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New Design Formulae for Equilateral Triangular Microstrip Antenna

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Abstract - *This paper presents a model for taking into account the effect of fringing fields on the sidelength of an equilateral triangular microstrip antenna (ETMSA). The model is straight forward, simple and accurate. It is propounded that the extension in physical sidelength of ETMSA is directly proportional to the physical sidelength itself and also to the normalized thickness of the dielectric substrate. At present several formulae are in use for this. The new model gives equally good results without any iteration. Novel results are that for all ETMSA (1) the ratio of extension in physical sidelength to physical sidelength itself is equal to H (2) the ratio of enlarged area to physical area of the patch is (1+2H) (3) the ratio of extension in physical sidelength to the height of the substrate is a constant 2/3 (4) a graphical tool for determining the physical sidelength of ETMSA has been put forward. Here H is the normalized substrate thickness. The results compare very well with the published measured and calculated data. Large number of ETMSA have been designed and simulated to validate the new thinking. Some typical data and simulation results have been incorporated in this paper. The work reported here is very useful for the antenna designer as the required value of S_p can be quickly and accurately found. The results are of great significance as these point to the similarities between MSA of various shapes.*

Key Words: **— Bhatnagar's Postulate, Design, Fringing Fields, Guide Wavelength, Physical Dimensions, Resonant frequency, Triangular Microstrip Antenna,**

1. INTRODUCTION

1.1 User Requirement

A microstrip antenna designer wants to know quickly and accurately the physical dimensions of the patch for his desired resonant frequency. The resonant frequency (f_r) depends on the electrical dimensions that are slightly larger than the physical dimensions due to fringing field effect. Vastly different models have been proposed and are in use for this. Resonant frequency values obtained from these formulae and also from simulation differ from one another as well as from physically measured values because of several assumptions that may not be true in every case.

1.2 The Classical Models

For microstrip antenna, equilateral triangular patch has been widely investigated and used. Either moment method or cavity model has been generally used. These covered a range of substrate dielectric constants (ε_r) and substrate thicknesses (h). The cavity model assumes electric walls on the top and bottom, and a magnetic wall around the sides. To take into account the effect of fringing fields, either effective side length or effective dielectric constant or both have been used with convenience for explaining the measured results of resonance frequency and other parameters. Dynamic permittivity has also been considered for the same reason. When an effective dielectric constant (ε_{eff}) is used the user has first to determine ε_{eff} then only he can estimate the physical sidelength. Formula for ε_{eff} is based on rectangular geometry of the patch. This formula uses a value of W (length of non-radiating side of the rectangular patch) that is discarded later on. If a triangular microstrip antenna (as shown in Fig-1)of side length S is constructed over an insulating substrate whose dielectric constant is ε_r then, using cavity model with perfect magnetic walls it can be shown that [1]

$$S = \frac{2c}{3 f_r \sqrt{\varepsilon_r}} \quad (1)$$

Where c is the velocity of electromagnetic waves in free space.

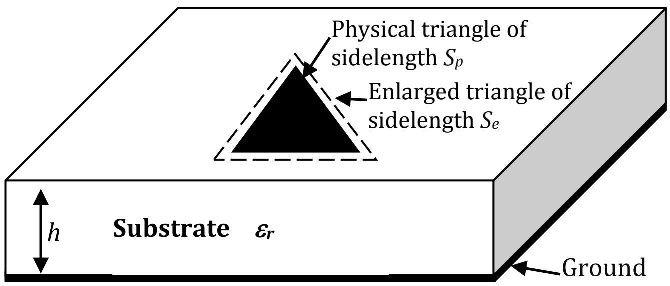

Fig-1: Basic structure of an equilateral triangular microstrip antenna

There have been a number of suggestions for taking into consideration the influence of fringing fields. According to one suggestion S in (1) should be replaced by its effective or enlarged value S_e. This gives

$$S_e = \frac{2c}{3f_r\sqrt{\varepsilon_r}} \quad (2)$$

[2] suggested that both S and ε_r be replaced by their effective values S_e and ε_{eff}. (1) then becomes

