A Study on Burn-In and Maintenance Policies

U. Rizwan[1], M. Sudhakaran[2]

[1,2]Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi – 635 752

Abstract - The concept of burn-in is given considering a burn-in process, using the bath-tub shaped failure rate, renewal results have been given. A cost model in product life cycle for burn-in system is also included.

Key Words: Inspection model, initially decreasing failure rate, eventually increasing failure rate, bathtub-shaped failure rate, optimal burn-in time, optimal replacement policy.

1. INTRODUCTION

Burn-in is a technique applied with the intention of eliminating early failures of a system or device. Without burn-in, defective components are more likely to be delivered to customers. We consider a burn-in procedure for a system that is maintained through periodic inspection and perfect repair at failure. We consider the problem of determining both the optimal burn-in time and optimal replacement policy under the assumption of a bathtub-shaped failure rate function.

Manufacturers guarantee the quality of their products by offering to repair or replace a faulty product free of charge for a certain length of time, referred to as the 'warranty period'. Expected warranty cost to a manufacturer generally increases in proportion to duration of warranty coverage. For consumers, longer period coverage and better warranty terms are an indication of higher reliability. Warranty coverage periods offered by manufacturers are progressively increasing and a large number of products are provided and sold with long-term warranties. A lifetime warranty protects consumers against unexpected failures occuring during the lifespan of a product and more products are offered with lifetime warranty policies (Murthy and Jack 2004).

Definition 1 ***Burn-in*** *is a manufacturing process applied to products to eliminate initial failures or weak components before their release on the market.*

Definition 2 *A failure rate function $r(t)$ is said to have a* ***bathtub shape,*** *if there exist $0 \le t_1 \le t_2 \le \infty$ such that*

$$r(t) = \begin{cases} \text{strictly decreases,} & if \quad 0 \le t \le t_1 \\ \text{is a constant, say } \lambda_0, & if \quad t_1 \le t \le t_2 \\ \text{strictly increases,} & if \quad t_2 \le t, \end{cases}$$

where t_1 and t_2 are called the (first and second) change points of $r(t)$.

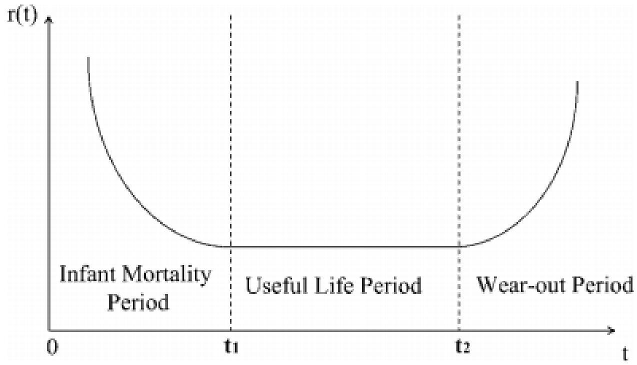

Figure : Bathtub-shaped failure rate function.

Definition 3 *The* ***optimal solution*** *is each burned-in product after the warranty is of age $b + w$. Let $u(\ge 0)$ be its post-warranty useful lifetime and L be its total useful lifetime. Therefore, the post-warranty useful lifetime is $u = L - b - w$. We assume that product failures over the post-warranty period are minimally rectified, and the cost is borne by the customer.*

Definition 4 ***Age replacement policy****, Under an age policy a system is replaced upon failure or at a fixed age T, whichever come earlier. Here, repair times are assumed to be negligible.*

Definition 5 ***Block replacement policy*** *under a block replacement policy, the system in operation is replaced upon failure and at fixed times $T, 2T, \ldots$ of the two replacement policies an age replacement policy is more difficult to excute as the age of the system must be recorded. Although block replacement policy is easier to execute as the age of the system need not be recorded, but we may have to frequently replace relatively new systems.*

Definition 6 *Under a* ***periodic replacement policy****, the system in operation is performed minimal repair at failure and replaced after a time T. The working age of a device at time t is the cumulative life-time for which the devce was working.*

Definition 7 *A failure rate function $r(x)$ is* ***initially decreasing (ID)*** *if there exists $0 < x_0 \le \infty$ such that $r(x)$ strictly decreases in $x \in [0, x_0]$. For an initially decreasing failure rate function $r(x)$, the first and second infancy points t_* and t_{**} are defined by*

$$t_* = \sup\{t \ge 0 : r(x) \text{ strictly decreases in } x \le t\}$$

$$t_{**} = \sup\{t \ge 0 : r(x) \text{ is nonincreasing in } x \le t\}.$$

Definition 8 *A failure rate function $r(x)$ is **eventually increasing (EI)** if there exists $0 \leq x_0 < \infty$ such that $r(x)$ strictly increases in $x \in [x_0, \infty)$. For an eventually increasing failure rate function $r(x)$, the first and second wear-out points t^* and t^{**} are defined by*

