Analysis of Two Phase Flow Induced Vibration in Piping Systems

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Abstract — *Flow induced vibration with internal fluid flow for long slender cylindrical pipe conveying fluid is studied in this thesis. The fourth order partial differential equation of motion for lateral vibration is employed to develop the stiffness and inertia matrix corresponding to two of the terms of the equations of motion. The Equation of motion further includes a mixed-derivative term that was treated as a source for a dissipative function. Four type of boundary condition namely pinned-pinned, clamped-pinned, clamped-clamped and clamped-free were considered for the pipe. Analytical approach is used which are based on Galkerin's method to finding the natural frequency at different boundary conditions and velocities. Further, ANSYS 15.0 is used as a tool for computational analysis for different boundary conditions and velocities. A comparison are made between these two methods and evaluate the percentage error to check the accuracy of the solutions. Pipe buckling or divergence is observed by increasing the flow velocity of fluid for different boundary conditions. The velocity at which the buckling starts is called critical velocity and natural frequency is diminishes at the onset of divergence for pinned-pinned, clamped-pinned, clamped-clamped boundary conditions. But very little effect was observed for the velocity ranges used. The instability of pipe in pinned-pinned, clamped-pinned, clamped-clamped boundary conditions are due to centrifugal forces in the pipe is observed. But, for clamped-free pipe the Coriolis component of force causes instability in pipe.*

Keywords- *cascade induced vibration; Mechanics of fluid; Momentum transfer; Slender cylindrical pipe; clamped free pipe.*

I. INTRODUCTION

1.1 **Overview of Internal Flow Induced Vibrations in Pipes** The vibration brought on by a liquid flowing in or around a body is known as Flow Induced Vibration (FIV). FIV best portray the cooperation that happens between the liquid's dynamic strengths and a structure's inertial, damping and flexible forces. The flow of a liquid through a pipe can force pressure on the walls of the pipe making it deflect under certain conditions. This deflection of the pipe may cause structural instability of the pipe. The fundamental natural frequency of a pipe for the most part reduces with increasing speed of liquid flow. There are sure situations where reduction in the natural frequency can be critical, for example, those utilized as a part of sustain lines to rocket engines and water turbines. The pipe ends up noticeably to resonance or fatigue failure if its natural frequency decreases up to certain limits. With expansive liquid speeds the pipe may become unstable. The most common type of this instability is the whipping of an unlimited garden hose. The investigation of dynamic reaction of a liquid passing on pipe in conjunction with the transient vibration of cracked pipes uncovers that if a pipe breaks through its cross area, then an adaptable length of unsupported pipe is left spewing out fluid and is allowed to whip about and affect different structures. In power plant plumbing pipe whip is one of probable condition for a failure of pipe. An investigation of the impact of the subsequent high speed liquid on the static and dynamic qualities of the pipe is accordingly vital.

1.2 Classification Of Flow Induced Vibration The classification of flow induced vibration is shown in the figure 1.The classification is categorize according to the type of flow:

1. Steady

2. Unsteady

In steady flow the vibration in pipes is due to two phenomenon. The first is instability and other is vortex induced. The instability in pipes occurs due to flutter, galloping and fluid elastic instability. Flutter basically occurs in pinned-pinned and pinned-clamped pipes. When fluid velocity reaches a critical velocity the buckling (divergence) phenomenon occurs due to centrifugal forces and pipe will become unstable. This phenomenon is called flutter. The fundamental frequency is zero due to flutter and instability in pipes occurs.

1.3 Slender structures in axial flow practically, the dynamics of the problems are often too difficult to fully describe starting purely from the fundamentals of Mechanics such as Newton's Laws or Hamilton's principle. Rather, a multi-faceted investigation involving empiricism, experience, numerical simulation, experiments and analogy to simpler problems is necessary; unfortunately, by being so specific, rarely can the results be generally applicable. Luckily, there exist more than a few cases for which general theory can be established under a set of reasonable assumptions, and from which analytical or semi-analytical solutions are realistically obtainable and verifiable using relatively straightforward experiments. Slender structures subjected to axial flow are one class of problems that fit these criteria.

The simplest examples of slender structures involving flow are pipes, cylinders, plates and shells. Idealized though they may be, these systems are in many cases directly applicable to common real-life uses such as pipelines, heat exchangers and fuselages. Moreover, their usefulness in establishing benchmarks and first estimates for more complex systems is equally important, and constitutes a major motivation for their analysis. Finally and perhaps most importantly according to some they are vital in helping to understand the fundamental nature of dynamical systems.

1.4 Pipes conveying fluid The present work is limited to the dynamics of pipes conveying fluids. Evidently, pipes can be found just about anywhere, transporting different kinds of fluid for different purposes. Whether they be utilized in reactors, heat exchangers, pipelines, mines, city streets or as the aforementioned garden hose, it is crucial to understand under what conditions these pipes will fail and in what way. In addition, more so than any other simple system mentioned above, the pipe conveying fluid has become a paradigm in dynamics, a well-understood stepping stone for tackling complex systems and gaining fundamental insight into their dynamics.

Even by restricting oneself to the study of pipes conveying fluid, the possibilities are almost endless, as witnessed by the production of several hundred publications over the last 50 years, with still many more emerging each year. Pipes conveying fluid have been studied using linear models to determine basic characteristics, and sophisticated non-linear models to predict complex motions, in some cases breaking down into chaos. They have been utilized to develop new methods of analysis, ranging from the purely analytical, to the semi-analytical, to the purely numerical. In short, research on pipes conveying fluid continues to fuel the imagination, while proving the usefulness of understanding fundamentals in a practical world.