$$S_e = \frac{2c}{3f_r\sqrt{\varepsilon_{eff}}} \quad (3)$$

2. Problem Formulation

The antenna designer is not interested in knowing S_e. He needs S_p quickly. The problem then becomes to find a relation between S_p and S_e and between ε_{eff} and ε_r. Table-1 gives some of the models that are being used for determining the resonance frequency of an equilateral triangular microstrip antenna. In some of the models the symbols have been redefined for the sake of uniformity. Most widely used model is:

$$\boldsymbol{S_e = S_p + \frac{h}{\sqrt{\varepsilon_r}}} \quad (4)$$

3. Methodology

For quick estimation of resonant frequency of rectangular microstrip patch antennas a new method had been proposed earlier [3]. This paper extends that concept to triangular patch geometry. First a new theory is propounded. A mathematical expression is derived. This is followed by development of a graphical tool. The emphasis is on estimating the value of physical sidelength (S_p) of the equilateral triangular microstrip antenna that will resonate at the desired frequency for the given substrate (ε_r *and* h). S_p is calculated by the new formula as well as by the classical formula to cover the usual range of the parameters. The values are compared with each other. The user is interested in f_r therefore for validating the new model simulations are done and the results are analyzed.

Novelties presented are (1) a very simple, straight forward yet accurate model for estimating S_p (2) a graphical tool for determining the physical sidelength (3) new theory

(i) the extension in S_p (due to fringing fields) has been directly related with S_p itself

(ii) The ratio of extension in physical sidelength to physical sidelength itself is equal to H.

(iii) The extension in physical sidelength is $2h/3$.

(iv) the ratio of fringing field area to the area of the patch is equal to $2H$

$$\text{where, } H = \frac{h}{\lambda_g} = \frac{1}{c} f_r h\sqrt{\varepsilon_r} \quad (5)$$

λ_g is the guide wavelength. For the given basic parameters f_r, h and ε_r, the physical sidelength S_p can be directly read from the graph. There is no need for assumptions and calculations for estimating it.

Table-1: The new and the classical models for estimating the physical sidelength of ETMSA

S. No.	Model	References
1	P1. Calculate physical area (A_p) of the triangle. P2. Determine the radius a_p of a circle whose area is equal to A_p P3. Calculate enlarged radius (a_e) of the circle $a_e = a_p * \left[1 + \frac{2h}{\pi a_p \varepsilon_r}\left(ln\left\{\frac{a_p}{2h}\right\} + 1.41\varepsilon_r + 1.77 + \frac{h}{a_p}(0.268\varepsilon_r + 1.65)\right)\right]^{0.5}$ P4. From this value of a_e *back calculate* S_e, $S_e = \frac{2\sqrt{\pi}a_e}{3^{0.25}}$	[2], [4], [5]
2	$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + \frac{12h}{a_p}\right)^{-0.5}$	[2]
3	$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1 + \frac{12h}{W_{equ}}\right)^{-0.5}$, Where $W_{equ} = \frac{3^{0.25}S_p}{2}$	[2]
4	$\boldsymbol{S_e = S_p + \frac{h}{\sqrt{\varepsilon_r}}}$	[1], [2], [6]
5	$S_e = S_p + 2\sqrt{m}\frac{h}{\sqrt{\varepsilon_r}} - 2\sqrt{mn}\frac{h}{\sqrt{\varepsilon_r}} + 197.1\frac{nh^2}{S_p\varepsilon_r}$ For m = 1 and n = 0 above equation reduces to $\boldsymbol{S_e = S_p + \frac{2h}{\sqrt{\varepsilon_r}}}$	[6]
6	$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1 + \frac{12h}{0.5S_p}\right)^{-0.5}$, $\boldsymbol{S_e = S_p + \frac{4h}{\sqrt{\varepsilon_{eff}}}}$	[7]
7	$\boldsymbol{S_e = S_p + h\left(0.1 + \frac{8}{\varepsilon_r^2}\right)}$	[8]
8	$\boldsymbol{S_e = S_p + h\left(1.2 + \frac{2.25}{\sqrt{\varepsilon_{eff}}}\right)}$ $\varepsilon_{eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1 + \frac{12h}{W_{equ}}\right)^{-0.5}$, Where $W_{equ} = \frac{3^{0.25}S_p}{2}$	[9], [10]
9	$S_p = \frac{20}{f_r\sqrt{\varepsilon_r}} - \frac{2}{3}h$	NEW

