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Dynamic Stability Analysis of Functionally Graded Materials

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Abstract – *This paper reports the dynamic stability analysis of Functionally Graded Materials subjected to in plane periodic load. A higher order shear deformation theory is used in conjunction with the finite element approach. A C_0 nine-noded isoparametric finite element with seven DOFs per node is used in present study. The temperature filed is assumed to be a uniform distribution over the plate. The boundaries of stability regions are obtained using Bolotin's approach and are conveniently represented in the non-dimensional excitation frequency to dynamic load amplitude. The influence of length to thickness ratio, volume fraction index, loading, boundary conditions along with temperature rise on the dynamic stability of the FGM plate is investigated. The evaluated results are compared with the available published results.*

***Key Words*: Bolotin's Approach, Dynamic Stability, Functionally Graded Materials (FGMs), Finite Element Method (FEM), Higher Order Shear Deformation Theory (HSDT)**

1. INTRODUCTION

A second order non-homogenous equation generally describes a resonant system. When the elastic system under goes normal resonance or forced resonance, the external excitation frequency is equal to natural frequency of the system. The phenomenon of dynamic stability is analyzed by second order homogenous equations. Parametric resonance refers to an oscillatory motion in a mechanical system due to time varying external excitation. The external applied loading terms appear as parameters or coefficients in the equation of motion of an elastic system. System undergoes parametric resonance when the external excitation is equal to an integral multiple of natural frequency of the system. In parametric resonance, systems amplitude increases exponentially and may grow without limit. This exponential unlimited increase of amplitude is potentially dangerous to the system. Parametric resonance is also known as parametric instability or dynamic instability. The system can experience parametric instability (resonance), when the excitation frequency or any integer multiple of it, is twice the natural frequency, that is to say

$$m\Omega = 2\omega_n$$

Where *m=1, 2, 3…n and* ω_n natural frequency of the system.

The case $\Omega = 2\omega_n$ is known as to be the most significant in application and is called main parametric resonance. Main objective of analysis of parametrically excited system is to establish the regions in the parameter space in which the system becomes unstable. These areas are known as regions of dynamic instability. The boundary separating a stable region from an unstable one is called a stability boundary. These boundaries drawn in the parameter space i.e. dynamic load amplitude, excitation frequency and static load component is called a stability diagram. Fig. 1 shows a typical dynamic stability diagram.

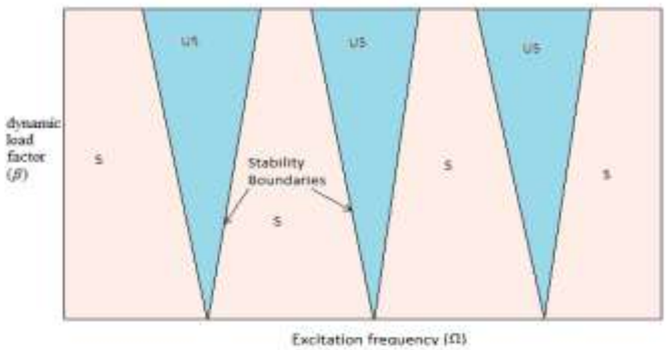

Fig -1: Stability Diagram of an excited system

The location of the unstable region closer to the dynamic load axis indicates that the system is more liable to dynamic instability, as the instability occurs at lower excitation frequencies. In contrast, if the unstable region is located farther from the dynamic load axis, it indicates that the system is less prone to dynamic instability. If the area of the instability region is large, it indicates instability over a wider frequency range. If the instability region shifts towards the dynamic load axis or there is an increase in its area, the instability of the system is said to be enhanced and when contrary to it happens, the stability is said to be improved. The parametric resonance may cause the loss of functionality of plate structures. One of the controlling methods of parametric resonance is by changing mass/stiffness. To reduce or prevent the structural vibration, the designer has to choose better materials with suitable mass/stiffness. Functionally Graded Materials (FGM) have successfully replaced the debonding and delamination problems of composite materials due to their gradual variation of properties. These types of materials also occur in nature. Bamboo and bones have functional grading. Due to the outstanding properties of FGMs they are used in many engineering applications such as aerospace, aircraft, defense, space shuttle, gas turbine blades, and rocket engine parts, biomedical and electronic industries. In future the availability and production cost of FGMs may be cheap, so that they can be used in helicopter rotor blades, turbo machinery parts and automobile parts etc. Functionally graded materials are new form of composites usually a mixture of metals and ceramics and are microscopically heterogeneous. The main role of the metal constituent in the FGM is to provide the structural support, while the other constituent is to provide heat shielding or thermal barrier when subjected to high temperature environments. The smooth transition of material provides thermal protection as well as structural rigidity. According to the material composition function specified, the volume fraction of one material constituent will be changed from 100% on one side to zero on another side, and that of another constituent will be changed the other way around as shown in figure 2.

