RESTRAINED LICT DOMINATION IN JUMP GRPHS

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ABSTRACT: A SET $D_r \subseteq V$ [n(J(G))] is a restrained dominating set of n[J(G)] where every vertex in V[n(J(G))] - D_r is adjacent to a vertex in D_r as well as vertices in V[n(J(G))] – D_r .

The restrained domination of lict jump graph n(J(G)) denoted by $\surd_m$(J(G)) is the minimum cardinality of a retrained dominating set of n(J(G)). In this paper we study its exact values for some standard graphs we obtain. Also it relation with other parameters is investigated.

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1. INTRODUCTION:

In this paper, all the graphs considered here are simple finite, nontrivial and connected. As usual p and q denotes the number of vertices and edges of a jump graph J(G). In this paper for any undefined terms or notations can be found in Harary [4]

As usual, the maximum degree of vertices I J(G)is denoted by Δ(J(G)).

The degree of an edge e=uv of J(G) is defined as ege = degu +degv – 2 and δ'(J(G))

(Δ'J(G)) is he Minimum(maximum)degree among the edges of J(G).

For any real number, x ⌈ x ⌉ denotes the smallest integer not less than x and ⌊ x ⌋ denotes the greatest integer not greater thn x. The complement J($\bar{G}$)of a jump graphJ(G) has V as its vertex set , but two vertices are adjacent in J($\bar{G}$) is they are non adjacent in J(G)..

A vertex (edge) cover it graph J(G)is a set of vertices that cover all the edges (vertices) of J(G). The vertex(edge) covering number α_0(JG)) (α_1(J(G))) is a minimum cardinality of a vertex(edge) cover in β_0(J(G)) (β_1(J(G))) is the maximum cardinality of independent se of vertices (edges) in J(G).

The greatest distance between any two vertices of a connected graph J(G) is called the diameter of J(G) and is denoted by diam (J(G)).

We begin by recalling some standard definition from domination theory.

A set D of a graph G= (V, E) is dominating set if every vertex in V – D is adjacent to some vertex in D. The domination number √(G) of graph G is the minimum cardinality of a minimal dominating set in G . The study of domination in graph was begun by Ore[7] and Berge[1].

A set D ⊆ V(L(G)) is dominating set of L(G) I every vertex not in D is adjacent to a vertex in D. The domination number of L(G) is denoted by √ (L(G)) is the minimum cardinality of dominating set in L(G).

A set F of edges in a graph G is called an edge dominating set of G. if every edge in E – F where E is the set of G if every edge in G is adjacent to at least one edge in F.
The edge domination number √'(G) of a graph G is the minimum cardinality of an edge dominating set of G.

The concept of edge domination number in graph were studied by Gupta[3]and S.Mitchell and S.T. Hedetineim[6]

A Set D of graph J(G)=(V, E) is a dominating set every vertex in V(J(G)) – D is adjacent to some vertex in D. The domination number √(J(G)) of J(G) is the minimum cardinality of a minimal dominating set in J(G).

Analogously we define restraind domination in lict graph as follows;

A dominating set D_r of lict graph is a restrained dominating set if every vertex not in D_r and adjacent in D_r and to a vertex in V – D_r. The restrained domination number of lict graph n(G) denotd by $\surd_m(G)$. Is the minimum cardinality of a restrained dominating set of n(G). The concept of restrained domination in graph was introduced by Domke et.al[2].

A dominating set D_r of lict jump graph is a restrained dominating set . I every vertex not in Dr is adjacent in D and to a vertex in V (J(G)) – D_r. The restrained domination number of lict jump graph n(J(G)) denoted by $\surd_m(J(G)))$ is the minimum cardinality of a restrained dominating set of n(J(G)).

In this paper many bounds on $\surd_m(J(G))$ are obtained and expressed in terms vertices, edges of J(G), but not the elements of n(J(G)) and express the results with other different domination parameter of J(G).

2. Results;

We need the following Theorems to establish our further results.

Theore A[5]: For any connected (p, q)graph G

$$\surd'(G) \geq \left\lceil \frac{q}{\Delta'(G) + 1} \right\rceil$$

Theorem B[5] if G is a graph with no isolated vertex then $\surd'(G)(\leq q - \Delta'(G)$.