4. New Theory

All the models given in literature have been suggested for explaining the measured values. These have been arrived at by curve fitting, semi-empirical or empirical approach. These explain the measured data and also predict new values reasonably well. Therefore these have been in extensive use. Choice of a model perhaps depends on the user rather than the theory. This paper first propounds a theory, derives a model and then validates it by comparing with the existing as well as the new data.

It has been shown that for a rectangular microstrip antenna the extension in physical length of the patch is directly proportional to H and also to its effective length (L_e) [3]. This is Bhatnagar's postulate. Mathematically

$$\boldsymbol{L_e = L_p + 2\beta H L_e} \quad (6)$$

Where L_p is its physical length, β is Constant of proportionality.

An earlier paper [11] had proposed to extend Bhatnagar's Postulate to the case of an equilateral triangular microstrip antenna. It was stated there that "the extension in length of the radiating side (ΔS) of an equilateral triangular patch of physical sidelength S_p is given by ΔS = β*H* S_e where S_e is the effective side length, H and β have the same meaning and β = 1. However, effective sidelength (S_e) and effective dielectric constant (ε_{eff}) are conceptual parameters only. The real parameters that can be measured are the physical sidelength (S_p) and the dielectric constant (ε_r). Therefore, the model should be based on these. Big data is now available in literature. Analysis of this data can lead to new results. This paper proposes to modify the earlier extension of Bhatnagar's postulate to the case of equilateral triangular microstrip antenna (ETMSA). According to the new thinking, it is propounded that for ETMSA, the ratio of effective sidelength S_e of the patch to its physical sidelength S_p should be *(1+βH)*. Mathematically,

$$\frac{S_e}{S_p} = 1 + \beta H \quad (7)$$

$$or \quad S_e - S_p = \beta H S_p \quad (8)$$

for triangular geometry of the patch $\beta = 1$.

It can then be stated that *"For an equilateral triangular microstrip antenna, the extension in physical sidelength, due to fringing fields, is proportional to the physical sidelength and also to the normalized substrate thickness".*

(8) can also be written as $\frac{S_e - S_p}{S_p} = \beta H$

This means that "for ETMSA, fractional extension in side length (due to fringing field) is directly proportional to the parameter H",

As per [11] S_p = (1- H) S_e

As per the model proposed now, (7) gives

$$S_p = \frac{S_e}{1+\beta H} = (1 - \beta H + \beta^2 H^2) S_e$$

This also gives, S_p = (1- H) S_e , (9)

Since H is very small and β = 1 for ETMSA. Thus the result is the same, therefore, there is no loss in calculations and validations.

For estimating S_p (2) and (4) are most commonly used. Therefore, these have been used for data generation. Each of the basic variables ε_r, *h and* f_r has been varied over the normal range to estimate the values of S_p, S_e *and H.* The data was analyzed to investigate extension in S_p *i.e.* (S_e - S_p) and related issues. No trend was visible. To derive meaningful conclusions the data was partitioned into groups on the basis of H parameter. Each group had constant value of H and this value varied from group to group. Very interesting results have been obtained that are discussed next.

Classically S_e is first calculated using (2) and then S_p is determined using (4), therefore, from the data S_p was plotted against S_e. Chart-1 shows the result.

1. H was kept constant at 0.04.
2. Slope of the line is {1/(1+H)} = 0.96 for H = 0.04.
3. The line passes through origin i.e. S_e is zero for S_p = 0.