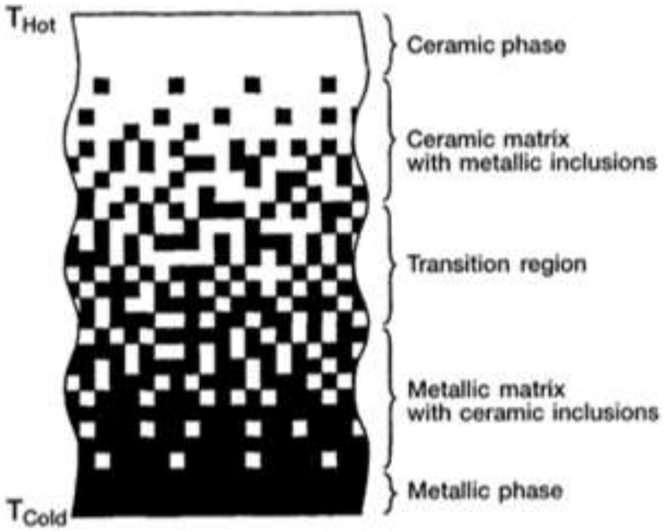

Fig -2: Microstructure of FGM

2. LITERATURE REVIEW

Pradyumna and Bandyopadhyay [1] evaluated the dynamic instability behavior of Functionally graded (FG) shells subjected to in-plane periodic load and temperature field using a higher-order shear deformation theory in conjunction with the finite element approach. Effect of material composition and geometrical parameters are studied on the dynamic instability characteristics of the five different forms of shells. Sahoo and Singh [2] analyzed dynamic stability analysis of laminated composite and sandwich Plate due to in-plane periodic load using recently developed Inverse trigonometric zigzag theory. They used C^0 continuous eight noded isoparametric element with seven field variables. Boundaries of instability region are determined using Bolotin's Approach. The influence of various parameters such as degree of orthotropy, span-thickness ratio's, boundary conditions, static load factors and thickness ratio on dynamic instability region is also studied. Balamurugan et al. [3] investigation the dynamic instability of anisotropic laminated composite plates considering geometric nonlinearity. The mathematical model is formulated using C^0 shear flexible, field consistent, QUAD-9 plate elements. The nonlinear governing equations are solved using the direct iteration technique. The effect of a large amplitude on the dynamic instability is studied for a simply-supported laminated composite plate. Detailed numerical results are presented for various parameters, namely, ply-angle, number of layers and thickness of plates. As the nonlinearity (i.e. w/h) increases, the dynamic instability region narrows down and shifts to higher frequencies. Increasing the aspect ratio, shifts the frequencies of instability region to higher values and reduces the dynamic stability strength. Increasing the number of layers and increasing the thickness of the plates results in better dynamic stability strength. Ng et al. [4] investigated the dynamic stability analysis of Functionally Graded Shells under Harmonic axial loading. Volume fraction profile was assumed as a normal mode expansion of the equation of motion yields a system of Mathieu-Hills equation the stability of which is analyzed by the Bolotin's Method. Effect of volume fraction and distribution of parametric response in particular the position and sizes of the instability regions were also studied. Darabi et al. [5] used deflection theory for solution of the dynamic stability of functionally graded shells under periodic axial loading. Material properties are assumed to be temperature dependent and graded in the thickness direction according to a simple power law distribution in terms of volume fraction of constituents. Zhu et al. [6] presented a three-dimensional theoretical analysis of the dynamic instability region of Functionally Graded (FG) piezoelectric circular cylindrical shells. The shell here is subjected to a combined axial compression and electrical field in the radial direction. Important result obtained here shows that piezoelectric effect slightly affects the unstable region. Ganapathi [7] studied the dynamic stability behavior of a clamped functionally Graded Materials spherical shell structure element subjected to external pressure load. Here the non-linearity is considered in the formulation using Von-Karman assumptions. Effect of power-law index of functionally graded materials on the axisymmetric dynamic stability characteristics of shallow spherical shells. Ramu and Mohanty [8] studied the parametric resonance characteristics of functionally graded materials (FGM) plates on elastic foundation under bi-axial in plane periodic load. It was observed that the increased foundation stiffness enhances the stability of the plates. Lanhe et al. [9] investigates the dynamic stability of thick functionally graded material plates subjected to aero-thermomechanical loads, using a novel numerical solution technique, the moving least squares differential quadrature method. The influence of various factors such as gradient index, temperature, mechanical and aerodynamic loads, thickness and aspect ratios, as well as the