ANALYTICAL AND NUMERICAL VALIDATION OF TRUNCATED CELLULAR LATTICE STRUCTURE WITH VARIOUS STRUT DIAMETERS

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Abstract – *To study to mechanical behavior of a lattice structure, it is required to consider a unit cell, which after being repeat in space then we are considering truncated cube lattice structure. The porous bio materials of mechanical properties made from the relative new unit cell, which is namely truncated cube. We present analytical solutions that relate the dimension of the repeat unit cell to the flow ratio of the elastic modulus Poisson and the buckling load of these porous structures. Analytical and numerical validation of truncated cellular lattice structure with various strut diameters of unit cell. Here Ansys work bench is using for numerical results. The Elastic of modulus, Poisson's ratio, Yield stress and buckling of truncated cube structures are obtain as functions of unit cell dimensions of Properties of unit cell .The beam theory used for obtaining the analytical relationships was Euler Bernoulli. In addition to the modulus of elasticity and Poisson's ratio of the porous structures, it has been found that they depend heavily on the ratio of the length of the inclined struts to that of the non-inclined vertical or horizontal struts Evaluation between the numerical and also analytical values of the truncated cellular lattice structure is good agreement between both the numerical and also analytical results which the percentage of change is 12%.Due to their many favorable properties, this is applications in the – orthopedic-surgery and other several other medical applications.*

***Key Words*: Keywords: lattice structure, Analytical, Numerical, truncated cube, Buckling.**

1. INTRODUCTION

Recent advancements in production technique should have allowed the manufacture from porous bio material usual with specifically defined micro architectures [1,2]. Micro architecture of the porous bio materials was shown on manage their mechanical properties including with the mechanical properties [3,4]. Fatigue resistances [5] and also permeable [6]. In attachment to the mechanical properties and geometric features of porous bio material on a small scales so as the curve of the holes and the indication shown to change the biological response to porous bio materials [7]. In specific, it has been notice that rate of the tissue regeneration depending on the features mention above [7-10]

Given the significance of micro-architecture to determine the physical properties and biological properties of porous bio materials, it is necessary to the regularly study the relation among the micro architecture of porous materials and their physical properties and biological properties. The purpose of the geometry of repeat unit cells applied for the (additive manufacturing) of the regular porous bio materials on these mechanical properties of structures needs to studied within there context. Mechanical properties of the porous bio materials that used to the bone replacement, hence received increase attention recently. Partially because it matches the unique mechanical properties of the porous bio materials for the people of the bone is at the present likely possible only by the taking the right type of cell unit and by adjust the unit cell dimension were selected. Similarly way, one can optimally dispense the mechanical properties of bio materials in orthopedic implants to reduce the effects of the phenomena of stress defensive and to mitigate unintended consequence associated. Described in the literature, see for example [11-13].

The pore of this porous bio material can be also used as a reservoir to deliver development factors [14] and the other potential types of the bio molecules.

The large area of surface with the porous structure is different opportunity combine with the best type of surface treatments and also the coating might add to increased bio-activity of the bio material and finally could lead to improved bone regeneration and integration performance [14].

The mechanical properties of the porous structure base upon different varieties of the unit cell has be studied previously [2.15 to 31]. Cubes lattice structure is a moderately new morphology that the properties should no be studied widely. The crushing performance closed-cell foam base on morphological cube cell, as a good represent from the conventional foam, has be studied numerically by the many researchers [32-36]. elasticity, Poisson's ratio, and stress results in periodic lattice open cell and also closed cell have also be investigate numerically in [34]. While here the number of numerical investigation of the porous structure based on truncated unit cube cells are not sufficient for the cell structure is closed and limited to just one study on the structure of open cell (in which these mechanical properties are obtained only for one new relative of density [34]) there is an analytical solution should be obtainable to predict the mechanical properties of these structures including the cells open (or) closed. Analytical solutions related to micro porous bio material architecture of their useful mechanical properties of various points. They can help out in knowing the mechanism of deformation and failure of their bio materials. Analytical solutions can be applied for the validation of the results of the calculation. Lastly, they can provide as a substitute for computing model when operation certain patient's optimization s algorithm that required a lot of estimation of the mechanical properties of the porous bio material. In this, analytical solutions are obtained for evaluating the elastic of modulus, yield stress and also Poisson's ratio, the buckling load of open cell of porous bio material made from the truncated cube unit cell. The FE model was developed to the compare the analytical solution with results of the calculation. Also, experimental data from one of our previous studies used for validation.

1.1 Euler buckling

The maximum load at the column tend to possess lateral displacement or tend to buckle is understood because the buckling Load (or) the crippling load, Load columns can be analyzing by Euler buckling formulas as

Columns and Struts:

(i) A structural member subjected to an axial compressive force is known a strut. As per definition, a strut may be the vertical, horizontal, or even inclined

(ii) The Vertical strut of a unit cell is called as column.

1.2 Literature review

Literature review is done from books, SAE papers, Journals, websites and magazines.

2. METHODOLOGY

The truncated hexahedron also called truncated cube (Fig. 2.1), one of limited number of unit cell that be able to produce lattice structure after repeated in the space. These cells are consider open-cell unit and the uninclined and inclined struts can have unequal (m ≠l) (or) equal (m = l) lengths (Fig. 2.1).The ratio of length of inclined struts to the no inclined struts (vertical or horizontal.) is given by α, it is examined how this affects the elastic properties obtained. Three subsections of this section deal with the derivations of analytical solutions, a description of the finite element (FE) model to estimate the mechanical properties of the porous structure.

