ(b∧d),μ(b∧e),μ(c∧d),μ(c∧e),μ(d∧e), If a fuzzy lattice L is fuzzy semi-super modular then for μ(c)≥μ(d) and μ(c∨e)=μ(d∨e), μ(c∧e)=μ(d∧e) for any μ(e) imply μ(c)=μ(d).

If L is a Fuzzy lattice, for μ (a) ≥μ (b), μ (a ∨c) =μ (b ∨c) and μ (a ∧c) = μ (b ∧c) forany μ (c) imply μ (a) =μ (b). Then L is Fuzzy modular but not a Fuzzy semi-super modular lattice.

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