boundary conditions on the dynamic instability region are carefully studied. Yang et al. [10] conducted a dynamic stability analysis of symmetrically laminated FGM rectangular plates with general out-of-plane supporting conditions, subjected to a uniaxial periodic in-plane load and undergoing uniform temperature change. Theoretical formulations are based on Reddy's third-order shear deformation plate theory, and account for the temperature dependence of material properties. The critical bucking load, vibration frequencies and dynamic stability behavior are found to be highly sensitive to the thickness ratio between the FGM layers and the middle homogeneous layer, the out-of-boundary conditions, the static load level, and the side-to-thickness ratio. The plate may even be totally unstable beyond a small range of dynamic loads when high-level static compression is applied. The presence of a temperature rise degrades the structure stiffness, and hence reduces the buckling strength, lowers the vibration frequencies and decreases the excitation frequencies. Jun-quing Zhu et.al [11] presented a three-dimensional theoretical analysis of dynamic instability region of functionally graded piezoelectric circular cylindrical shells. Obtained results show that the unstable region of the structure is controlled by its geometric parameters, rigidity of material and the imposed loading. The converse piezoelectric effect only slightly affects the unstable region. However, the direct piezoelectric effects play a significant role in changing unstable regions corresponding to high order circumferential modes. T.Y. Ng et al [12] presented a formulation for the dynamic stability analysis of functionally graded shells under harmonic axial loading. The study examines the effects of the volume fraction of the material constituents and their distribution on the parametric response, in particular the positions and sizes of instability regions. Andrzej Tylikowski [13] a study of parametric vibrations of functionally graded plates subjected to in-plane time-dependent forces is presented. Material properties are graded in the thickness direction of the plate according to volume fraction power law distribution. An oscillating temperature causes generation of in-plane time- dependent forces destabilizing the plane state of the plate equilibrium. The asymptotic stability criteria involving damping coefficient and loading parameters are derived using Liapunov's direct method. Effects of power law ex-ponent on the stability domains are studied. Burney and Jaeger [14] have used stability analysis of parametrically excited systems to determine the region of the dynamic instability of a uniform column for different end conditions. They assumed the column to be consisting of different segments, each segment being considered as a massless spring with lumped masses. Piovan and Machado [15] used Bolotin's method to determine the dynamic instability regions of a functionally graded thin-walled beam subjected to heat conduction. Abbas and Thomas [16] studied the dynamic stability of beams by using finite element method for different end conditions. Shastry and Rao [17] used finite element method to plot the stability boundaries of a cantilever column acted upon by an intermediate periodic load at different positions. S. C. Mohanty et al. [18] presents the evaluation of static and dynamic behavior of functionally graded ordinary (FGO) beam and functionally graded sandwich (FGSW) beam for pined-pined end condition. The variation of material properties along the thickness is assumed to follow exponential and power law. A finite element method is used assuming first order shear deformation theory for the analysis. Sheng G.G and Wang X [19] developed a theoretical model to study the dynamic stability of the stiffened functionally graded cylindrical shell in thermal environment. FSDT and Bolotin's method are used to model stiffened FG cylindrical shells. The effects of thermal environment, stiffness number, material characteristics on the dynamic stability are examined. Briseghella et al. [20] studied the dynamic stability of elastic structures like beams and frames using finite element method.