2.1 Analytical solutions of Truncated cube unit cell:

2.1.1. Relative density:

The Relative density is expressed as the density of ratio of porous structures to density of solid material and this study, the analytical relationship presented with the mechanical properties of the truncated cube lattice structure has struts and also (i.e. square, Circular) the relative density relationship was obtained for the 3 different types cross sections shown. Each truncated cubic unit cell consists of 24 slanted edges of length (one each shared by the adjacent unit cell with another unit cell) and 12 non-sloped edges of length $m = 2\alpha l$ (each shared by three unit cells of adjacent unit cells), see fig. 1. Each truncated cubic elementary cell occupied, hence a volume of $v_{uc} = \left(2\alpha + \sqrt{2}\right)^3 l^3$.The relative density of structure can be obtain by the divided the volume of material by total volume of the elementary cell for the segment of the circle that we have

$$u = \frac{\frac{24}{2}\{\pi r^2 l\} + \frac{12}{4}\{2\pi r^2 \alpha l\}}{\left(2\alpha + \sqrt{2}\right)^3 l^3} = \frac{\pi(12 + 6\alpha)}{\left(2\alpha + \sqrt{2}\right)^3}\left(\frac{r}{l}\right)^2 \quad (1)$$

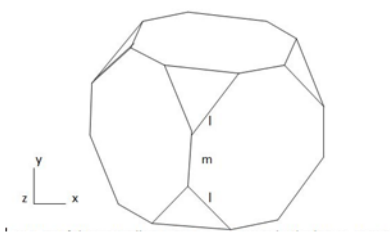


Figure2.1 one of the unit cells constructing cubic lattice structure

For α=0.5 (i.e. m=l), and correspond to the equilateral truncated cubic unit cell are below

$$u = \frac{V_{struts}}{V_{uc}} = \frac{15\pi}{(1+\sqrt{2})^3}\left(\frac{r}{l}\right)^2 \quad (2)$$

The side length is b, for square cross-section we have here:

$$u = \frac{V_{struts}}{V_{uc}} = \frac{\frac{24}{2}\{b^2 l\} + \frac{12}{4}\{2b^2 \alpha l\}}{(2\alpha+\sqrt{2})^3 l^3} = \frac{12+6\alpha}{(1+\sqrt{2})^3}\left(\frac{r}{l}\right)^2 \rightarrow u = \frac{15}{(1+\sqrt{2})}\left(\frac{b}{l}\right)^2, \rightarrow \alpha = 0.5 \quad (3)$$

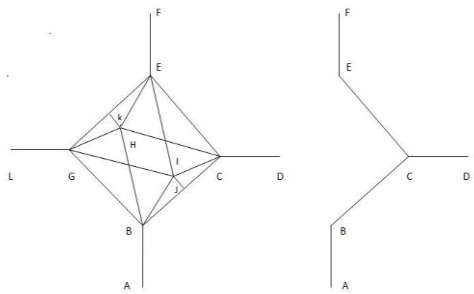


Figure2.2 (a) separately shown unit cell. (b) Some struts of unit cell consider for more thorough examination

And for triangular cross-section, we have:

$$u = \frac{V_{struts}}{V_{uc}} = \frac{\frac{24}{2}\left\{\frac{\sqrt{3}}{4}b^2 l\right\} + \frac{12}{4}\left\{2\frac{\sqrt{3}}{4}b^2 \alpha l\right\}}{(2\alpha+\sqrt{2})^3 l^3} = \frac{\sqrt{3}(12+6\alpha)}{4(2\alpha+\sqrt{2})^3}\left(\frac{b}{l}\right)^2 \rightarrow u = \frac{15}{4(1+\sqrt{2})^3}\left(\frac{b}{l}\right)^2, \rightarrow for\ \alpha = 0.5 \quad (4)$$

2.1.2. Stiffness matrix derivation

In this subsection, the analytical relation for the elastic modulus, yield stress, Poisson's ratio, and stress is a truncated cube structure buckling taken as a function of the dimensions of the unit cell

To study the mechanical behavior of a lattice structure, it is necessary to consider a unit cell, which after being repeated in space, constructs a truncated cube lattice structure. Because the cube lattice structure is truncated cube, the edges of the unit cell (Fig. 2.1) divided by the neighboring cells, the mechanical behavior of a single cell unit with a full cross-sectional area would not represent a mechanical response from the unit cell lattice structure. Given the unit cells thick in Fig. 2.2a as repeating cell unit instead of the unit cell porous structure is presented in Ensure that members of the unit cell that is not owned by the adjacent unit cells and the elastic properties obtained for this unit cell can be generalized to those of the lattice structure. Because the vertical and horizontal struts of the unit cell shown in Fig. 2.2b divided by four adjacent cells (see eight colored gray cells in Fig. 2.2a), for reasons of symmetry, they cannot have any rotations or bends over their entire length and are only allowed to contract or expand. For reasons of symmetry, they cannot have any rotations or bends over their entire length and are only allowed to contract or expand. In addition, all the corner points of the lattice structure are permitted to be consider rigid and since they are all related to the non-inclined struts (no rotation), and also the inclined edges are only allows to be rotate at their central parts and not at there goals. In addition, for each of the connections FECDBA, FEGLBA, FEHKBA and also FEIJBA (Figure. 2.2a) are only allows to move in the plane in which their were originally located. The examination of the deformation of one of the above mentioned relations in the plane is sufficient to investigate the mechanical behavior of the unit cell due to the equilibrium of geometry and deformation between the four connections of the unit cell. Therefore consider only one connection, FECDBA (Figure 2.1 c). Link FECDBA consists of six corner points, each with three degrees of freedom for deformation in the plane, creating a total of 18 degrees of freedom. However, degrees of freedom 18 can be reduced if the following consideration are taken into account. First, as mention above, corner points are not allows to rotate since the non-inclined struts have an orthogonal intersection between two planes between neighboring cells, shown in gray in (Figure 2.1 a) (and all corner points have a non-inclined end). In adding, points A, B, E and F are only allows to move vertically because of the same plane symmetry. As the values of horizontal displacement of the points C and D may be dissimilar, they have the same vertical shift, since members of the CD should always remain horizontal. Point A is assumed to be staple, and not allowed to move transitionally or rotational direction. The boundary condition described leaving only six degrees of freedom (shown in Fig. 2.3).Because the system is consider for the derivation of analytical solution is linear in both sanity of geometric and material, common deformation which involve the transfer of all six DOFs (Fig. 2.3) can be regarded as a superposition of particular separate displacement degrees of freedom. Taking benefit of the superposition principle, the stiffness matrix can be construct that regulate cell deformation truncated cube unit. Though, (18 degrees of freedom) can be greatly reduced if the following consideration are taken into account. First, as mention above, corner point are not allowed to rotate because the non-inclined struts have an orthogonal intersection between two planes between neighboring cells, shown in (Figure 2.1 a) (and all corner points have a non-inclined end). In adding, points A, B, E and F are only allows to move vertically since of the same plane symmetry.

