BENDING ANALYSIS OF FUNCTIONALLY GRADED BEAM CURVED IN ELEVATION USING HIGHER ORDER THEORY

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ABSTRACT:- This paper presents bending analysis of functionally graded beam curved in elevation using higher order theory, which includes both shear deformation and thickness stretching effects. Various symmetric and non-symmetric sandwich beams with FG material in the core or skins under the uniformly distributed load are considered. MATLAB code and Navier solutions are developed to determine the displacement and stresses of FG sandwich beams for various power-law index, skin-core-skin thickness ratios and boundary conditions. Numerical results are compared with those predicted by other theories to show the effects of shear deformation and thickness stretching on displacement and stresses.

Keywords: *Functionally Graded, Sandwich beam, Navier solution, Numerical Analysis*

1. INTRODUCTION

The concept is to make a composite material by varying the microstructure from one material to another material with a specific gradient. This enables the material to have the best of both materials. If it is for thermal, or corrosive resistance or malleability and toughness both strengths of the material may be used to avoid corrosion, fatigue, fracture and stress corrosion cracking. The transition between the two materials can usually be approximated by means of a power series. Beams can be analysed using classical beam theory, Timoshenko beam theory and equivalent single layer theories [1-11].

Huu-Tai Thai et. al. [12] presented static behaviour of functionally graded (FG) sandwich beams by using a quasi-3D theory, which included both shear deformation and thickness stretching effects. Finite element model (FEM) and Navier solutions were developed to determine the displacement and stresses of FG sandwich beams for various power-law index, skin-core-skin thickness ratios and boundary conditions. It concluded that the effect of normal strain is important and should be considered in static behaviour of sandwich beams. Nguyen et al. [13] presented bending, buckling and free vibration of axially loaded rectangular functionally graded beams using the first-order shear deformation theory. Effects for the power-law index, material contrast and poisson's ratio on the displacements, stresses, natural frequencies, critical buckling loads and load-frequency curves as well as corresponding mode shapes are investigated. Jiang and Ding [14] presented analytical solutions for orthotropic density functionally graded cantilever beams using the superposition principle and the trial and error method. It is recommended that these analytical solutions can serve as benchmarks for numerical methods such as finite element method, the boundary of method, etc. Li [15] developed a new unified approach for analysing the static and dynamic behaviour of functionally graded beams considering rotary inertia and shear deformation. In this study authors have reduced the Euler-Bernoulli and Rayleigh beam theories from the Timoshenko beam theory. Benatta et al. [16] applied high order flexural theories for short functionally graded symmetric beams under three point bending. The general solutions for displacement and stresses are obtained. Benatta et al. [17] also presented an analytical solution for static bending of simply supported functionally graded hybrid beams subjected to transverse uniform load based on higher order shear deformation beam theory. Analytical method is by Sallai. Sallai et al.[18] for bending analysis of simply supported sigmoid functionally graded material beam subjected to a uniformly distributed transverse loading using various shear deformation theories. Li et al. [19] develop the higher order shear deformation theory for bending of functionally graded beams. The FG beams of various end conditions including free, hinged, lamped and elastically restrained are considered. The general solutions for displacement and stresses are presented and concluded that not only deflection but also internal stresses strongly depend on the gradient variation of material properties. Giunta et al.[20] propose the several higher order refined theories for the linear static analysis of functionally graded beams via a unified formulation. It is observed that Bernoulli-Euler and Timoshenko theories are the particular cases of a unified formulation. A Navier type, closed form solution is obtained for bi-directional FGM beam.

1.1 Functionally graded materials

There are two types of graded structures which can be prepared in case of FGM, continuous structure and stepwise structure In case of continuous graded structure, the change in composition and microstructure occurs continuously with position on the other hand in case of stepwise, microstructure feature changes in stepwise manner, giving rise to a multi-layered structure with interface existing between discrete layers.

1.2 FGM Application

FG materials are preferred due to delamination, matrix cracks, stress concentration and other damage mechanisms which are often observed in fibrous composite laminates. Most commonly used FG materials are ceramic and metal. Functionally graded materials are having attractive properties such as high thermal resistance, high impact resistance, increases the bond strength and reduce the residual stress, thermal stress and crack driving forces. A low-cost ceramic-metal functionally graded material would be ideal for wear-resistant linings in the mineral processing industry. Such a material would comprise a hard ceramic face on the exposed side, a tough metal face on the rear side that can be bolted or welded to a support frame, and a graded composition from metal to ceramic in between. The gradation would enhance the toughness of the ceramic face and also prevent ceramic-metal de-bonding.