3. MATERIAL PROPERTIES

Consider a Functionally Graded Material Plate of Width *b*, length *a*, and thickness *h* subjected to Uniform dynamic load as shown in figure 3.

The plate is assumed to be subjected to uniaxial in-plane dynamic loading represented as

$P(t)= P_s + P_t$

P_s, P_t where are the static and dynamic load components, respectively.

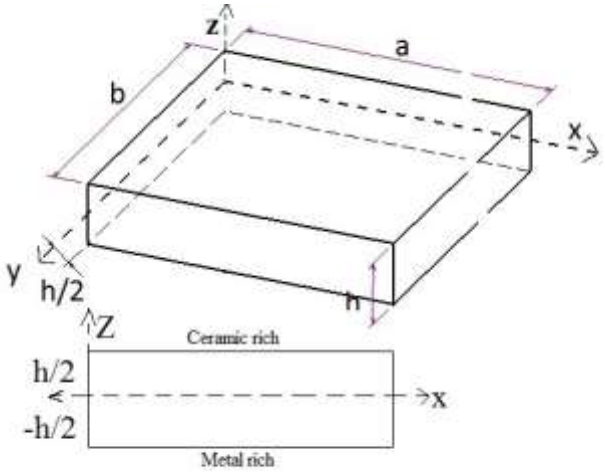

Fig -3: Geometry of Functionally Graded Plate

Assuming power law distribution in the thickness direction, the volume fraction of ceramic constituent may be written as:

$$V_{c(z)} = \left(\frac{z}{h} + \frac{1}{2}\right)^n$$

where '*z*' varies from metal surface *-h/2* to ceramic surface *+h/2*.

The materials property as a function of temperature is given as

$$P(T)= P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3)$$

where $P_0, P_1, P_2, and\ P_3$ are the coefficients of temperature *T* in Kelvin and each constituent have unique value.

$$T=T_0 + T(z)$$

Where T(z) is temperature rise through the thickness direction and T_0 is room temperature.

Based on the volume fraction of the constituent materials, the effective material properties such as Young's modulus (E_z), Poison's ratio(υ_z), mass density (ρ_z), and the coefficient of thermal expansion (α_z) of the temperature-dependent material properties are obtained using the following expressions:

$$E(z,T) = E_m(T) + [E_c(T) - E_m(T)]\left(\frac{2z+h}{2h}\right)^n$$

$$\alpha(z,T) = \alpha_m(T) + [\alpha_c(T) - \alpha_m(T)]\left(\frac{2z+h}{2h}\right)^n$$

$$v(z,T) = \upsilon_m(T) + [\upsilon_c(T) - \upsilon_m(T)]\left(\frac{2z+h}{2h}\right)^n$$

$$\rho(z,T) = \rho_m + [\rho_m - \rho_c]\left(\frac{2z+h}{2h}\right)^n$$

$$K(z,T) = K_m + [K_m - K_c]\left(\frac{2z+h}{2h}\right)^n$$

Table -1: Properties of Si_3N_4 and SUS304

Types of Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
SUS304	E(Pa)	201.04e+9	0	3.079e-4	-6.534e-7	0
	α (1/K)	12.330e-6	0	8.086e-4	0	0
	V	0.3262	0	-2.002e-4	3.797e-4	0
Si3N4	E(Pa)	348.43e+9	0	-3.070e-4	2.016e-7	-8.946e-7
	α (1/K)	5.8723e-6	0	9.095e-4	0	0
	v	0.2400	0	0	0	0

Functionally graded material considered in this study includes ceramic material as silicon- nitride and metal materials as stainless steel. Various properties considered for Si3N4- SUS304 are as given in Table 1 and Table 2.

