

# SOFT HYPERFILTER IN COMPLETE JOIN HYPERLATTICES

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**Abstract:** Firstly, hyper filter of complete join hyper lattices is introduced and several interesting examples of them are given. Secondly, soft hyper filter is proposed, which are generalizations of hyper filters and soft hyper filters in complete join hyper lattices. Finally, under the soft homomorphism of complete join hyper lattices, the image and pre-image of soft hyper filter are studied.

**Keywords:** complete join hyper lattices, filter, complete join hyper filter, soft hyper filter, hyper operation

## I. INTRODUCTION

In this paper we introduce soft hyper filter and complete join hyper filter, and study some properties of them. Some of the authors already proved definition of hyper lattices and few theorems related to hyper lattices, here we give some definition and theorem soft hyper filter incomplete join hyper lattice.

## II. PRELIMINARIES

### 2.1 Definition:

Let  $X$  be a universe set and  $E$  be a set of parameters. Let  $P(X)$  be the power set of  $X$  and  $A \subseteq E$ . A pair  $(F, A)$  is called a soft set over  $X$ , where  $A$  is a subset of the set of parameters  $E$  and  $F: A \rightarrow P(X)$  is a set-valued mapping.

### 2.2 Definition:

A pair  $(f, A)$  is called a fuzzy soft set over  $X$ , where  $A$  is a subset of the set of parameters  $E$  and  $f: A \rightarrow I^X$  is a mapping. That is, for all  $a \in A$ ,  $f(a) = f_a: X \rightarrow I$  is a fuzzy set on  $X$ .

### 2.3 Definition:

Let  $(L, \leq)$  be a non-empty partial ordered set and  $\vee : L \times L \rightarrow \rho(L)^*$  be a hyperoperation, where  $\rho(L)$  is a power set of  $L$  and  $\rho(L)^* = \rho(L) \setminus \{\emptyset\}$  and  $\wedge : L \times L \rightarrow L$  be an operation. Then  $(L, \vee, \wedge)$  is a hyperlattices if for all  $a, b, c \in L$ ,

- (i)  $a \in a \vee a, a \wedge a = a$ .
- (ii)  $a \vee b = b \vee a, a \wedge b = b \wedge a$ ;
- (iii)  $(a \vee b) \vee c = a \vee (b \vee c) ; (a \wedge b) \wedge c = a \wedge (b \wedge c)$ ;
- (iv)  $a \in [a \wedge (a \vee b)] \cap [a \vee (a \wedge b)]$ ;
- (v)  $a \in a \vee b \Rightarrow a \wedge b = b$ ;

Where for all non-empty subset  $A$  and  $B$  of  $L$ ,  $A \wedge B = \{a \wedge b \mid a \in A, b \in B\}$  and  $A \vee B = \cup \{a \vee b \mid a \in A, b \in B\}$ .

### 2.4 Definition:

Let  $(L, \vee, \wedge)$  is a hyper lattice. A partial ordering relation  $\leq$  is defined on  $L$  by  $x \leq y$  if and only if  $x \wedge y = x$  and  $x \vee y = y$ .

### 2.5 Definition:

A nonempty subset  $F$  of a hyper lattices  $L$  is called a filter of  $L$  if (i)  $a \wedge b \in F$  and  $a \vee x \in F$ . (ii)  $a \in F$  and  $a \leq b$  then  $b \in F$ .

**2.6 Definition:**

A filter  $F$  of  $L$  is a subset  $F \subseteq L$  followings are properties of filters:

- (i)  $1 \in F$
- (ii)  $a \in F$  and  $a \leq b, b \in L$  then  $b \in F$
- (iii) if  $a, b \in F$  then  $ab \in F$ .

**2.7 Definition:**

Let  $(L, \vee, \wedge)$  is hyper lattices. For any  $x \in L$  the set  $\{x \in L \mid a \leq x\}$  is a filter, which is called as a principal filter generated by  $a$ .

**2.8 Definition:**

Let  $L$  be a nonempty set and  $P^*(L)$  be the set of all nonempty subsets of  $L$ . A hyper operation on  $L$  is a map  $\circ: L \times L \rightarrow P^*(L)$ , which associates a nonempty subset  $a \circ b$  with any pair  $(a, b)$  of elements of  $L \times L$ . The couple  $(L, \circ)$  is called a hypergroupoid.

**2.9 Definition:**

Let  $L$  be a nonempty set endowed with two hyper operation " $\otimes$ " and " $\oplus$ ". The triple  $(L, \otimes, \oplus)$  is called a hyper lattices if the following relations hold: for all  $a, b, c \in L$ ,

- (1)  $a \in a \otimes a, a \in a \oplus a$ ;
- (2)  $a \otimes b = b \otimes a, a \oplus b = b \oplus a$ ;
- (3)  $(a \otimes b) \otimes c = a \otimes (b \otimes c), (a \oplus b) \oplus c = a \oplus (b \oplus c)$ ;
- (4)  $a \in a \otimes (a \oplus b), a \in a \oplus (a \otimes b)$ ;

**III. MAIN RESULT****3.1 Definition:**

Let  $(L, \otimes, \oplus)$  be a hyperlattice and  $S$  be a non-empty subset of  $L$ .  $S$  is called a  $\oplus$ -complete join hyper filter of  $L$  if for all  $a, b \in A$  and  $x \in L$ ,

- (1)  $a \oplus b \subseteq S$  and  $a \otimes x \subseteq S$
- (2)  $a \in S$  and  $a \leq b$  then  $b \in S$ .