Initially we begin with restrained domination number of Lict jump graph of some standard graphs which are straight forward in the following theorem.

Theorem 1.

i) For any cycle C_n with n≥3 vertices
$\surd_m(J(C_n)) = n - 2 \lfloor \frac{n}{3} \rfloor$

ii) For any path p_n with n>2 vertices $\surd_m(J(P_{2n-1})) = k$
$\surd_m(J(P_{2n}) = k$ when n = 2,3,4. 5….. then k= 1, 2, 3, 4,……

iii) For any star $K_{1,p}$ where p ≥ 2 vertices $\surd_m(J(K_{1,p})) = 1$

iv) For any wheel W_p with p ≥ 4 vertices $\surd_m(J(W_p)) = 1 + \lceil \frac{p-3}{3} \rceil$

v) For any complete graph K_p with p≥3 vertices $\surd_m((K_p)) = \lfloor \frac{p}{2} \rfloor$

In the following theorem we etblish the upper bound for $\surd_m(J(T))$ in terms of vertices of the J(G)

Theorem 2: For any tree T with p>2 vertices m end vertices $\surd_m(J(T)) \leq p - m$. Equality holds ifT = $K_{1,p}$ with p ≥2 vertices .

Proof: If diam(J(G)) ≤ 3, then the result is obvious, Let diam(J(T))>3 and $V_1 = \{v_1, v_2, v_3 \ldots\ldots v_p\}$ be set of all end vertices of J(T) where v_1= = m Further E={e_1, e_2,e_3,……e_q} C= {c_1,c_{2+},c_3,……..c_i} be the set of edges and cut vertices in J(G). In N(J(G)), V(n(J(G)) = E(J(G))∪ C(J(G)) and in J(G). ∀ e_i incident with c_i 1≤j ≤ I forms a complete induced subgraph as a block in n(J(G)) such that the number of blocks in n(J(G)) = | C |. Let { e_1,e_2,$e_{3=}$,……… e_j} in n(J(G)). Let C_1' ≤ C' be a restrained dominating set in n(J(G)) such that | C' | ≤$\surd_m(J(G))$ for any non trivialtree p>q and |C"| ≤ p – m which gives $\surd_m(J(T)) \leq p - m$ which gives $\surd_m(J(T)) \leq p - m$.
Further equality hods if T = $K_{1,p}$ then n(J($K_{1,p}$))= K_{p+1} and $\surd_m$ (J($K_{1,p}$))= p – m.
The following corollaries are immediate from the above theorem.
Corollary 1; for any connected (p,q)jump gaph J(G)
$\surd_m(J(G)) + \surd(J(G) \leq \alpha_0(J(G)) + \beta_0(J(G))$. Equality holds if J(G) is isomorphic toJ(C_3) or J(C_5).

Corollary 2; For any connected (p,q) jump graph J(G)
$\surd_m$(J(G)) + $\surd$(J(G) ≤ α_1(J(G))+ β_1(J(G)) equality holds if J(G) is isomorphic to J(C_3) of J(C_5)

Theorem 3 .; For any connected (P,q) jmp graph J(G) with p>2 vertices $\surd_m$(J(G)) ≤ $\lceil \frac{p}{2} \rceil$, equality holds if J(G) is J(C_4) or J(C_5) or J(C_8) or K_p if p is even.

Proof:Let E={e_1, e_2 e_3,………e_p} be the edge set of J(G) such that V[n9J(G))]=E(J(G)) ∪ C(J(G)) by definition of lict jump graphwhere C(J(G)) is the set of cutvertices in J(G). LetD_r = {v_1,v_2,………..v_n} ⊆ V[n(G)] be the rstrained dominatingset of n(G). Suppose if |V[n(G)- D_r|≥ 2, then {V[n(G)- D_r} contains atleast two vertices which gives $\surd_m$(G) < $\frac{p}{2}$≤ $\lceil \frac{p}{2} \rceil$
For the quality, i) If J(G) is isomorphicto J(C_4) or J(C_5) or J(C_8) For any cycle C_p with p≥3 vertices n(J(C_p)) = C_p which gives |D_r| = $\lceil \frac{p}{2} \rceil$
Therefore $\surd_m$(J(C_p))= $\lceil \frac{p}{2} \rceil$
ii) if J(G) is isomorphic to J(K_p) where p is even then byTheorem1, $\surd_m$(J(K_p)) = $\lceil \frac{p}{2} \rceil$
In the followed by Theorem, we obain the relation between $\surd_m$(J(G)) and diameter of J(G).