In 2D deformation, deformation of the free end of the cantilever beams can be consider as the sum of the four type of displacement or rotation including the axial contraction / expansion (u), the lateral displacement (v), rotation (θ), and rotate (φ) .Forces and moments necessary to create the pure displacement and rotation depict .This figure will be referred to many times in the follow section for determining troop and when apply to the struts of the unit cell. It should noted that the statement V_c = 1, we mean that the point C and the subsequent to his points I, G, and H in the unit cell (figure 2.1b) all displaces laterally by unity. The same thing apply to the point D (and the corresponding points J, L, and k),

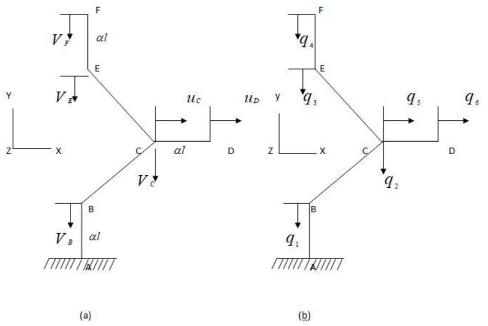


Figure 2.3 (a) unit cell Degrees of freedom (b) generalization of degree of freedom

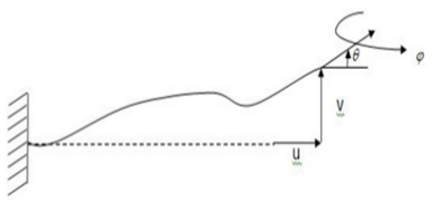
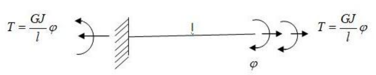


Figure 2.4a cantilever beam have four types of the displacements/rotation at its free end

The other point in unit cell are clear and the necessary forces obtain at point C and D must be multiply by the four to give total force exerted on the second and also sixth DOF, correspondingly.

In arrange to align



the notation of the Degree of freedom (DOFS), their all denote by the qi of this point:

$$V_B = q_1$$

$$V_c = V_1 = V_H = V_G \rightarrow q_2$$

$$V_F = q_4$$

$$V_E = q_3$$

$$u_c = u_I = u_H = u_G \rightarrow q_5$$

$$u_D = u_J = u_K = u_G \rightarrow q_5 \qquad (5)$$

The external force that needs to be exerted on a DOF qi is known as Qi. The force displacement relationships of this system has the follow form below education:

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{43} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix} \qquad (6)$$

Where the stiffness matrix of elementsK_{ij}

Have to be decided to obtained displacements, forces, rotations and also moments of functions of the exerted external force F with which as various of elastic properties of the unit cell be able to be derived."

First DOF: $q_1 = 1$

We develop the element of the 1st column of stiffness matrix of unit cell,

Taking displacements $q_1 = 1$

In to account and by setting

$$q_2 = q_3 = q_4 = q_5 = q_6 = 0$$

.

This deformation shifts point B down by one below figure. Since of this deformation, AB is contracted by unity and applies below.

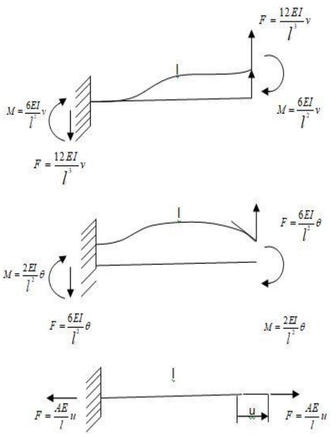


Figure 2.5 Forces and moments to be applied to cause the illustrate deformations

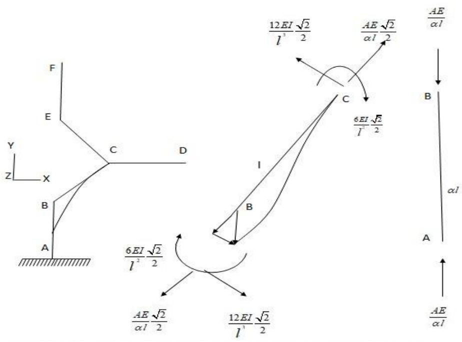


Figure 2.6.Forces and moments to be applied to cause the illustrate deformations

Elastic modulus of bulk material. The BC beam deformations can be regarded because a superposition of two different deformation of an axial expansion and the lateral deflection of point B (both having the same value, i.e. (figure 2.6). $\frac{\sqrt{2}}{2}$ The load apply by the beams BC to point B in y direction is

$$\left(\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\frac{12E_sI}{l^3}+\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\frac{AE_sI}{l^3}\right)$$

Hence, total load apply by the beams BC, BI, BH and BG to the point B is $\frac{24E_sI}{l^3}+\frac{2AE_s}{l}$

Below equation, Equilibrium of the forces at the point B in y direction yields,

$$\sum f_{By}=0\to 4\left(6\frac{E_sI}{l^3}+\frac{AE_s}{2l}\right)+\frac{AE_s}{\alpha l}-F_B=0\to Q_1=k_{11}=F_B=\frac{24E_sI}{l^3}+\frac{2AE_s}{l}+\frac{AE_s}{\alpha l}$$

(7)

Equilibrium of the forces at the point C in x direction yield, below equation:

$$\sum f_{cx}=0\to\frac{6E_sI}{l^3}-\frac{AE_s}{2l}+F_{c.x}=0\to Q_5=k_{51}=4F_{cx}=-\frac{24E_sI}{l^3}+\frac{2AE_s}{l}$$

(8)

While the fifth Degree of freedom consists of point (C, I, H, and G), according to the fifth degree of freedom is quadruple of the force to be apply at point C,

F_{cx}, i.e. $Q_5 = 4F_{cx}$.