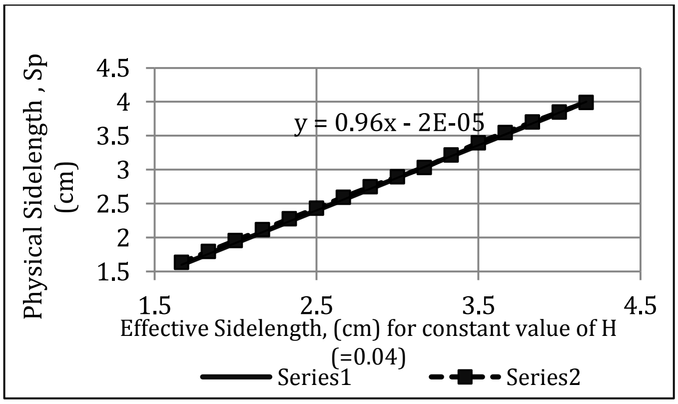

Chart-1: Comparison of the results of the new method (series 1) and the classical method (Series 2)

This graph can be interpreted and written mathematically as

$$S_p = \left(\frac{1}{1+H}\right) S_e$$

All plots of S_p against S_e were found to be straight line passing through origin and having a slope of 1/(1+H).

(2) and (9) give, $S_p = \frac{2c}{3f_r\sqrt{\varepsilon_r}} - \frac{1}{c} f_r h\sqrt{\varepsilon_r} * \frac{2c}{3f_r\sqrt{\varepsilon_r}}$

and for h in cm and f_r in GHz, $S_p = \frac{20}{f_r\sqrt{\varepsilon_r}} - \frac{2}{3}h$, (10)

(10) is the new model. It gives the physical sidelength directly in terms of the basic parameters f_r, ε_r *and* h. No need to compute any effective ε_r or effective sidelength or anything like that. In Charts- 2, 3 and 4, series 1 to 6 correspond to different h values. h = 0.05 cm, 0.1 cm, 0.15 cm, 0.2 cm, 0.25 cm and 0.3 cm for series 1 to 6 respectively.

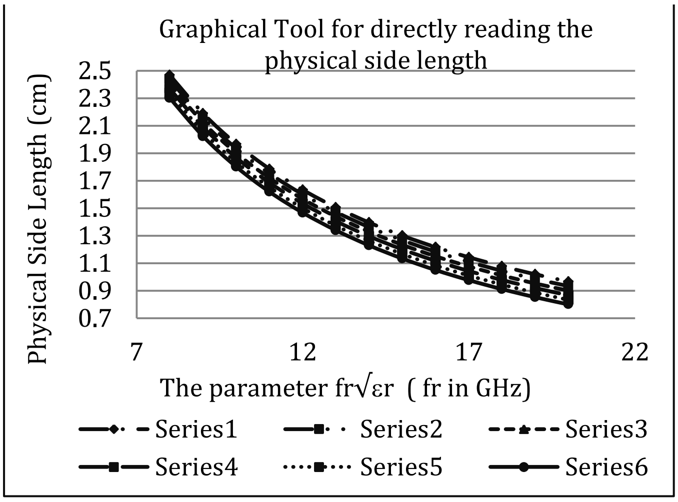

Chart-2: Tool for determining S_p

Select the curve corresponding to the desired substrate thickness h, for the value corresponding to given f_r and ε_r read S_p directly from the tool. Simplest, quickest and accurate method for determining S_p.

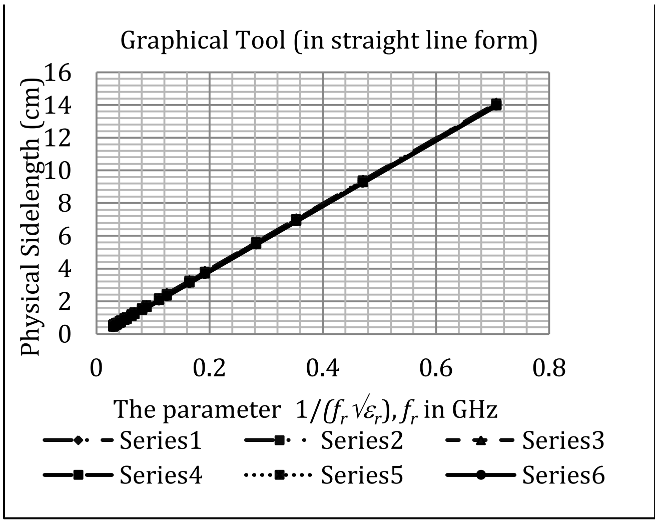

Chart-3: The Graphical Tool (straight line form) for determining S_p.