$$t^* = \inf\{t \geq 0: r(x) \text{ is non decreasing in } x \geq t\}$$

$$t^{**} = \inf\{t \geq 0: r(x) \text{ strictly increases in } x \geq t\}.$$

2. Optimal Burn-In for Systems with ID and/or EI Failure Rate Functions

Let a system have random life X which has distribution function $F(x)$, density $f(x)$, and failure rate function $r(x) = f(x)/\overline{F}(x)$, where $\overline{F}(x) = 1 - F(x)$ is the survival function of X. Throughout, we assume that $r(x)$ is continuous. Furthermore, let the lifetime of a system which has survived the burn-in time and cumulative distribution function (CDF) be X_b and $F_b(x)$, respectively. Then

$$F_b(x) = P(X_b \leq x)$$

$$= P(X \leq b + x | X > b)$$

$$= \frac{\overline{F}(b) - \overline{F}(b+x)}{\overline{F}(b)}, \ b, x \geq 0,$$

and

$$\overline{F}_b(x) \equiv 1 - F_b(x)$$

$$= \frac{\overline{F}(b+x)}{\overline{F}(b)}$$

$$= \exp\left(-\int_b^{b+x} r(u)du\right),$$

$b, x \geq 0$ denote the failure rate function of X_b as $r_b(x)$, which is given by $r_b(x) = r(b + x), b, x \geq 0$.

Theorem 2.1 *Suppose that the lifetime distribution function $F(t)$ has an eventually increasing failure rate function $r(t)$ with the first wear-out point t^*.*

- If $s_1 - t^* < \tau$ then $0 \leq b^* \leq t^*$;
- If $s_1 - t^* \geq \tau$ then $0 \leq b^* \leq s_1 - \tau$,

where s_1 is defined by $s_1 \equiv \sup\{t \geq t^*: r(t) = r(t^*)\}$.

Theorem 2.2 *Suppose that the failure rate function $r(t)$ is both initially decreasing and eventually increasing. Let $r^* \equiv sup\{r(t): t \geq t_{**}\}$ and assume that $r^* < r(0)$.*

- If $s_1 - t^* < \tau$, then $t_0 \leq b^* \leq t^*$;
- If $s_1 - t^* \geq \tau$, then $t_0 \leq b^* \leq s_1 - \tau$,

where $s_1 \in [t^*, t^{**}]$ is defined by $s_1 \equiv \sup\{t \geq t^*: r(t) = r(t^*)\}$ and $t_0 \in [0, t_{**}]$ is defined by $t_0 \equiv \sup\{t \leq t_{**}\ r(t) = r^*\}$.

Theorem 2.3 *Suppose that the lifetime distribution function $F(t)$ has an eventually increasing failure rate function $r(t)$ with first wear-out point t^*. Then the optimal burn-in time b^* satisfies $0 \leq b^* \leq t^* + \tau$.*

Theorem 2.4 *Suppose that the failure rate function $r(t)$ is both initially decreasing and eventually increasing. Assume that $r^* < r(0)$, where r^* is as for Theorem 2.2. Then the optimal burn-in time b^* satisfies $b_0 \leq b^* \leq t^* + \tau$, where $b_0 \equiv max\{t_0 - \tau, 0\}$ and $t_0 \in [0, t_{**}]$ is defined by $t_0 \equiv sup\{t \leq t_{**}: r(t) = r^*\}$.*

Theorem 2.5 *Suppose that the lifetime distribution function $F(t)$ has a bathtub-shaped failure rate function $r(t)$, which has change points $0 \leq t_1 \leq t_2 \leq \infty$.*

CASE I: $(b < t_1)$

In this case we define $n_2(b) \equiv \lfloor s_2(b)/\tau \rfloor$, where $s_2(b) \equiv t_2 - b$.

- If $n_2(b) = 0$, then $n^*(b) \geq 1$. Otherwise, if $n_2(b) \geq 1$, then $n^*(b) \geq n_2(b)$.
- Suppose further that $r(b) < r(\infty) \leq \infty$.

(a) If $n_3(b) \geq 1$, then $n^*(b) \leq n_3(b) + 1$;

(b) If $n_3(b) = 0$, then $n^*(b) = 1$,

where $n_3(b) \equiv \lfloor s_3(b)/\tau \rfloor$ and $s_3(b) > 0$ is uniquely determined by the equation $r(b + s_3(b)) = r(b)$.

CASE II: $(t_1 \leq b < t_2)$

Define $n_2(b) \equiv \lfloor s_2(b)/\tau \rfloor$ as in Case I. If $n_2(b) = 0$, then $n^*(b) = 1$. Otherwise, if $n_2(b) \geq 1$, then $n^*(b)$ could be any one of $\{1, 2, \ \dots, n_2(b)\}$.