And Equilibrium of the forces at the point C in y direction yield,

$$\sum f_{cy}=0\to\frac{6E_sI}{l^3}-\frac{AE_s}{2l}+F_{cy}=0\to Q_2=k_{21}=4F_{cy}=-\frac{24E_sI}{l^3}-\frac{2AE_s}{l}$$

(9)

Here, since the beam of the (EC, EF, and the CD) do not take deformation and their do not form any of force to point (E, F, and D), which means that:

$$Q_3 = Q_4 = Q_6 = k_{31} = k_{41} = k_{61} = 0$$

Second DOF: $q_2 = 1$

This type of the deformation can be displacing the beam (CD, IJ, GL, and HK) downwards by

The unity (Fig. 2.7a). Y direction load applied by the beams BC to point B is

$$-\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\frac{12E_sI}{l^3}-\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\frac{AE_sI}{l}$$

(Fig.2.7c).the total load exerted on point B by the beams (BC, BI, BG and BH) is

$$-\frac{24E_sI}{l^3}-\frac{2AE_s}{l}.$$

The equilibrium of the forces at the point B in y direction results from the following equation:

$$\sum f_{By} = 0 \rightarrow 4\left(\frac{6E_sI}{l^3}+\frac{AE_s}{2l}\right) - F_{By} = 0 \rightarrow Q_1 = k_{12} = F_{By} = -\frac{24E_sI}{l^3} - \frac{2AE_s}{l} \quad (11)$$

Similarly, the load apply by the carrier EC to the point E is

$$-\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\frac{12E_sI}{l^3} - \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\frac{AE_sI}{l}$$

In y direction (FIG. 2.7b). Hence, total load carry by the carriers (EC, EI, EG) and $-\frac{24E_sI}{l^3} - \frac{2AE_sI}{l}$

EH to point E, Equilibrium of the forces at the point E in y direction yields is:

$$\sum f_{Ey} = 0 \rightarrow -4\left(\frac{6E_sI}{l^3}+\frac{AE_s}{2l}\right) - F_{Ey} = 0 \rightarrow Q_3 = k_{32} = F_{Ey} = -\frac{24E_sI}{l^3} - \frac{2AE_s}{l} \quad (12)$$

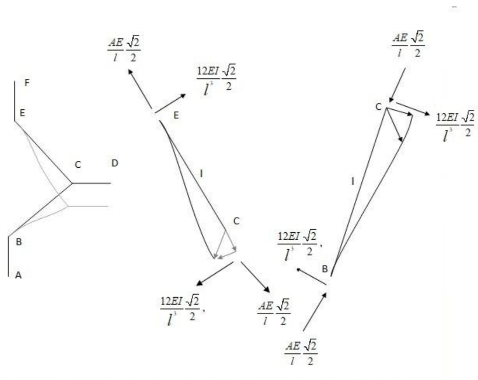


Figure 2.7 (a) $q_2 = 1$ for deformation of unit cell, (b) Free body diagrams for the strut EC, (c) Free body diagrams for the strut BC.

The Equilibrium of the forces in x direction at the point C yields is in (fig.2.7b-2.7c):

$$\sum f_{cx} = 0 \rightarrow \frac{6E_sI}{l^3} - \frac{AE_s}{2l} + \frac{AE_s}{2l} - \frac{6E_sI}{l^3} + F_{cx} = 0 \rightarrow Q_5 = k_{52} = F_{cx} = 0 \quad (13)$$

On other hand, the vertical loads apply by the beams (EC and BC) to point C is

$$2\left(\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\frac{12E_sI}{l^3} + \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\frac{AE_sI}{l}\right)$$

The forces in the y direction at the point C is equilibrium:

$$\sum f_{cy} = 0 \rightarrow 2\left(\frac{6E_sI}{l^3}+\frac{AE_s}{2l}\right) - F_{cy} = 0 \quad (14)$$

Since second Degree of freedom consists of all points(C, I, H, and G), the force corresponding to the second Degree of freedom is quadruple of the force which must be apply at point C is, F_{cy},

$$Q_2 = k_{22} = 4F_{cy} = \frac{48E_sI}{l^3} + \frac{4AE_s}{l} \quad (15)$$

The remaining two Degree of freedom are not affected by these deformations, therefore:

$$Q_4 = Q_6 = k_{42} = k_{62} = 0 \quad (16)$$

Third DOF: $q_3 = 1$

This deformation of displaces at point E downward by (Fig.2.8a). The y direction load applied by the beam EC to the point E is

$$\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\frac{12E_sI}{l^3} + \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\frac{AE_sI}{l}$$

Hence, total load apply by the beams EC, EI, EG, and EH to point E is below equation

$$\frac{24E_sI}{l^3} + \frac{2AE_s}{l}$$

The beam EF imposes a load $\frac{AE_s}{\alpha l}$ to point E. all the forces of Equilibrium are apply to point E.

$$\sum f_{Ey} = 0 \rightarrow 4\left(\frac{6E_sI}{l^3}+\frac{AE_s}{2l}\right) + \frac{AE_s}{\alpha l} - F_{Ey} = 0 \rightarrow Q_3 = k_{33} = F_{Ey} = \frac{24E_sI}{l^3} + \frac{2AE_s}{l} + \frac{2AE_s}{\alpha l} \quad (17)$$

Equilibrium of the forces apply to the point C in x direction yields is (Fig.2.8b):

$$\sum f_{cx} = 0 \rightarrow -\frac{6E_sI}{l^3} + \frac{AE_s}{2l} + F_{cx} = 0 \rightarrow Q_5 = k_{53} = 4F_{cx} = \frac{24E_sI}{l^3} - \frac{2AE_s}{l} \quad (18)$$

And Equilibrium of the forces apply to the point C in x direction yields is (Fig.2.8b):

$$\sum f_{cy} = 0 \rightarrow -\frac{6E_s I}{l^3} - \frac{AE_s}{2l} - F_{cy} = 0 \rightarrow Q_2 = k_{23} = 4F_{cy} = -\frac{24E_s I}{l^3} - \frac{2AE_s}{l} \quad (19)$$