2. METHODOLOGY

Functionally graded curved sandwich beam under consideration

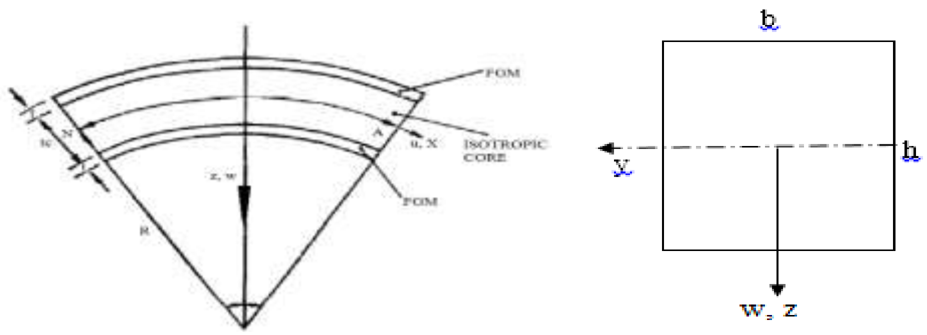

Fig.2.1 Functionally graded curved sandwich beam

Consider a functionally graded sandwich beam curved in elevation with length L and rectangular cross-section b × h, with b being the width and h being the height and with radius of curvature R.

2.1 Displacement fields

The displacement field of the present higher order shear deformation theory is given by,

$$u = \left(1 + \frac{z}{R}\right) u_0 - z\frac{\partial w_0}{\partial x} + f(z)\,\phi_x$$
$$w = w_0 + \beta\, g(z)\,\phi_z \qquad (1)$$

where, u, w are the axial and transverse displacements, u_0, w_0 are the axial displacements of a point on the neutral axis, $\frac{\partial w_0}{\partial x}$ is bending slope and ϕ_x, ϕ_z are the shear slopes.

2.2 Strains

The non-zero normal and transverse shear strains associated with the displacement field in equation are obtained within the framework of linear theory of elasticity,

$$\in_x = \frac{\partial u}{\partial x} + \frac{w}{R} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} + f(z)\frac{\partial \phi_x}{\partial x} + \frac{w_0}{R} + \beta\frac{g(z)\phi_z}{R}$$

$$\in_z = \frac{\partial w}{\partial z} = \beta\frac{\partial[g(z)]}{\partial z}\phi_z \qquad (2)$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} - \frac{u_0}{R} = \frac{u_0}{R} - \frac{\partial w_0}{\partial x} + g(z)\phi_x + \frac{\partial w_0}{\partial x} + \beta g(z)\frac{\partial \phi_z}{\partial x} - \frac{u_0}{R} = g(z)\left[\phi_x + \beta\frac{\partial \phi_z}{\partial x}\right]$$

2.3 Stresses

The stress-strain relationship at any point in the beam is given by the two dimensional Hooke's law as follows,

$$\{\sigma\} = [Q]\{\in\}$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_z\\ \tau_{xz}\end{Bmatrix} = \begin{pmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{33}\end{pmatrix}\begin{Bmatrix}\in_x\\ \in_z\\ \gamma_{xz}\end{Bmatrix} \qquad (3)$$

$$Q_{11} = Q_{22} = \frac{E(z)}{(1-\mu^2)} \quad Q_{12} = \mu\frac{E(z)}{(1-\mu^2)} \quad Q_{33} = \frac{E(z)}{2(1+\mu)}$$

where, σ_x is normal stress, τ_{xz} is transverse shear stress, E is Young's modulus and ε_x is normal strain, μ is Poisson's ratio.

2.4 Principle of virtual work

$$\int_0^L q\delta w_0 dx = \int_0^L\int_{-h/2}^{+h/2}\left(\sigma_x\delta\in_x + \sigma_z\delta\in_z + \tau_{xz}\delta\gamma_{xz}\right)\, dz dx$$

$$= \int_0^L\int_{-h/2}^{+h/2}\sigma_x\left(\frac{\partial\delta u_0}{\partial x} - z\frac{\partial^2\delta w_0}{\partial x^2} + f(z)\frac{\partial\delta\phi_x}{\partial x} + \frac{\delta w_0}{R} + \beta\frac{g(z)\delta\phi_z}{R}\right)dz\,dx \qquad (4)$$