Table -2: Properties of Si_3N_4 and SUS304

Property	Notation	Value
Thermal conductivity of Si_3N_4	k_c	9.19 W/mK
Thermal conductivity of SUS304	k_m	12.04 W/mK
Elastic moduli for Si_3N_4 (T=300K)	E_{c0}	322.2715e+9 Pa
Elastic moduli for SUS304 (T=300K)	E_{m0}	207.7877e+9 Pa
Density of Si_3N_4	ρ_c	2370 kg/m^3

4. DYNAMIC STABILITY ANALYSIS

The equation for the conservative system, with application of Bolotin's principle yields to the equation of motion of structure under in-plane load, which may be expressed in matrix form as:

$$[\mathrm{M}]\{\ddot{\delta}\} + [[\mathrm{K}] - \mathrm{P(t)}[\mathrm{K_G}]]\{\delta\} = 0 \qquad (1)$$

where $[\mathrm{M}]$, $[\mathrm{K}]$ and $[\mathrm{K_G}]$ denote the Global mass matrix, Global linear stiffness matrix, and Geometric stiffness matrix, respectively.
Above equation is general governing equation, which can be reduced to have governing equations for the Eigen value problem of buckling, Vibration and dynamic Stability of the Plate structure.

For dynamic stability problem we consider uniform loading hence we use equation above mentioned as a governing equation in Dynamic Stability Analysis.

Here load 'P' is expressed as $P(t) = P_0 + P_t$

Where P_0=Static portion of P(t)

P_t=Amplitude of dynamic portion of P(t).

The quantities P_0 and P_t are expressed in terms of static elastic buckling load P_{cr} of panel as:

$$P(t) = \alpha P_{cr} + \beta P_{cr} \qquad (2)$$

Where α=$\frac{P_0}{P_{cr}}$ and β=$\frac{P_t}{P_{cr}}$ are termed as Static and dynamic load factors, respectively.

Substituting above equation in main governing equation we get,

$$[\mathrm{M}]\{\ddot{\delta}\} + [[\mathrm{K}] - \alpha P_{cr} - \beta P_{cr}[\mathrm{K_G}]]\{\delta\} = 0 \qquad (3)$$

Above equation is a set of Mathew type equation, governing the instability behavior of the plate structure. For given values of parameters in above equation, the solution of the equation may be either bounded or unbounded. The spectrum of these values of parameters has unbounded solutions for some regions of the plane due to parametrically excited resonance. This phenomenon is known as dynamic instability and these regions are known as DIRs (Dynamic Instability Regions)

A more generalized form is achieved by presenting'δ' in trigonometric form and the governing Equation simplifies to:

$$\left[[K] - \alpha P_{cr}[K_G] \pm \tfrac{1}{2}\beta P_{cr}[K_G] - \tfrac{\omega^2}{4}[M]\right] = 0 \quad (4)$$

Above equation is basically a generalized eigenvalue problem of the systems for the known values of α, β and P_{cr}. The two conditions under a plus and minus sign indicates to two boundaries of the DIR.

The solutions of Eq. (3) yield

[1] Static buckling factor under thermal load when β = ω = 0,

[2] Free vibrations under thermal load when α = 0, β = 0,

[3] Free vibrations under thermal and axial constant mechanical load when β = 0,

[4] Regions of unstable solutions with constant α and varying β.

In this study, various types of boundary conditions namely simply supported (SSSS), clamped (CCCC), two opposite edges clamped and other two simply supported, CFFF, hinged (HHHH), SFSF, CFCF, (CSCS) can be used for investigation.

Table -3: Boundary conditions of FGMs Plates

Boundary Condition Type	Condition
All edges simply supported (SSSS)	v=w=θ_y=ψ_y=0 at x=0, a; u=w=θ_x=ψ_x=0 at y=0, b
All edges clamped (CCCC)	u=v=w=θ_y=θ_x=ψ_x=ψ_y=0 at x=0, a and y=0, b

Two opposite edges clamped and other two simply supported (SCSC)	u=v=w=θ_y=θ_x=ψ_x=ψ_y=0 at x=0 and y=0;
	v=w=θ_y=ψ_y=0 at x=a;
	u=w=θ_x=ψ_x=0 at y=0, b.