In fact (10) leads to a graphical tool for estimating *Sp* from the three basic parameters. This tool is shown below in Chart-2. In Chart-3 different series correspond to different h values. h = 0.05 cm, 0.1 cm, 0.15 cm, 0.2 cm, 0.25 cm and 0.3 cm for series 1 to 6.

Little more effort, just find inverse of $f_r\sqrt{\varepsilon_r}$. Then the curves in the tool become straight lines as shown in Chart-3. It covers a very wide range of $(1/f_r\sqrt{\varepsilon_r})$ values. At that scale all the series appear to have merged. In practice only a small range is required. This is depicted in Chart-4 which becomes the enlarged view of the straight line tool. Different series are clearly visible.

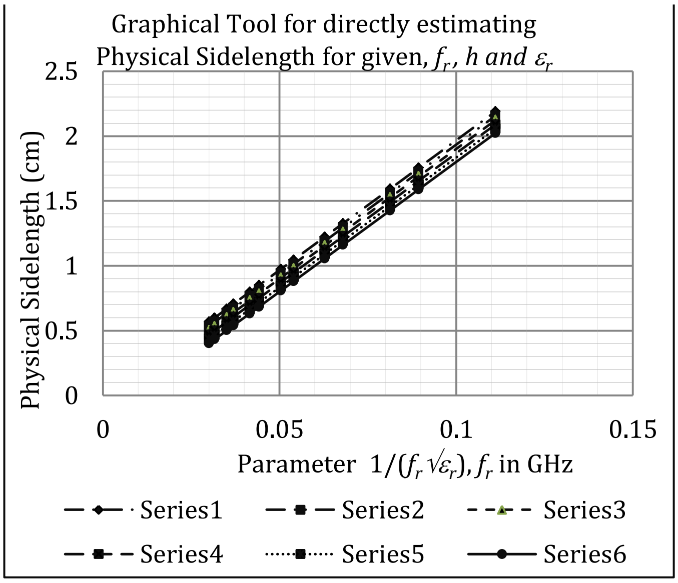

Chart-4: Enlarged view of the straight line tool.

Fringing Field Area: It is clear from model #1 of Table I that as far as the fringing is concerned the shape of the patch perhaps does not matter much. It is the total area of the patch that appears to be enlarged due to the effect of the fringing field. Therefore, for finding enlarged value of the sidelength of the triangle, different shapes (square, circle) were created. The area of the new shape was taken to be equal to the area of the triangle. The physical area and the apparently enlarged area were therefore considered.

Physical area A_p of the equilateral triangle is given by

$A_p = (\sqrt{3}/4)\, S_p^2$

The enlarged area, A_e is $A_e = (\sqrt{3}/4)\, S_e^2$

Therefore, $\frac{A_e}{A_p} = (1+\beta H)^2 = 1 + 2H$ (10)

Since, β = 1 for equilateral triangle and H^2 is negligible.

This is a very important inference. Right hand side of (10) is independent of S_p and S_e. This means that irrespective of the size of the patch, for all the equilateral triangular microstrip antennas the ratio of enlarged area (due to fringing fields) to the physical area of the patch is always equal to (1 + 2H). Earlier this ratio has been assumed and then used in calculating the $\varepsilon_{\varepsilon eff}$.