The load that must be applied to fourth degree of freedom to maintain in the equilibrium is (Fig.2.8c):

$$Q_4 = k_{43} = F_{Fy} = -\frac{AE_s}{\alpha l} \quad (20)$$

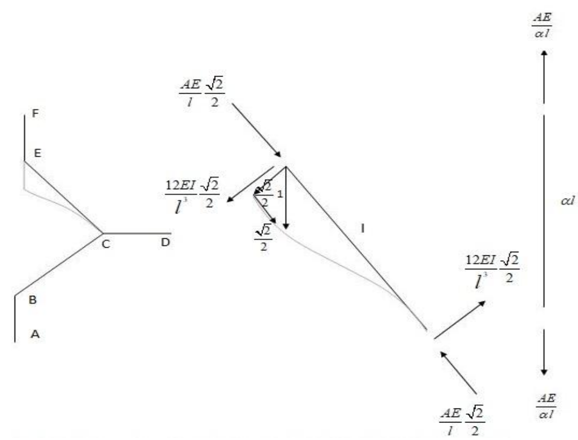


Figure 2.8 (a) $q_3 = 1$ for deformation of unit cell, (b) Free body diagrams for the strut EC, (c) Free body diagrams for the strut EF.

Here, since the beams BC, AB and also CD do not deform and also their do not inflict any force to points D and B, which means that,

$$Q_1 = Q_6 = k_{13} = k_{63} = 0 \quad (21)$$

Fourth DOF: q_4=1

This step, point F is moved downwards by the unity. The load required to be applied to the 3rd degree of freedom, to be maintain the equilibrium is

$$Q_3 = k_{34} = F_{Ey} = -\frac{AE_s}{\alpha l} \quad (22)$$

And also the load required to applied to 4th degree of freedom of unit, to be keep in the equilibrium is

$$Q_4 = k_{44} = F_{Fy} = \frac{AE_s}{\alpha l} \quad (23)$$

And the other points where not affected by this deformation:

$$Q_1 = Q_2 = Q_5 = Q_6 = k_{14} = k_{24} = k_{54} = k_{64} = 0 \quad (24)$$

Fifth DOF: $q_5 = 1$

This step, point C is displacement in x the direction by unity (fig.2.9a) and the force that must be apply to the point D in x direction to maintaining the equilibrium is $-\frac{AE_s}{\alpha l}$.

Since, the 6th degree of freedom consists of four points are D, J, L, and K, the 6th force is calculating below equation:

$$Q_6 = k_{65} = F_{Dx} = -\frac{4AE_s}{\alpha l} \quad (25)$$

The y direction load apply by the beam BC to the point B is

$-\frac{6E_s I}{l^3} + \frac{AE_s}{2l}$ Here, the total load apply by the beams BC, BI, BG, and BH to point B is

$-\frac{24E_s I}{l^3} + \frac{2AE_s}{l}$ And the equilibrium of the forces at the point B in y direction yields below equation:

$$\sum f_{By} = 0 \rightarrow -\frac{24E_s I}{l^3} + \frac{2AE_s}{l} - F_{By} = 0 \rightarrow Q_1 = k_{15} = 4F_{By} = -\frac{24E_s I}{l^3} - \frac{2AE_s}{l} \quad (26)$$

Similar, the equilibrium of the forces in y direction at the point E gives (Fig.2.9b):

$$Q_3 = k_{35} = F_{Ey} = \frac{24E_s I}{l^3} - \frac{2AE_s}{l} \quad (27)$$

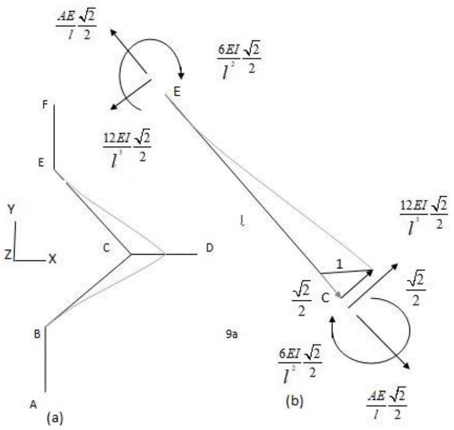


Figure 2.9 (a) $q_5 = 1$ for deformation of unit cell, (b) Free body diagrams for the strut EC

Similar way, considering the equilibrium of the forces at the point C in y direction gives:

$$Q_2 = k_{25} = 0 \tag{28}$$

When the point C is displaces in x direction by the unity, and the symmetrically points of G, I, and also H are correspondingly displaces in in $-x$, z, and $-y$

Horizontal struts are IC, CH, HG, and IG go over pure tension. Their lengths hence increase by$\sqrt{2}$.

This increase in length generates an axial load of$\frac{\sqrt{2}AE_s}{l}$.

In all the four struts. Sum of x-direction forces apply by the struts IC, CH to the point C is $\frac{\sqrt{2}AE_s}{l}\frac{\sqrt{2}}{2}2 = \frac{2AE_s}{l}$

.The Sum of x direction loads apply by the struts EC, C and the point C is $-2\frac{6AE_s}{l^3} = -2\frac{AE_s}{l}$

Finally, the x direction load applied by the strut CD to this point is $-\frac{AE_s}{\alpha l}$

Therefore, equilibrium of forces at the point C in x direction yields:

$$\sum f_{cx} = 0 \rightarrow -\frac{12E_sI}{l^3} - \frac{AE_s}{l} - \frac{AE_s}{\alpha l} - \frac{AE_s}{2l} + F_{cx} = 0 \tag{29}$$

$$\rightarrow Q_5 = k_{55} = 4F_{cx} = \frac{48AE_sI}{l^3} + \frac{12AE_s}{l} + \frac{4AE_s}{\alpha l}$$

And Beam EF is not in deformation type. Hence, load that must be apply at the point F is zero,

$$Q_4 = k_{45} = 0 \tag{30}$$

Sixth DOF: $q_6 = 1$

Point D is displaced by unity in x direction. The load required to be apply to the 6th degree of freedom to keep the equilibrium is gives: $Q_6 = k_{66} = 4F_{Dx} = \frac{8AE_s}{\alpha l}$