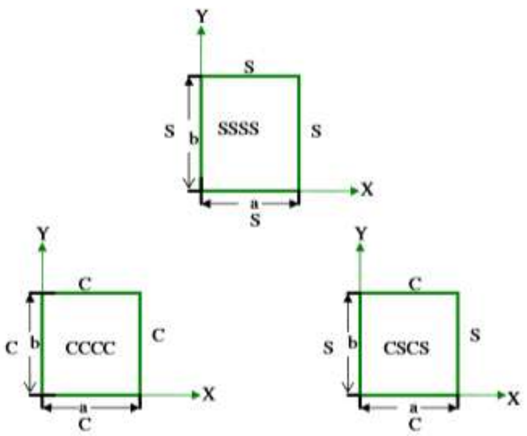

Fig -4: Schematic diagram of various boundary conditions

Following steps are used for Dynamic Stability Analysis:

1. Define the plate dimensions (length, width, thickness)
2. Define number of elements, degree of freedom per element and degree of freedom per node.
3. Define amplitude ratio. (Wmax)
4. Define nodal connectivity of selected element type.
5. State the boundary conditions (ex. Simply Supported, Clamped etc.)
6. State the Material properties and define as Temperature Independent or Dependent accordingly.
7. Define Element Stiffness matrix, Element mass matrix and element geometric matrix.
8. Assemble the element matrices to form global matrices.
9. Apply boundary conditions to obtain reduced global matrices.
10. Obtain natural frequency and buckling load.
11. Given the value of α i.e. Static load factor and known values of natural frequency and buckling load from step 10. Solve the governing equation for various values of dynamic load factor (β) to obtain lower and upper boundary limits of instability regions.
12. Obtain the non-dimensional parameter and plot the same with value of dynamic load factor (β), taking Non-dimensional parameter as X-axis and Dynamic load factor as Y-axis.
13. Study the trend of graphs obtained from different conditions like change in length to thickness ratio, power law index, temperature etc. to obtain conclusions of occurrence of dynamic stability for particular situations.

For the convenience of presentation of the present results and for the convenience of comparison with other numerical results, a non-dimensional frequency parameter is used.

$$\Omega = \omega \frac{a^2}{h} \sqrt{\frac{\rho_m (1 - v^2)}{E_m}}$$

5. RESULTS AND DISCUSSION

The dynamic stability of FGM plates under parametric excitation has been investigated for a plate with Si_3N_4 and SUS304. The power law index value, the length, the width and the thickness of the FGM plates are varied to assess their effects on the parametric instability behavior. For dynamic stability study the first, second and third mode instability regions are represented. Consider a Functionally graded plate made up from Si3N4 and SUS304 as ceramic and metal components respectively. Temperature field is assumed to be uniformly distributed over the plane of plate and varied in the thickness direction only. Material properties are assumed to be varying according to power law.

Figs.5 and 6 plot the dynamic stability results for simply supported FGM plate having uniform temperature rise and no loading condition and loading condition (α is taken as 0.2) respectively. It is seen that as we increase the value of gradient index n, the origin of instability shifts to lower forcing frequency and simultaneously the width of instability region decreases with respect to dynamic load. It is also found that for mechanically pre-stressed plate (α is taken as 0.2) origin of instability is lower than that of a previous condition (i.e. α=0) At higher dynamic load there is overlapping of boundaries of Instability regions Thus there are more chances of system to become unstable at various operating conditions.

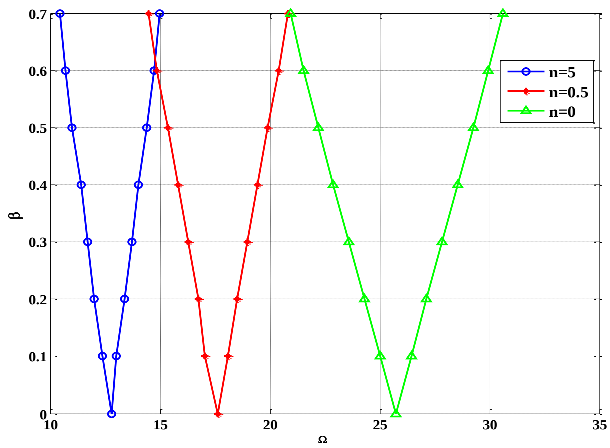

Fig-5: Effect of gradient index n on the instability region for a simply supported FGM square plate (a/h=20, α=0.0, Tc=300,Tm-300,Load=0).