CASE III: $(t_2 \ \leq b)$

In this case, $n^*(b) = 1$.

Theorem 2.6 *Suppose that the lifetime distribution function $F(t)$ has a bathtub-shaped failure rate function $r(t)$ which has change points $0 < t_1 \leq t_2 < \infty$. Then $t_1 + \tau$ is an uniform upper bound for optimal burn-in time, that is, $0 \leq b^*(n) \leq t_1 + \tau$, for all $n \geq 1$.*

Theorem 2.7 *Suppose that the lifetime distribution function $F(t)$ has a bathtub-shaped failure rate function $r(t)$ which has change points $0 < t_1 < t_2 = \infty$. Then* $A(b', \ n) < A(t_1, n) = A(b'', n)$, forall $b' < t_1, b'' > t_1$, for each fixed $n \geq 1$. That is, optimal burn-in time $b^*(n) = t_1$, for all $n \geq 1$.

3. Cost Models in Product Life Cycle

Let $E[h(b)]$ be the total expected burn-in cost for a repairable product with burn-in time b. This cost includes the cost of burn-in procedure and the expected minimal repair cost during the bum-in period $[0, b]$, that is,

$$E[h(b)] = c(b) + E[M_b], \quad (1)$$

where $c(b)$ is the burn-in cost and $E[M_b]$ is the expected minimal repair cost during the burn-in period. We assume that the burn-in cost is a sum of the fixed setup cost of the burn-in period of product and the variable cost which is proportional to the length of the burn-in time with coefficient of proportionality $c_b > 0$, that is,

$$c(b) = c_s + c_b b.$$

The total expected burn-in cost for a product with burn-in time b, $E[h(b)]$ is

$$E[h(b)] = c_s + c_b b + \int_0^b c_1(t) \times r(t)dt \quad (2)$$

Each product that survives the burn-in procedure enters the market at age b. If the item fails at time t_w ($\in [0, w]$) during the warranty period (t_w is a calendar time over the warranty period), then it is repaired instantly by a minimal repair with cost of $c_2(t_w)$. We assume that $c_2(t) \geq c_1(t)$ for all $t \geq 0$.

Let $E[c_w(b)]$ be the expected warranty servicing cost of the burned-in product with burn-in time b. Then, the expected warranty servicing cost is

$$E[c_w(b)] = \int_0^w c_2(t_w) \times r_b(t_w)dt_w, \quad (3)$$

where $r_b(\cdot)$ is the failure (hazard) rate function of the product after the burn-in procedure. Since we assumed that failures during burn-in are corrected by minimal repair, we have $r_b(t_w) = r(b + t_w)$. Then

$$E[c_w(b)] = \int_b^{b+w} c_2(t-b) \times r(t)dt. \quad (4)$$

Let $E[c_p(b,w)]$ be the expected penalty cost for the manufacturer during the post-warranty period. Then

$$E[c_p(b,w)] = \int_0^u c_p(t_u) \times r_b(w + t_u)dt_u. \quad (5)$$

By replacing $r_b(w + t_u) = r(b + w + t_u)$, we have

$$E[c_p(b,w)] = \int_{b+w}^L c_p(t-b-w) \times r(t)dt. \quad (6)$$

where the penalty cost depends on the failure time t_u and $c_p(t_u)$ is a continuous non-increasing function of t_u.

Then $E[c(b,w)]$ be the total mean servicing cost per unit of product, subjected to a burn-in procedure with time b and sold under warranty of length w. Therefore,

$$E[c(b,w)] = E[h(b)] + E[c_w(b)] + E[c_p(b,w)]. \quad (7)$$

By substituting equations (2), (4) and (6) into (7), the following non-linear optimization problem is considered

$$\text{Min} c_s + c_b b + \int_0^b c_1(t) \times r(t)dt + \int_b^{b+w} c_2(t \;\; b) \times r(t)dt + \int_{b+w}^L c_p(t-b+w) \times r(t)dt$$

$$s.t: b \geq 0, \quad w \geq 0 \text{ and } b + w \leq L. \quad (8)$$

By taking the derivatives of $E[c(b, w)]$ with respect to b and w, we obtain

$$\frac{d}{db}E[c(b,w)] = c_b + c_1(b) \times r(b) + \left[c_2(w) \times r(b+w) - c_2(0) \times r(b) + \int_b^{b+w} r(t) \times \frac{dc_2(t-b)}{db}dt\right] + \left[-c_p(0) \times r(b+w) + \int_{b+w}^L r(t) \times \frac{dc_p(t-b-w)}{db}dt\right]$$

and

$$\frac{d}{dw}E[c(b,w) = c_2(w) \times r(b+w) + \left[-c_p(0) \times r(b+w) + \int_{b+w}^L r(t) \times \frac{dc_p(t-b-w)}{dw}dt\right].$$

$$c_b + [c_1(b) - c_{2m}] \times r(b) + \int_b^{b+w} r(t) \times \frac{dc_2(t-b)}{db}dt + \int_{b+w}^L r(t) \times \frac{dc_p(t-b-w)}{db}dt = [c_{p0} - c_2(w)] \times r(b+w)$$

$$\int_{b+w}^L r(t) \times \frac{dc_p(t-b-w)}{dw}dt = [c_{p0} - c_2(w)] \times r(b+w) \quad (9)$$