Theorem 4; For any connected (p,q) jump graph J(G)

$\surd_m$(J(K_p)) ≥ $[\frac{diam(J(G)+1}{3}]$
proof: Let D_r be restrained dominating set of n(J(G)) such that |D_r| = $\surd_m$(J(G)) consider an arbitrary path of length which is a diam (J(G)). This diamaterial path induces at most three edges from the induced subgraph < N(V)> for each v ∈ D_r Further more since D_r is $\surd_m$-set.
The dia meterial path induces at most $\surd_m$(J(G)) – 1 dges joining the neighbor hood of the vertices of D_r
Hence diam(J(G)) ≤ 2$\surd_m$(J(G)) + $\surd_m$(J(G)) – 1
Hence diam(J(G)) ≤ 3$\surd_m$(J(G)) – 1 Hence the result follows
The following theorem results domination number of J(G)and restrained domination number n(J(G))).

Theorem5: For any (p,q) jump graph J(G) with p≥3 vertices

$\surd_m$(J(G)) ≤ p - $\surd$(J(G))
Equality holds if J(G) ≅ $J(C4) Or\ j(C5)$.
Proof: Let D = {u_1, u_2,u_3……….u_n } be a minimal dominating set of n(J(G)) such that | D | =$\surd$(J(G)). Further let F_1={ e_1 e_2,e_3 e_4…………e_n} be the set of all edges which are incident to the vertices of D and F_2 = E(J(G)) – F_1.
Let C = { c_1, c_2,……c_n} be the cutvertex set o J(G). By definition of Lict jump graph
V[n(J(G)) = E(J(G)) ∪ C(J(G)) and F_1 <⊆ V[n(J(G)] Let I_1 ={ e_1, e2, e3……e_k}; 1≤k≤I where I_1⊆ F_1 and I_2⊆ F_2 since each induced subgraph which is complete in n(J(G))may contain at least one vertex of either F_1 or F_2. Then (I_1 U I_2) forms a minimalrestrained dominating set in n(J(G)) such that | I_1 U I_2 | = |D_r|=$\surd_m$(J(G)). Clearly |D| U | I_1 U I_2|≤p
Thus it follows that $\surd$(J(G)) + $\surd_m$(J(G)) ≤ p.
For equality If G ≅ C_p for p=4 or 5 then by definition of lict jump graph n(J(C_p)≅ C_p. Then in this case
|D| U | D_r|=$\frac{p}{2}$ clealy it follows that $\surd_m$(J(G)) + $\surd$(J(G)) ≤ p.
For equality If J(G) = J(C_p) forp=4 or 5 then by definitionof Lict jump graph n(J(C_p))≅ J(C_p), Then in this case |D| = |D_r|=$\frac{p}{2}$ clearly it follows that $\surd_m$(J(G)) + $\surd$(J(G)) = p
In[5] they related $\surd$'(J(G) with the line domination of G. In the following theorem we establish our result with edge domination of J(G)

Theorem 6: For any non trivial connected (p,q) jump graph J(G).

$\surd_m$(J(G)) ≤ $\surd$' (J(G))
Proof: Let E= {e_1, e_2, e_3……….e_n be the edge set of J(G and C = {c_1,c_2, c_3,….c_n} be the set of cut vertices in J(G)
$\surd$[n(J(G))]=E(JG)) U C(J(G)) Let F={e_1,e_2 e_3…..e_n} ∀ e_i where 1≤ i≤n be the minimal edge dominating set of J(G) such that |F|= $\surd$' (J(G)). Since E(J(G))≤$\surd$[n(J(G))], every edge

$e_i \in F$: $\forall e_i$ $1 \leq i \leq n$ forms a dominating set in n(J(G)). Suppose$F_1 = E(J(G)) - F \subseteq \surd[n(J(G))$, we consider $I_1 = \{e_1, e_2, e_3, ...e_n\}$ $\forall$ $1 \leq k \leq n$ whee $I_1 \subseteq F$ and $I_2 \subseteq F_1$. Since eachinduces sub graph which is completein n(J(G)) may contain at least one vertex of either F or F_1 Then
| I_1 UI_2 |forms a minimal restrained dominating set in n(J(G)) clearly it follows that |F| ⊆ |I_1 U I_2| in n(J(G)) Hence √'(J(G)) ≤ √m(J(G)).In the next theorem we obtain the relation between domination number of J(G) and restrained domination number of n(J(G)) in terms of vertices and diameter of J(G).