(31)

And the load required to be applied to 6th degree of freedom, to keep the equilibrium is, below equation: $Q_5 = k_{56} = 4F_{Cx} = -\frac{8AE_s}{\alpha l}$

And also the other points are not affected by deformation. We have below equation,

$$Q_1 = Q_2 = Q_3 = Q_4 = k_{16} = k_{26} = k_{36} = k_{46} = 0 \tag{33}$$

3.1.3. Stiffness matrix derivation

In this case, using the stiffness matrix elements are obtain in section 2.1.2, we can see below equation:

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{Bmatrix} = \begin{bmatrix} \frac{24E_sI}{l^3}+\frac{2AE_s}{l}+\frac{AE_s}{\alpha l} & -\frac{24E_sI}{l^3}-\frac{2AE_s}{l} & 0 & 0 & -\frac{24E_sI}{l^3}+\frac{2AE_s}{l} & 0 \\ -\frac{24E_sI}{l^3}-\frac{2AE_s}{l} & \frac{48E_sI}{l^3}+\frac{4AE_s}{l} & -\frac{24E_sI}{l^3}-\frac{2AE_s}{l} & 0 & 0 & 0 \\ 0 & -\frac{24E_sI}{l^3}-\frac{2AE_s}{l} & \frac{24E_sI}{l^3}+\frac{2AE_s}{l}+\frac{2AE_s}{\alpha l} & -\frac{AE_s}{\alpha l} & \frac{24E_sI}{l^3}-\frac{2AE_s}{l} & 0 \\ 0 & 0 & -\frac{AE_s}{\alpha l} & \frac{AE_s}{\alpha l} & 0 & 0 \\ -\frac{24E_sI}{l^3}-\frac{2AE_s}{l} & 0 & \frac{24E_s}{l}-\frac{2AE_s}{l} & 0 & \frac{48AE_sI}{l^3}+\frac{12AE_s}{l}+\frac{4AE_s}{\alpha l} & -\frac{4AE_s}{\alpha l} \\ 0 & 0 & 0 & 0 & \frac{4AE_s}{\alpha l} & -\frac{4AE_s}{\alpha l} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix}$$

(34)

3. ANAL YTICAL AND NUMERICAL SOLUTIONS

Case (1): 3.1 Strut diameter of Circle cross section with 0.5mm

Analytical solutions:

The Elastic modulus, poison's ratio, and buckling stress of the truncated cubic structures are obtain as function of unit cell dimensions, in this structure, all the degree of freedom corresponding to point A were constant in area and points B, E, and F had been only allowed to move vertically. None of the vertices of the unit cell have been allowed to rotate. A concentrated force, F, was applied to the top point of the simple structure. The analytical results are obtained as per boundary conditions such as

At point A: fixed (x, y, z) Displacement, At point D: 0, 0, Free (x, y, z) Displacement, At point F: 0, -100, 0 (x, y, z) Displacement, At point L: 0, 0, free (x, y, z) Displacement

Elastic Modulus =3.5 GPa, Poisson ratio =0.3, Area= 0.78 mm^2 Moment of inertia=0.049 mm^4 Force=100 mm Length =5mm Radius =0.50mm

$$\begin{Bmatrix}0\\0\\0\\F\\0\\0\end{Bmatrix}=\begin{bmatrix}2216.928 & -1124.928 & 0 & 0 & 1059.072 & 0\\1124.928 & 2249.856 & -1124.928 & 0 & 0 & 0\\0 & 1124.928 & 2216.928 & -1092 & -1059.072 & 0\\0 & 0 & -1092 & 1092 & 0 & 0\\1059.072 & 0 & -1059.072 & 0 & 8801.856 & -4368\\0 & 0 & 0 & 0 & -8736 & 8736\end{bmatrix}\begin{Bmatrix}q_1\\q_2\\q_3\\q_4\\q_5\\q_6\end{Bmatrix}$$

By solving above get below deformations

$q_1 = 0.09158$ $q_4 = 0.50626$

$q_2 = 0.25313$ $q_5 = 0.07718$

$q_3 = 041468$ $q_6 = 0.07718$

$q_1, q_2, q_3, q_4 q_5, q_6$, Are deformed at each point.

Strut diameter of Circle cross section with 0.5mm:

Finite element models (FEM) of the porous structures had be created in the commercial finite element package (ANSYS)). That implementations of the Euler's beam elements in ANSYS makes use of linear interpolation and take transverse shear deformation into account. The beams had be rigidly connected at vertices. The matrix material was consider linear elastic with an elastic modulus of E_s=3.5GPA, yield stress of σ_{ys}=980 MPa, and Poisson's ratio of Vs=0.33 [37]

The static nonlinear implicit solver accessible in ANSYS used to be used for running the simulations. Two types of models have be used for the numerical modeling. The first model use to be a small model consisting of eighteen struts similar to the configuration regarded for the analytical study In this structure, all the degree of freedom corresponding to point A were constant in area and points B, E, and F had been only allows to move vertically. None of the vertices of the unit cell have been allowed to rotate. A concentrated force, F, was apply to the top point of the simple structure

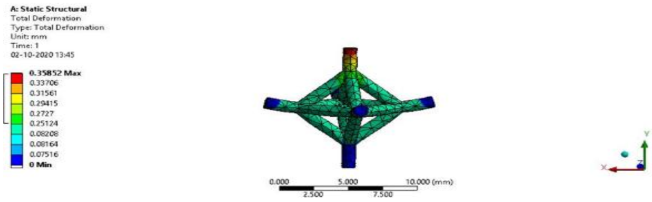


Figure 3.1 Strut diameter of Circle cross section with 0.5mm:

Table (3.1): Comparison between Analytical Vs Numerical:

	q_1	q_2	q_3	q_4	q_5	q_6
Analytical	0.09158	0.25313	0.41468	0.50626	0.07718	0.07718
Numerical	0.08164	0.25124	0.33706	0.35852	0.08208	0.07516
Percentage of change	11	1	19	30	6	5