An upper bound for the optimal burn-in time : Assume that the product has a bathtub-shape failure rate with two change points t_J and $t_V (0 \leq t_J \leq t_V < \infty)$.

Suppose b and b' such that $t_J \leq b < b'$. From equation (2), it is obvious that $E[h(b)]$ is strictly increasing in $b > 0$. Now, we consider

$$\varsigma(b,\ b',\ w) = (E[c_w(b')] - E[c_w(b)]) + \left(E[c_p(b',\ w)] - E[c_p(b,\ w)]\right).$$

By using equations (3) and (5), we have

$$\varsigma(b,\ b',\ w) = \left(\int_0^w c_2(t)\ \times\ [r(t+b') - r(t+b)]dt\right)$$
$$+ \left(\int_0^u c_p(t)\ \times\ [r(t+w+b') - r(t+w+b)]\ dt\right).$$

Since $c_2(t) > 0, c_p(t) > 0$ and $r(t)$ is non-decreasing in $t > t_J$, we have $\varsigma(b,\ b',\ w) > 0$. This implies that the total mean servicing cost $E[c(b,w)]$ is strictly increasing in $b > t_J$, and the optimal burn-in time occurs not later than the first change point of bathtub failure rate, that is, $0 \leq b^* \leq t_J$.

Let w° be the optimal length of the warranty period such that minimises $E[c(0,w)]$,that is,w° is the solution of the following equation:

$$\int_{w^\circ}^{L} r(t) \times \frac{dc_p(t-w)}{dw} dt = \left[c_{p0} - c_2(w^\circ)\right] \times r(w^\circ),$$
$$0 \leq w^\circ \leq L.$$

That is,

$$E[c(b^*, w^*)] < E[c(0, w^\circ)], \qquad (10)$$

where $E[c(b^*, w^*)]$ can be obtained by replacing the optimal burn-in time (b^*) and optimal length of the warranty period (w^*) in equation (8), and $E[c(0,w^\circ)]$ is obtained by replacing $b = 0$ and w^* in equation (8) as follows:

$$E[c(0,w^\circ)] = \int_0^{w^\circ} c_2(t) \times r(t)dt + \int_{w^\circ}^{L} c_p(t-w^\circ) \times r(t)dt.$$

Therefore, the burn-in procedure is beneficial if

$$c_s + c_b b^* + \int_0^{b^*} c_1(t) \times r(t)dt + \int_{b^*}^{b^*+w^*} c_p(t-b^*) \times r(t)dt$$
$$+ \int_{b^*+w^*}^{L} c_p(t-b^*-w^*) \times r(t)dt$$
$$< \int_0^{w^\circ} c_2(t)\ \times r(t)dt$$
$$+ \int_{w^\circ}^{L} c_p(t-w^\circ)\ \ \times r(t)dt.$$

Conclusion

In this paper, we discussed the concept of burn-in process using the bath-tub shaped failure rate, renewal results and also we investigated the cost model in product life cycle for burn-in system.

References

[1] Cha J-H and Na MH, (2009), Accelerated burn-in procedures and system maintenance policies, *Commun StatisticalTheory Methods*, **38**, 719 – 733.

[2] G. Klutke, P.C. Kiessler, and M.A. Wortman, (2003), A critical look at the bathtub curve, *IEEE Trans. Reliab.*, **52**, 125 – 129.

[3] Murthy DNP and Jack N, (2004), Warranty servicing strategies to improve customer satisfaction, *IMA J Manag Math*, **15**, 111 – 124.

[4] J. Sarkar and S. Sarkar, (2000), Availability of a periodically inspected system under perfect repair, *J. Statis. Plan. Inf.*, **91**, 77 – 90.

[5] L.K. Wong, (1988), The bathtub does not hold water any more, *Qual. Reliab. Eng. Int.*, **4**, 279 – 286.

[6] L.K. Wong, (1989), The roller-coaster curve is in, *Qual. Reliab. Eng. Int.*, **5**, 29 – 36.

[7] Ye ZS, Tang LC and Xie M, (2012), Degradation-based burn-in with preventive maintenance, *Eur J Oper Res*, **221**, 360 – 367.