5. Results

Validation: The aim of present work is to compute physical sidelength from the material parameters ε_r and h and the desired resonance frequency f_r. This is what has been done in arriving at the new model. There should be no need to compute ε_{eff} and S_e .Therefore; the results have been compared with the models #4 (of Table-1) only. Basic parameters have been varied over large ranges — ε_r from 2 to 10, f_r from 1 GHz to 10 GHz and h from 0.05 cm to 3.0 cm. 24 results are given in Table-2. The results match very well.

Simulation Results: Very large number of ETMSA have been designed for various values of h and ε_r for resonant frequency varying from 1 GHz to 10 GHz. This leads to variations in S_e and hence S_p. The antenna structure was designed. Physical sidelength was calculated by the new formula (10) as well as by the classical formula (4). The structure was simulated, using HFSS software, for classical as well as new designs. Typical result is shown in Chart-5 which shows the S11 v/s frequency plot. Target frequency was 6.0 GHz. Referring to Table I, the dotted curve is for model #4 (simulated f_r = 5.8604 GHz), the solid curve is for the new model #9 (f_r= 5.9200 GHz) and the dashed curve is for model #7 (f_r= 6.1377 GHz).This set has high value of H (> 0.06).

6. Analysis and Discussions

Some of the models mentioned in Table I have been obtained by using Artificial Neural Network (ANN). The models differ widely even when they are given by the same author. Considering the extension in physical sidelength i.e. (S_e - S_p), the values given by existing models are $h/\sqrt{\varepsilon_r}$, $2h/\sqrt{\varepsilon_r}$, $4h/\sqrt{\varepsilon_{eff}}$, $h\left(0.1 + \frac{8}{\varepsilon_r^2}\right)$ ***and*** $h\left(1.2 + \frac{2.25}{\sqrt{\varepsilon_{eff}}}\right)$. Estimation of ε_{eff} also differs from researcher to researcher. The common thinking is that the extension is directly proportional to h and has some inverse relationship with ε_r. The new result is that the extension is equal to *2h/3.*

As the designer wants easy and quick results, model #4 gets more users. Almost everyone attributes it to [12]which is wrong. According to [12], the semiempirical relation $S_e = S_p + ph$ is a good approximation for S_e. It has been further added that with the substrate dielectric material alumina (ε_r = 10) p ≈ 1/3, and with garnet (ε_r =15) p ≈ 1/4, for S_p /h ≥ 4. Somehow p has been taken as $1/\sqrt{\varepsilon_r}$ and the condition S_p /h ≥ 4 has been forgotten. These models have been arrived at by empirical or semiempirical means and by curve fitting techniques. The new model does not suffer from such limitations.

Table-2: Comparison of the new and the classical results

S. No.	f_r	ε_r	h (cm)	S_e (cm)	S_p (cm) New	S_p (cm) Classical	S_p (new) - S_p(Classical)
1	1	10	0.1	6.3246	6.258	6.293	-0.035
2	3	8	0.1	2.357	2.2904	2.3216	-0.0312
3	5	6	0.1	1.633	1.5664	1.5922	-0.0258
4	6	4	0.1	1.6667	1.6001	1.6167	-0.0166
5	8	2	0.1	1.7678	1.7012	1.6971	0.0041
6	10	2	0.1	1.4142	1.3476	1.3435	0.0041
7	1	10	0.15	6.3246	6.2247	6.2772	-0.0525
8	3	8	0.15	2.357	2.2571	2.304	-0.0469
9	2	6	0.15	4.0825	3.9826	4.0213	-0.0387
10	3	4	0.15	3.3333	3.2334	3.2583	-0.0249
11	7	2	0.15	2.0203	1.9204	1.9142	0.0062
12	6	2	0.15	2.357	2.2571	2.2509	0.0062
13	2	10	0.2	3.1623	3.0291	3.0991	-0.07
14	3	8	0.2	2.357	2.2238	2.2863	-0.0625
15	2	6	0.2	4.0825	3.9493	4.0009	-0.0516
16	2	4	0.2	5	4.8668	4.9	-0.0332
17	4	2	0.2	3.5355	3.4023	3.3941	0.0082
18	5	2	0.2	2.8284	2.6952	2.687	0.0082
19	1	10	0.25	6.3246	6.1581	6.2455	-0.0874
20	2	8	0.25	3.5355	3.369	3.4471	-0.0781
21	2	6	0.25	4.0825	3.916	3.9804	-0.0644
22	1	4	0.25	10	9.8335	9.875	-0.0415
23	3	2	0.25	4.714	4.5475	4.5372	0.0103
24	4	2	0.25	3.5355	3.369	3.3587	0.0103