Case (2): 3.2 Strut diameter of Circle cross section with 0.75mm

Analytical solutions

At point A: fixed (x, y, z) Displacement At point D: 0, 0, Free (x, y, z) Displacement, At point F: 0,-100, 0 (x, y, z) Displacement, At point L: 0, 0, free (x, y, z) Displacement

Elastic Modulus =3.5 GPa, Poisson ratio =0.3, Area= 1.76 mm^2 Moment of inertia=0.248 mm^4 Force=100 mm Length =5mm Radius =0.75mm

$$\begin{Bmatrix}0\\0\\0\\F\\0\\0\end{Bmatrix}=\begin{bmatrix}5094.656 & -2630.656 & 0 & 0 & 2296.344 & 0\\-2630.656 & 5261.312 & -2630.656 & 0 & 0 & 0\\0 & -2630.656 & 5094.656 & -2464 & -2297.344 & 0\\0 & 0 & -2464 & 2464 & 0 & 0\\2630.656 & 0 & -2297.344 & 0 & 24973.312 & -9856\\0 & 0 & 0 & 0 & -19712 & 19712\end{bmatrix}\begin{Bmatrix}q_1\\q_2\\q_3\\q_4\\q_5\\q_6\end{Bmatrix}$$

By solving above get below deformations

$q_1 = 0.04059$ $q_4 = 0.18254$

$q_2 = 0.09128$ $q_5 = 0.01451$

$q_3 = 0.14196$ $q_6 = 0.01451$

$q_1, q_2, q_3, q_4 q_5, q_6$, Are deformed at each point.

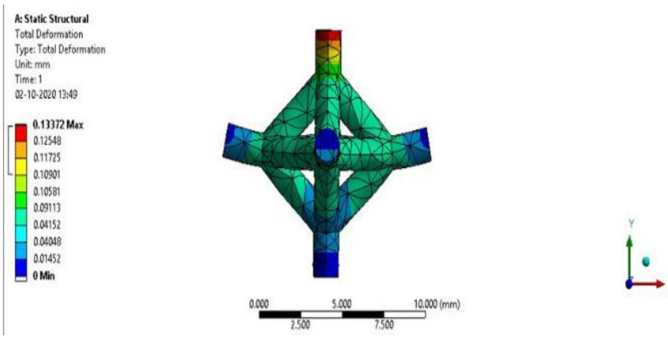


Figure 3.2 Strut diameter of Circle cross section with 0.75mm:

Table (3.2): Comparison between analytical Vs Numerical:

	q_1	q_2	q_3	q_4	q_5	q_6
Analytical	0.04059	0.09128	0.14196	0.18254	0.01451	0.01451
Numerical	0.04048	0.09113	0.10581	0.13372	0.04152	0.01452
Percentage of change	1	1	26	27	60	1

Case (3): 4.3 Strut diameter of Circle cross section with 1.0mm

Analytical solutions

Boundary conditions:

At point A: fixed (x, y, z) Displacement, At point D: 0, 0, Free (x, y, z) Displacement, At point F: 0,-100, 0 (x, y, z) Displacement, At point L: 0, 0, free (x, y, z) Displacement

Elastic Modulus =3.5 GPa, Poisson ratio =0.3, Area= 1.76 mm^2 Moment of inertia=0.248 mm^4 Force=100 mm Length =5mm Radius =1.00mm

$$\begin{Bmatrix} 0 \\ 0 \\ 0 \\ F \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 9324.408 & -4926.0148 & 0 & 0 & 3870.4372 & 0 \\ -4926.0148 & 9852.0296 & -4926.0148 & 0 & 0 & 0 \\ 0 & -4926.0148 & 9324.2408 & -4398.226 & -3870.4372 & 0 \\ 0 & 0 & -4398.226 & 4398.226 & 0 & 0 \\ 3870.43720 & 0 & -3870.4372 & 0 & 45037.8376 & -17592.904 \\ 0 & 0 & 0 & 0 & -35185.808 & 35185.808 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix}$$

By solving above get below deformations

$q_1 = 0.02274$ $q_4 = 0.09765$

$q_2 = 0.04882$ $q_5 = 0.00736$

$q_3 = 0.07491$ $q_6 = 0.00736$

$q_1, q_2, q_3, q_4 q_5, q_6$, Are deformed at each point.

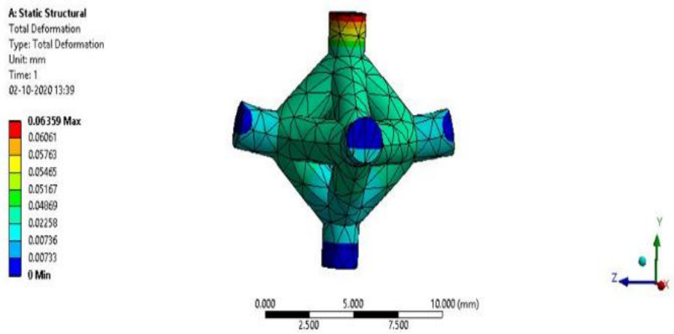


Figure 3.3 Strut diameter of circle cross section with 1.00mm

Table (3.3): Comparison between Analytical Vs Numerical:

	q_1	q_2	q_3	q_4	q_5	q_6
Analytical	0.2274	0.04882	0.07491	0.09765	0.00736	0.00736
Numerical	0.02258	0.04869	0.05465	0.06359	0.00736	0.00733
Percentage of change	1	1	28	35	0	1

Case (4): 3.4 Strut diameter of Circle cross section with 1.25m m

Analytical solutions

Boundary conditions:

At point A: fixed (x, y, z) Displacement, At point D: 0, 0, Free (x, y, z) Displacement, At point F: 0,-100, 0 (x, y, z) Displacement, At point L: 0, 0, free (x, y, z) Displacement

Elastic Modulus =3.5 GPa, Poisson ratio =0.3, Area= 4.90 mm^2 Moment of inertia=1.917 mm^4 Force=100 mm Length =5mm Radius =1.25mm

$$\begin{Bmatrix} 0 \\ 0 \\ 0 \\ F \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 15008.224 & -8148.224 & 0 & 0 & 5571.776 & 0 \\ -8148.224 & 16296.448 & -8148.224 & 0 & 0 & 0 \\ 0 & -8148.224 & 15008.224 & -6860 & -5571.776 & 0 \\ 0 & 0 & -6860 & 6860 & 0 & 0 \\ 8148.224 & 0 & -5571.776 & 0 & 71176.448 & -27440 \\ 0 & 0 & 0 & 0 & -54880 & 54880 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix}$$

By solving above get below deformations

$q_1 = 0.01458$ $q_4 = 0.05749$

$q_2 = 0.02873$ $q_5 = 0.00275$

$q_3 = 00.04288$ $q_6 = 0.00275$

$q_1, q_2, q_3, q_4 q_5, q_6,$ Are deformed at each point.