Theorem 7: For connectd (p,q) jump gaph J(G) with p≥2 vertices $\surd_m(J(G)) \leq p + \surd(J(G) - diam(J(G))$

Proof; LetV= {v_1,v_2,v_3,…….} be the set of vertices in J(G).
Suppose there exists two vertices u,v ∈ V(J(G) such that dist(u,v)=diamJ(G) Let D ={v_1,v_2,……..v_p} 1 ≤p≤n e a minimal dominating set in n(J(G)). Now we consider F={e_1,e_2,e_3,…….e_n}; F⊆E(J(G)) and ∀e_i ∈ V[n(J(G))] 1 ≤i ≤n
In n(J(G)). Then V[n(J(G))] = E(J(G)) UC(J(G)) where C(J(G))is the set of cut vertices in J(G) suppose F_1, C_1 are the subsets of F and C. then there exists a set {M}∈ V[F_1(J(G))]- {F_1 UC_1} such that <M> has no isolates. Cclealy |F_1 UC_1| = $\surd_m$(J(G)) let u,v∈ V(J(G)) d(u,v)=diam(J(G)) then{F_1UC_1} U diam(J(G))< p U |D| Hence $\surd_m$(J(G)) + diam(J(G))≤ p + √(J(G)) which implies
$\surd_m J(G)) \leq p + \surd(J(G)) - diam\ (J(G)$.

Theorem 8 For any connected (p,Q) jump graph J(G)with p>2 vertices

$\surd_m(J(G)) \leq \alpha_0(J(G))$.
Proof; Let B={v_1,v_2,v_3……..v_m} ⊂ V(J(G))be the minimum number of vertices which covers all the edges such that |B|=α_0(J(G)) and E_1={e_1,e_2,e_3,…..e_k} ⊂ E(J(G)) such that
∀ v_i ϵ B; 1 ≤ I ≤ n is incident with e_i,for 1 ≤ I ≤ n we consider the following case;
case(i); suppose for any two vertices v_1, v_{2+} ∈ B and v_1 ∈N9v_2) then an edge e incident with $v_1$and v_2 overs all edges incident withv_1nd v_2.Hence e belongs to v_m-set of J(G).Further for any vertex v_i∈ B covering the edge e∈ E_1 incident with a vertex v_i of J(G) e_i belongs to the set $\surd_m$ set of J(G). Thus $\surd_m(J(G)) \leq |B| = \alpha_0(J(G))$
case(ii) Suppose for any two vertices v_1,v_2∈ B and v∉N(v_2). Then e_1,e_2∈ E_1 covers all the edges incident with v_1 and v_2. Since B consist of the vertices which covers the edges that are incident all the cut vertices of J(G), the corresponding edgs in E covers the cu vertices of J(G).
Thus $\surd_m(J(G)) \leq |B| = \alpha_o J(G)$.
Next we obtain a bound of retrained lict domination number in terms of number of edges and maximum edges degree of J(G).

Theorem 9: For any connected (p,q) jump grph J(G) with p≥ 3 $\surd_m(J(G)) \leq q - \Delta'(J(G))$.

Proof; we consider the following cases,
Case i) Suppose J(G) is non separale using theorem 6 and theorem B the resul follows
Case ii) suppose J(G) is separable Let be anedge with degree Δ′ and M be the set of edges adjacent to e in J(G)
Then E(J(G)) – M covers all the edges and all the cut vertices of J(G_. But some of the e_i' s ∈ E(J(G)) – M for
1 ≤ I ≤ n forms a minimal restrained dominating set in n(J(G)).
$\surd_m(J(G)) \leq |\ E(J(G)) - M\ |$ which gives
$\surd_m(J(G)) \leq q - \Delta'(J(G))$.