2.5 Governing Equations

The governing equations can be obtained by integrating the derivatives of the varied quantities by parts and collecting the coefficients of δu_0, δw_0, $\delta\phi_x$ and $\delta\phi_z$

$$\delta u_0 : \frac{\partial N}{\partial x} = 0$$

$$\delta w_0 : q + \frac{\partial^2 M^b}{\partial x^2} - \frac{N}{R} = 0$$

$$\delta\phi_x : Q_x - \frac{\partial M^s}{\partial x} = 0 \qquad (5)$$

$$\delta\phi_z : Q_z + \frac{Q_{xz}}{R} - \frac{\partial Q_x}{\partial x} = 0$$

2.6 Method of solution

The Navier method is used for static analysis in the simply supported sandwich beam. Field can be assumed

$$\begin{Bmatrix} u_0 \\ w_0 \\ \phi_x \\ \phi_z \\ q \end{Bmatrix} = \begin{Bmatrix} u_{mn}\cos\alpha x \\ w_{mn}\sin\alpha x \\ \phi_{xmn}\cos\alpha x \\ \phi_{zmn}\sin\alpha x \\ q_{mn}\sin\alpha x \end{Bmatrix} \quad ....\text{where } \alpha = (m\pi / L) \tag{6}$$

By substituting these equation into equations (5), four differential equations can be obtained as

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \begin{Bmatrix} u_{mn} \\ w_{mn} \\ \phi_{xmn} \\ \phi_{zmn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ q \\ 0 \\ 0 \end{Bmatrix} \tag{7}$$

Where ,

$$\begin{aligned} & K_{11} = A_{11}\alpha^2, \; K_{12} = -(B_{11}\alpha^3 + \psi(A_{11}/R)\alpha) = K_{21} \\ & K_{13} = C_{11}\alpha^2 = K_{31}, \; K_{14} = -(\psi(K_{11}/R) + D_{12})\alpha\beta = K_{41} \\ & K_{22} = E_{11}\alpha^4 + 2\psi(B_{11}/R)\alpha^2 + \psi(A_{11}/R^2) \\ & K_{23} = -(F_{11}\alpha^3 + \psi(C_{11}/R)\alpha) = K_{32} \\ & K_{24} = (\psi(M_{11}/R)\beta + G_{12}\beta)\alpha^2 + \psi(AK_{11}/R^2)\beta + \psi(D_{12}/R)\beta = K_{42} \\ & K_{33} = H_{11}\alpha^2 + L_{33}, \; K_{34} = -(\psi(N_{11}/R) + I_{12} - L_{33})\alpha\beta = K_{43} \\ & K_{44} = L_{11}\psi(\beta^2/R^2) + 2O_{12}\psi(\beta^2/R) + J_{22}\beta^2 + L_{33}\alpha^2\beta^2 \end{aligned} \tag{8}$$

3. NUMERICAL RESULTS AND DISCUSSION

The material properties of metal, ceramic and FGM layers are as the following

E_{metal} = 70 GPa and $E_{ceramic}$ = 380 GPa

Non-dimensional maximum axial and transverse deflection of the beam are considered as

$$U_N = \frac{100E_m h^3 U}{L^4 Q_0}, \quad W_N = \frac{100E_m h^3 W}{L^4 Q_0}$$

Non-dimensional maximum axial and shear stresses of the beam are considered as

$$\sigma_{Nx} = \frac{h\sigma_x}{LQ_0} \quad \text{and} \quad \sigma_{Nz} = \frac{h\sigma_z}{LQ_0} \quad \tau_N = \frac{h\tau_{zx}}{LQ_0}$$

Table 1: The maximum transverse deflection of single layer FG curved beam

L/h	p	w					
		R=5	R=10	R=20	R=50	R=100	Straight beam
5	0	2.4776	2.4778	2.4778	2.4778	2.4778	2.4778
	1	4.8587	4.8456	4.8391	4.8352	4.8339	4.8326
	2	6.2263	6.2094	6.201	6.1961	6.1944	6.1928

	5	7.6173	7.5993	7.5904	7.585	7.5833	7.5815
	10	8.5354	8.5148	8.5046	8.4985	8.4965	8.4944
10	0	2.3191	2.3192	2.3192	2.3193	2.3193	2.3193
	1	4.592	4.5785	4.5717	4.5677	4.5664	4.5651
	2	5.8515	5.8341	5.8255	5.8204	5.8187	5.817
	5	6.9954	6.977	6.9679	6.9624	6.9606	6.9588
	10	7.7705	7.7495	7.739	7.7327	7.7307	7.7286
100	0	2.2664	2.2666	2.2666	2.2666	2.2666	2.2666
	1	4.5034	4.4897	4.483	4.4789	4.4776	4.4762
	2	5.727	5.7095	5.7009	5.6957	5.694	5.6923
	5	6.7891	6.7705	6.7613	6.7558	6.754	6.7522
	10	7.5167	7.4955	7.4849	7.4786	7.4765	7.4744