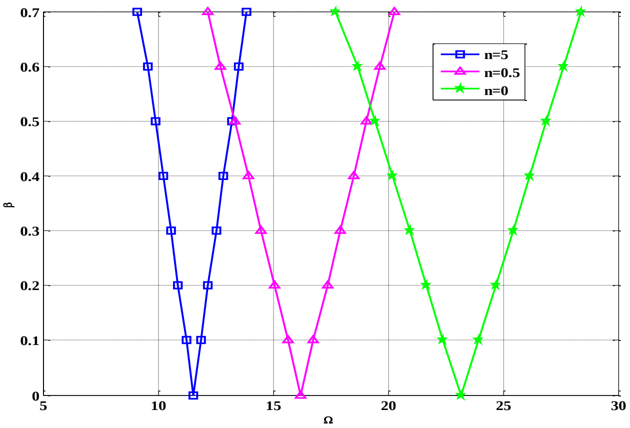

Fig-6: Effect of gradient index n on the instability region for a simply supported FGM square palte (a/h=20, α=0.0, Tc=300,Tm-300,Load=0).

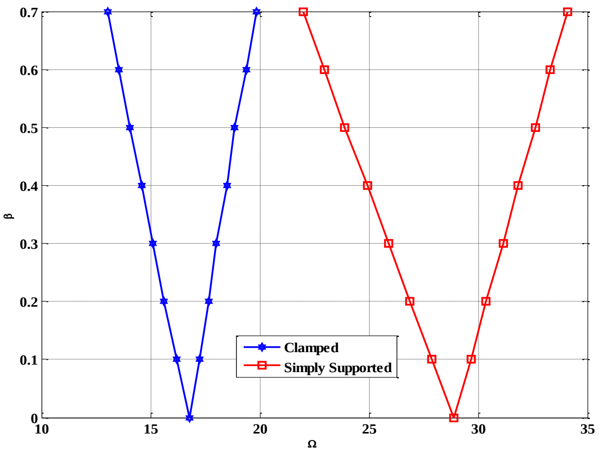

Fig-7: Effect of boundary conditions on the instability region for a square FGM plate (a/b = 1, n = 0.2, Tc = 500, Tm = 300, Load = 200, *n* = 5.0).

Figs.7 and 8 plot the dynamic stability results for simply supported FGM plate having value of gradient index n as 0.5 and 5 respectively. It is observed from the figures that Resonance occurs in case of clamped case at lower excitation frequencies as compared to that of simply supported boundary condition. Band of instability decreases in case of clamped boundary condition as compared to simply supported condition. Origin of instability in case of clamped case is lower than that of the simply supported case.

6. CONCLUSIONS

In this paper, the dynamic stability of functionally graded plates subjected to periodic in plane load is carefully studied.

Important concluions found out in the dynamic stability analysis were as follows:

1. Increase in gradient index (n) decreases the width of instability region and also the origin of unstable region.

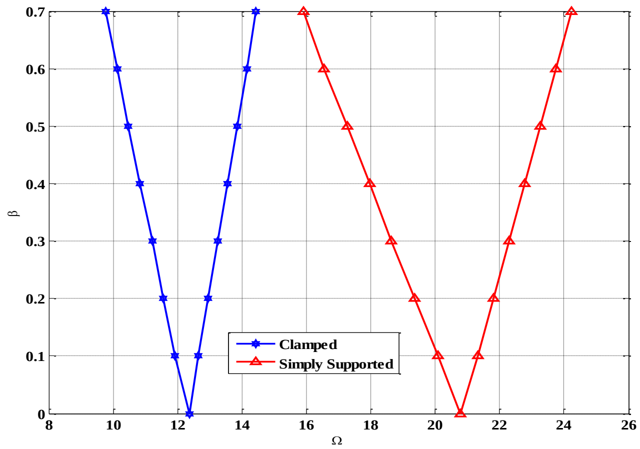

Fig-8: Effect of boundary conditions on the instability region for a square FGM plate (a/b = 1, n = 0.2, Tc = 500, Tm = 300, Load = 200, *n* = 5.0).

2. Overlapping of instability regions occurs at higher values of dynamic load factor (β) for different values of gradeint index.
3. Increase in, boundary conditions and thickness increase the excitation frequency to higher values hence decrease chances of resonance but overlapping of instability regions is at low dynamic load factor.

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