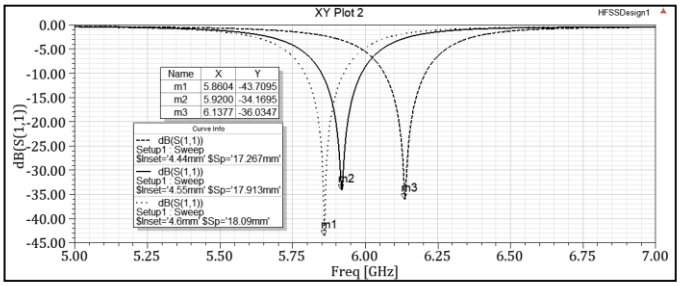

Chart-5: S_{11} v/s frequency plot for the new model (model #9) as well as for the models #4 and #7 of Table-1.

There have been two approaches , say A and B. In Approach- A, calculations are done first for ε_{eff}, then for S_e and finally for S_p using appropriate formulae and given values of ε_r, h and f_r. Approach-B takes several steps in arriving at the value of S_p. These are:

Step 1. Calculate physical area (A_p) of the triangle,

Step 2. Determine the side L_p of a square whose area is equal to A_p, Step 3. Due to fringing fields the side of the square will be effectively enlarged. Calculate enlarged side (L_e) of the square $L_e = L_p + 2\Delta L$

$$\Delta L = 0.412h\frac{(\varepsilon_{eff} + 0.3)}{(\varepsilon_{eff} - 0.258)}\frac{\left(\frac{W}{h} + 0.264\right)}{\left(\frac{W}{h} + 0.8\right)}$$

Step 4. From this value of L_e back calculate S_e , $S_e = \frac{2L_e}{3^{0.25}}$

Step 5. From this value of S_e calculate f_r using appropriate formulae.

Table-3: Validation of the new formula

H	Enlarged Area/ Physical Area		1 + 2H
	New	Classical	
0.01054	1.0214	1.01007	1.02108
0.01581	1.03236	1.01516	1.03162
0.01667	1.03415	1.02548	1.03334
0.02449	1.0508	1.03067	1.04898
0.02635	1.05481	1.02549	1.0527
0.02667	1.05549	1.04123	1.05334
0.02828	1.059	1.03073	1.05656
0.03	1.06275	1.04657	1.06
0.03266	1.06859	1.04121	1.06532
0.03536	1.07457	1.07945	1.07072
0.03771	1.07983	1.08505	1.07542
0.03771	1.07983	1.08506	1.07542
0.04	1.08498	1.06281	1.08
0.04082	1.08684	1.05191	1.08164
0.04082	1.08684	1.05196	1.08164
0.04216	1.08988	1.0412	1.08432
0.04243	1.09048	1.04654	1.08486
0.04243	1.09048	1.0965	1.08486
0.04714	1.10128	1.10802	1.09428
0.04714	1.10128	1.10802	1.09428
0.04714	1.10128	1.05195	1.09428
0.04714	1.10128	1.10805	1.09428
0.0495	1.10675	1.11393	1.099
0.05657	1.12338	1.0628	1.11314

A variation of Approach-B converts the triangle into a circle of equal area, calculates enlarged area of the circle, assumes it to be equal to the enlarged area of the triangle, uses it to calculate first S_e and then f_r.