Table 2: The maximum normal stress σ_x of single layer FG curved beam

L/h	P	σ_x					
		R=5	R=10	R=20	R=50	R=100	Straight Beam
5	0	3.1112	3.1254	3.1325	3.1368	3.1382	3.1396
	1	4.8295	4.8467	4.855	4.8598	4.8614	4.863
	2	5.6526	5.6708	5.6796	5.6848	5.6865	5.6882
	5	6.6694	6.6904	6.7006	6.7066	6.7086	6.7105
	10	7.9737	7.9991	8.0113	8.0186	8.021	8.0233
10	0	6.1442	6.176	6.1918	6.2014	6.2046	6.2077
	1	9.5303	9.5684	9.5869	9.5978	9.6013	9.6049
	2	11.1312	11.1718	11.1913	11.2028	11.2066	11.2104
	5	13.0739	13.1207	13.1433	13.1567	13.1611	13.1655
	10	15.6496	15.706	15.7334	15.7495	15.7549	15.7602
100	0	61.1879	61.5167	61.6811	61.7797	61.8126	61.8455
	1	94.886	95.2795	95.4701	95.5825	95.6197	95.6566
	2	110.7456	111.1645	111.3666	111.4856	111.5248	111.5639
	5	129.8731	130.356	130.5903	130.7286	130.7743	130.8198
	10	155.5221	156.1043	156.3874	156.5547	156.61	156.6651

Table 3: The maximum normal stress σ_z of single layer FG curved beam

L/h	P	σ_z					
		R=5	R=10	R=20	R=50	R=100	Straight Beam
5	0	-0.3121	-0.3074	-0.3052	-0.3039	-0.3035	0.303
	1	0.3211	0.3507	0.3656	0.3745	0.3775	0.3805
	2	0.3696	0.407	0.4258	0.437	0.4408	0.4446
	5	-0.3821	0.3932	0.4137	0.426	0.4302	0.4343
	10	0.3683	0.4155	0.4393	0.4537	0.4584	0.4632

10	0	-0.456	-0.4456	-0.4406	-0.4377	-0.4367	0.4358
	1	-0.6581	-0.6404	-0.6314	-0.6259	-0.6241	-0.6222
	2	-0.7472	-0.7295	-0.7205	-0.715	-0.7132	-0.7113
	5	-0.6834	-0.6899	-0.6934	-0.6955	-0.6962	-0.6969
	10	-0.5941	-0.6015	-0.6054	-0.6078	-0.6086	-0.6094
100	0	-4.0057	-3.8981	-3.8466	-3.8164	-3.8064	3.7965
	1	-6.905	-6.73	-6.6404	-6.586	-6.5677	-6.5494
	2	-7.7549	-7.5798	-7.4902	-7.4358	-7.4176	-7.3993
	5	-6.5667	-6.6298	-6.663	-6.6834	-6.6903	-6.6972
	10	-5.61	-5.6812	-5.7188	-5.742	-5.7498	-5.7577

Table 4: The maximum shear stress τ_{xz} of single layer FG curved beam

L/h	P	τ_{zx}					
		R=5	R=10	R=20	R=50	R=100	Straight Beam
5	0	0.4914	0.4914	0.4914	0.4914	0.4914	0.4914
	1	0.5317	0.5316	0.5316	0.5315	0.5315	0.5315
	2	0.5501	0.55	0.55	0.5499	0.5499	0.5499
	5	0.509	0.5089	0.5089	0.5089	0.5089	0.5089
	10	0.4386	0.4385	0.4385	0.4385	0.4385	0.4385
10	0	0.4919	0.4919	0.4919	0.4919	0.4919	0.4919
	1	0.5322	0.5321	0.5321	0.5321	0.5321	0.5321
	2	0.5507	0.5507	0.5506	0.5506	0.5506	0.5506
	5	0.5098	0.5097	0.5097	0.5097	0.5097	0.5097
	10	0.4393	0.4392	0.4392	0.4392	0.4392	0.4392
100	0	0.4921	0.4921	0.4921	0.4921	0.4921	0.4921
	1	0.5324	0.5323	0.5323	0.5322	0.5322	0.5322
	2	0.551	0.5509	0.5508	0.5508	0.5508	0.5508
	5	0.5101	0.51	0.51	0.5099	0.5099	0.5099
	10	0.4395	0.4394	0.4394	0.4394	0.4394	0.4394

CONCLUSION:

Based on a higher order shear deformation theory, MATLAB code and Navier solutions are developed to determine the displacement and stresses of FG sandwich beams. This theory includes both shear deformation and thickness stretching effects. Single layer functionally graded straight and curved beams are considered. Numerical results are compared with those predicted by other theories to show the effects of shear deformation and thickness stretching on the displacement and stresses.

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