**3.2 Definition:**

Complete join hyperlattice

Let  $(L, \vee, \wedge)$  be a join hyperlattice then  $L$  is called join hyperlattice if for every  $S \subseteq L$  and subset  $S^U = \{x \in L \mid (\text{for all } s \in S) s = s \wedge x\}$ ,  $S^l = \{x \in L \mid \text{for all } s \in S) s \in S \vee x\}$   $S^U$  has least element and  $S^l$  has a greatest element with the order relation  $\leq$  on  $L$ .

**3.3 Theorem:**

Any  $\otimes$  hyper filter  $S$  of a complete join hyper lattice  $L$  satisfies, If  $a \in S$  and  $a \leq b$  then  $b \in S$ .

**Proof:**

Given  $(L, \otimes, \oplus)$  be a complete join hyper lattice and  $A$  is a  $\oplus$  hyper filter of  $L$ . Assume that for all  $a \in S$  and  $a \leq b$ .

Take  $(ab) = 1 \in S$  implies  $ab \in S$  so that  $b \in S$  when  $a \in S$ .

Hence proved.

### 3.4 Theorem:

In a hyper lattice  $(L, \otimes, \oplus)$ , Every filter is a  $\otimes$  hyper filter and complete join hyper filter.

#### Proof:

Given  $(L, \otimes, \oplus)$  be a hyper lattice and  $A$  be any filter of  $L$ .

Let  $a, b \in A$ .

Take  $b(a \otimes b) = (ba) \otimes (bb) = ba \otimes 1 = ba \geq a$  implies that  $b(a \otimes b) \geq a$  and  $b(a \oplus b) \in A$  implies that  $a \otimes b \in A$ ; similarly when  $x \in L$  implies  $a \otimes x \in A$  (by the previous theorem) for all  $a \in A$  and  $a \leq b$  implies that  $b \in A$ .

From the above two result  $A$  is a  $\otimes$  hyper filter. Also, if  $a, b \in S$  then  $a \otimes b \in S$ ,  $a \in S$  and  $a \leq b$  then by previous theorem  $b \in B$  so it is complete join hyper lattices.

Hence proved.

### 3.5 Definition:

Let  $(L, \otimes, \oplus)$  be a hyper filter and  $(F, A)$  be a softest over  $L$ ,  $(F, S)$  is called a soft  $\otimes$  complete join hyperfilter over  $L$ , if  $F(x)$  is  $\otimes$  hyper filter of  $L$  for all  $x \in \text{sup}(F, A)$

### 3.6 Specimen:

Let  $\mu$  be a fuzzy  $\otimes$  hyperfilter of complete join hyperlattice  $(S, \oplus, \otimes)$  the fuzzysset of  $\mu$  satisfies the following condition:

For all  $x, y \in S$  (i)  $\vee \mu(z) \leq \mu(x) \vee \mu(y)$

(ii)  $\vee \mu(z) \leq \mu(x) \wedge \mu(y)$

Clearly,  $\mu$  is a fuzzy  $\oplus$  hyperfilter of  $L$ .if and only if for all  $t \in [0,1]$  with  $\mu_t \neq 0$ . Let  $\mu_t = \{x \in L | \mu(x) \leq t\}$  is a  $\otimes$  complete join hyper filter of  $S$ . and  $F(t) = \{x \in L | \mu(x) \leq t\}$  for all  $t \in [0,1]$  and  $F(t)$  is a  $\otimes$  complete join hyper filter of  $S$ .

#### Note:

Every fuzzy hyper  $\otimes$ filter can be interpreted as soft  $\otimes$  hyperfilter.

### 3.7 Theorem:

If  $(S, \otimes, \oplus)$  is a complete join hyper lattice and  $(F, L)$  denote softest over  $L$ , then  $(F, S)$  is a soft  $\otimes$  hyperfilter of  $S$ .

#### Proof:

By hypothesis,

$(L, \otimes, \oplus)$  be a hyperlattice. so clearly  $(L, \wedge, \vee)$  be a lattice. Now define hyper operation on  $L$ . For all  $a, b \in L$   $a \otimes b = \{x \in L | a \vee b \leq x\}$

For all  $a \in S$  define a principal filter generated by  $a$ ,  $F(a) = \{x \in L | x \geq a\} = \uparrow a$ , Hence  $F(a)$  is a  $\otimes$  hyperfilter of the hyperlattice  $L$ , also For all  $b \in S$  define a principal complete join filter generated by  $b$ ,  $G(b) = \{x \in L | x \geq b\} = \downarrow b$ , hence  $G(b)$  is  $\oplus$  hyper filter of the hyperlattice  $L$ , Now define a map  $F: L \rightarrow P(L)$  by,  $F(a) = F(a) = \uparrow a$  for all  $a \in L$  and also a map  $G: L \rightarrow P(L)$  by,  $G(b) = G(b) = \downarrow b$ .

So that,  $(F, S)$  is a soft  $\otimes$  hyper filter of  $S$ .

Hence proved.

**IV. REFERENCES**

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