The new approach presented in this paper directly gives S_p for the given values of ε_r, h and f_r. The graphical tool further simplifies it. In Approach-B, the enlarged area of the triangle is calculated either by first converting it into a square or a circle of equivalent physical radius, calculating the enlarged area of the square or the circle by classical means and then assuming that the enlarged area of the triangle is equal to this area. Without mentioning explicitly this assumes that the enlargement in area does not depend on the shape of the patch although it depends on the area of the patch. The new theory enunciates it — irrespective of the size of the patch, for all the equilateral triangular microstrip antennas the ratio of enlarged area (due to fringing fields) to the physical area of the patch is always equal to $(1 + 2H)$. There is no need for assumptions and calculations for estimating this area ratio. Different values of the parameter H were obtained by varying f_r, ε_r and h over a wide range. For these H values the ratio of enlarged area to the physical area of the triangle was calculated by the new approach as well as by the classical approach. The table below shows these values and also $(1+2H)$ for comparison.

It is clear from the table that

$$\frac{Enlarged\ area\ of\ the\ triangle}{Physical\ area\ of\ the\ triangle} = (1 + 2H)$$

Conclusion

For the first time a graphical tool has been developed for determining the physical sidelength of an ETMSA. Bhatnagar's Postulate has been extended to include ETMSA. Instead of curve fitting and using empirical or semiempirical approach, a theory has been propounded and then used for arriving at various formulae including that for the enlarged area of the triangle.

REFERENCES

1. R. Garg, P. Bhartia, I. Bahl and A. Ittipiboon, "Microstrip Antenna Design Handbook", Artech House, Boston, 2001
2. K. Güney, "Resonant frequency of a triangular microstrip antenna," Microwave Opt. Technol. Lett., vol. 6, pp. 555–557, July 1993.
3. D. Mathur, S. K. Bhatnagar and V. Sahula, "Quick Estimation of Rectangular Patch Antenna Dimensions Based on Equivalent Design Concept" IEEE Antennas and Wireless Propagation Letters, Vol 13,(2014), pp 1469 – 1472.
4. R. Garg and S. A. Long, "An improved formula for the resonant frequency of the triangular microstrip patch antenna," IEEE Trans. Antennas Propagat., vol. AP-36, p. 570, Apr. 1988.
5. N. Kumprasert and K. W. Kiranon, "Simple and accurate formula for the resonant frequency of the equilateral triangular microstrip patch antenna," IEEE Trans. Antennas Propagat., vol. 42, pp. 1178–1179, Aug. 1994

6. A. Gadda, S. Bedra, C. Agaba, S. Benkouda, R. Bedra and T. Fortaki, "Computer-Aided Design of Superconducting Equilateral Triangular Patch on Anisotropic Substrates" , Progress In Electromagnetics Research M, Vol. 86, 203–211, 2019
7. G. Kumar and K. P. Ray, "Broadband Microstrip Antennas", Artech House, Boston, 2003
8. D. Karaboğa, K. Güney, A. Kaplan and A. Akdağli, "A new effective side length expression obtained using a modified Tabu search algorithm for the resonant frequency of a triangular microstrip antenna." International Journal of RF and Microwave Computer-Aided Engineering, 8: 4-10. (1998)
9. D. Karabo˘ga, K. Güney, N. Karabo˘ga, and A. Kaplan, "Simple and accurate effective sidelength expression obtained by using a modified genetic algorithm for the resonant frequency of an equilateral triangular microstrip antenna," Int. J. Electron., vol. 83, pp. 99–108, Jan. 1997.
10. C. S. Gurel and Erdem Yazgan, "New Computation of the resonant frequency of a tunable equilateral triangular microstrip patch" IEEE Transactions on microwave theory and techniques, Vol 48, No. 3 March 2000, pp 334 – 338.
11. S. Swami, S. K. Bhatnagar, A. Vats and M. Mathur, "Equilateral Triangular Microstrip Antenna- A New design Formula Based on Bhatnagar's Postulate", SKIT Research Journal Vol 5, Issue 1, pp 40 (2015)
12. J. Helszajn and D. S. James, "Planar triangular resonators with magnetic walls," IEEE Trans. Microwave Theory Tech., vol. MTT-26, pp. 95–100, Feb. 1978.