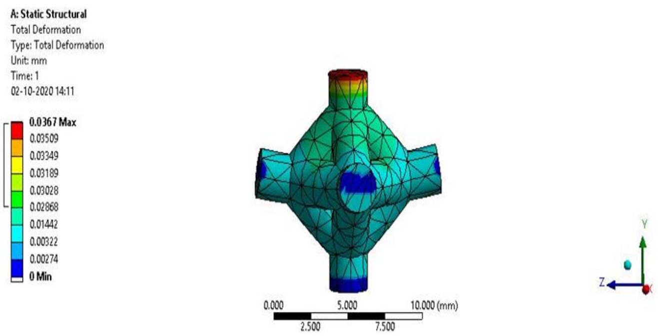


Figure 3.4 Strut diameter of circle cross section with 1.25mm:

Table (3.4): Comparison between Analytical Vs Numerical:

	q_1	q_2	q_3	q_4	q_5	q_6
Analytical	0.01458	0.02873	0.04288	0.05746	0.00275	0.00275
Numerical	0.01442	0.03028	0.0367	0.02868	0.02868	0.00274
Percentage of change	2	6	26	37	5	1

4. RESULTS AND DISCUSSION

4.1 Comparison of Analytical and numerical Results:

By taking Elastic modulus, Poisson ratio for each case of truncated cellular lattice structure,

Case (1):

Analytical: $q_1 = 0.09158, q_2 = 0.25313, q_3 = 0.41468,$

$q_4 = 0.50626 q_5 = 0.07718, q_6 = 0.07718,$

Numerical: $q_1 = 0.09158, q_2 = 0.25313, q_3 = 0.41468,$

$q_4 = 0.50626 q_5 = 0.07718, q_6 = 0.07718,$

Are deformed at each point.

Overall Percentage of change: 11%

Case (2):

Analytical: $q_1 = 0.04059, q_2 = 0.09128, q_3 = 0.14196,$

$q_4 = 0.18254 q_5 = 0.01451, q_6 = 0.01451,$

Numerical: $q_1 = 0.04048, q_2 = 0.09113, q_3 = 0.10581,$

$q_4 = 0.13372, q_5 = 0.04152, q_6 = 0.04152,$

Are deformed at each point.

Overall Percentage of change: 19%

Case (3):

Analytical: $q_1 = 0.02274, q_2 = 0.04882, q_3 = 0.07491,$

$q_4 = 0.09765, q_5 = 0.00736, q_6 = 0.00736$

Numerical: $q_1 = 0.02258, q_2 = 0.04869, q_3 = 0.05465,$

$q_4 = 0.06359 q_5 = 0.00736, q_6 = 0.00736$

Are deformed at each point.

Overall Percentage of change: 11%

Case (4):

Analytical: $q_1 = 0.01458, q_2 = 0.02873, q_3 = 0.04288,$

$q_4 = 0.05746, q_5 = 0.00275, q_6 = 0.00275$

Numerical: $q_1 = 0.01442, q_2 = 0.03028, q_3 = 0.03189,$

$q_4 = 0.0367, q_5 = 0.02868, q_6 = 0.02868$

Are deformed at each point.

Overall Percentage of change: 12%

Overall percentage of change for truncated cellular lattice structure with various strut diameter of unit cell.

4.2 Discussions

The main involvement of this study is that the source of analytical relations for the elastic properties of porous structures support the truncated cube unit cells. These analytical solutions are useful for quick evaluation of the properties of these structures without need of making and solve of finite element model. The obtainable analytical relations could hence save time, cost, and energy and release computational resource for other steps of the structural analysis like optimization of the sharing of mechanical properties within an establish. The analytical solutions obtain during this study might be also useful for bench marking computational results and for appreciative the physical phenomena behind complex mechanical behaviors that these sorts of structures frequently demonstrate [42]. Especially, analytical solutions make it's easier to know the influence of various design parameter of the porous structures of unit cell.

4.2.1 Cross section type effect

It was to know how the type of cross-section affect the mechanical properties of structures. Hence, three different types of cross-section be consider and it have been see that the cross-section type just affects the Poisson's ratio value and it's nearly unsuccessful on the coefficient of elasticity or yield stress. This is often a key conclusion for the truncated cube lattice structure, since it's been practical in other morphology (such as diamond, rhombic dodecahedron, and Wearied Phelan) that the cross-section type can severely affect yield stress [43].

5. CONCLUSIONS

Open-cell for porous bio materials of mechanical properties are made from an exceptionally new kind of unit cell, then truncated cube had been studied. Most importantly, the analytical solutions had been derived for estimating the elastic modulus, Poisson's ratio, yield stress, and buckling restriction of the porous structure given the micro architecture of the unit cell. The analytically calculated mechanical properties have been in contrast with their corresponding analytical and numerical. The numerical and analytical solutions were inappropriate agreement with each other. The ratio between lengths of inclined struts to the uninclined struts, a, sequentially approached zero and infinite, the analytical for mechanical properties are approached and the analytically calculated of the cubic lattice structures, therefore imparting extra validation for the analytical solution bought here. the analytical solutions obtainable here for the truncated cube lattice can be considered as a more extra general form of analytical solutions for mechanical properties of cubic lattices and also octahedral. The analytical solutions to deviate from numerical results for very large α values. Those should now not consequently be used for structures with large α values.

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