Theorem 10 ; For any connected graph J(G) with p>2 vertices

$\surd_m(J(G)) \leq q - \surd[L(J(G))]$
Proof: Let E ={e_{1+},e_2,……..e_n} be the edge set of J(G) and
C ={c_1,c_2,c_3……c_n} be the cutvertex set of J(G) then V[n(J(G))] =E(J(G)) U C(J(G)) and V[L(J(G))] = E(J(G) by definition suppose M={u_1,u_2,……u_n}⊆ V[L(J(G)] be the set of vertices of degree,deg(u_i)≥2, 1≤ I ≤ n, then D' ⊆ H forms minimal dominating set of L(J(G)) such that |D'| = √[L(J(G))]
Further let H'= {u_1', u_2'…….u_i'} ; 1≤ I ≤ n, where H'⊆ H then H' U D' forms a minimal restrained dominating set in n{J(G)].Sin V[L(J(G))] =E(J(G))=q and lso V[L(J(G))] ⊆V[n(J(G))] Clearly it follows that
| D' U H'| U |D'| ≤ q Thus $\surd_m(J(G)) + \surd[L(J(G))] \leq q$
We gives the following observations;

Observation 1; For a connected (p,q) graph J(G) $\surd_m$(J(G)) ≤ q – 2.
Proof ;Suppose D_r is a restrained dominating set of n(J(G)).Then by definition of restrained domination [√(n(J(G))] ≥ 2, Further by definition of n(J(G)). q - $\surd_m$ (J(G)) ≥ 2 Clearly it follows that $\surd_m$(J(G))≤q – 2 .
Observation 2: Suppose D_r be any restrained dominating set of n(J(G)), such that |D_r| =$\surd_m$(J(G))
Then | √[n(J(G)) + D_r]| ≤ $\sum_{vi\in Dr} \deg vi$
Proof: Since every vertex in √[n(J(G))] + D_r is adjacent to at least one vertex in √[n(J(G))] + D_r contributes at least one of the sum of degrees of vertices of D_r. Hence the proof.

Theorem 11: For any connected (p,q) jump graph J(G)

$\frac{q}{\Delta'(J(G))+1}$ ≤ $\surd_m$(J(G) ≤ q – δ'(J(G)).
Proof: let e ∈E(J(G)), now without loss of generality by definition of lict gaph
$e = u$ ∈ √[n(J(G))]and let d_r be the restrained dominting set of n(J(G)) such that |D_r|=$\surd_m$(J(G)). If δ(J(G)) ≤ 2, then by observation 1 . $\surd_m$(J(G))≤ q – 2 ≤ q – δ'(J(G)). If δ'(J(G)).≥ 2 then for any edge f ϵ N 9e0and by definition of n(J(G)) f = w ϵN(J(G)). D_r ⊆ {[V(n(J(G))] – N(J(G))} U {w}
Then $\surd_m$(J(G)) ≤ [q – (δ'(J(G)) + 1) + 1] = q – δ'(J(G)).
Now for the lowe bound we have by observation 2 and the fact that any edge e ∈ E(J(G)) and degree ≤ Δ'(J(G)) we have ,
q -$\surd_m$(J(G)) ≤ | V(n(J(G)) + n(J(G)) | ≤ $\sum_{v\in Dr} \deg v$ ≤ $\surd_m$(J(G)) . Δ'(J(G))
there fore $\frac{q}{\Delta'(J(G))+1}$ ≤ $\surd_m$(J(G)).

Theorem 12: For any connected non trivial (p,q) graph J(G) $\surd_m$(J(G)).≥ $\frac{q}{\Delta'(J(G))+1}$

Proof: Using theorem 6 and Theorem A the result follows.
Finally we obtain the Nordhus -Gaddum type result.
Theorem 13: Let J(G) be a connected (p,q) jump graph such that J(G) and J($\overline{G}$) are connected then
i) $\surd_m$(J(G)) + $\surd_m$(J($\overline{G}$)) ≥ $\lceil \frac{p}{2} \rceil$
ii) $\surd_m$(J(G)) . $\surd_m$(J($\overline{G}$)) ≥ $\lceil \frac{3p}{2} \rceil$

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