1.5 Objective The main objective of the thesis is to implement numerical solutions method, more specifically the Finite Element Analysis (FEA) to obtain solutions for different pipe configurations and fluid flow characteristics. The governing dynamic equation describing the induced structural vibrations due to internal fluid flow has been formed and discussed. The governing equation of motion is a partial differential equation that is fourth order in spatial variable and second order in time. The analytic approach is used to find natural frequencies of cylindrical beam with different boundary conditions and further verified by using Ansys 15.0. Further, how various parameters like velocity of fluid affects the natural frequency of pipe and involve in the dynamic instability of the pipe has also been discussed.

II. Fundamentals of Flow Induced Vibration In Pipes Conveying Fluids

2.1 Defining the pipe conveying fluid The pipe conveying fluid is one of many dynamical systems consisting of axially moving continua, a class of problems involving momentum transport that also includes high speed magnetic and paper tapes, aerial cable tramways, band-saw blades, power transmission chains and belts and extrusion processes. In being relatively easy to understand, model and conduct experiments on, the pipe conveying fluid can be employed to display a wide array of interesting yet fundamental dynamical behaviour. Consequently, it can be used to glean generic but important characteristics of more complex systems involving flowing fluid, such as shells or cylinders in axial flow, or other similar problems. For these reasons has the pipe conveying fluid come to be considered a model dynamical problem.

A pipe conveying fluid, depicted in Fig. 2.1, is, effectively a hollow, flexible beam transporting fluid from one end to the other. The pipe is slender, implying that its length is on the order of 20 times greater than its diameter ($L/D \geq 20$), and has a considerable thickness when compared to the diameter, thereby differing from its relative the cylindrical shell. In general, the pipe may be either extensible or inextensible; however, this discussion will be restricted to the latter. Finally, the movement of the pipe is assumed to be restricted to a plane. The pipe may thus be modelled as an Euler-Bernoulli beam; in more complicated cases, resorting to the Timoshenko beam theory may become necessary.

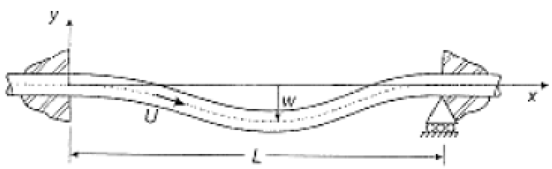

Figure 2.1 Diagram of a pipe conveying fluid, positively clamped at the upstream end and clamped with axial sliding permitted at the downstream end.

2.2 Studying the pipe conveying fluid Though there exist several motivations for the investigation of a pipe conveying fluid, the primary concern is nearly always that of stability: specifically, under what conditions will the system become unstable and therefore exhibit often undesirable or even catastrophic behaviour? A linear model for the pipe can usually accurately predict its most important stability characteristics, as well as indicate what kind of circumstances need to be avoided to conserve stability. However, to explore the more complicated and arguably more interesting – features of the system, including the amplitude, the type of motion, the detailed nature of the instability, or the post-instability behaviour, a non-linear model is absolutely essential. The theoretical scope of this thesis is linear in nature, with references to non-linear behaviour appearing only briefly and superficially; for a detailed discussion of the non-linear dynamics of pipes conveying fluid.

2.3 Added Mass Effect It is a phenomenon in which the fluid flow inside a pipe exerts a force due to the mass of fluid entrained by the cylinder. This force is called added mass or hydrodynamic mass and it acts in the direction of fluid acceleration. The fluid added mass increases the effective structural mass for dynamic analysis. The magnitude of the effect depends on the density of fluid relative to the mass of structure.

2.4 Equation of Motion for Pipe Conveying Fluid A fluid flows through the pipe at pressure ‘p’ and density ρ at a constant velocity ‘v’ through the internal pipe cross-section of area ‘A’. As the fluid flows through the internal pipe it is accelerated, because of the changing curvature of the pipe and the lateral vibration of the pipeline. The vertical component of fluid pressure applied to the fluid element and the pressure force F per unit length applied on the fluid element by the tube walls oppose these accelerations. Referring to figure (2.2), balancing the forces in the Y direction on the fluid element for small deformations, gives

$$F - \rho A \frac{\partial^2 y}{\partial x^2} = \rho A\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)^2 Y \qquad (2.1)$$

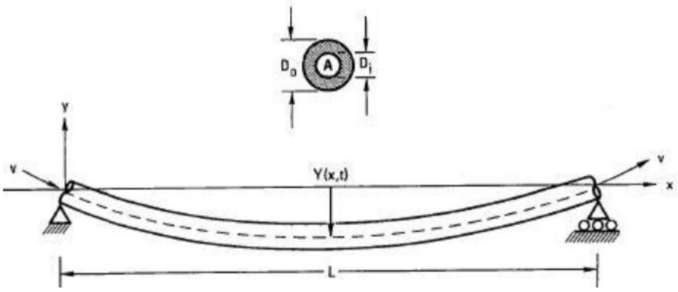

Figure 2.2 Pinned-Pinned Pipe Carrying Fluid

The pressure gradient in the fluid along the length of the pipe is opposed by the shear stress of the fluid friction against the tube walls. The sum of the forces parallel to the pipe axis for a constant flow velocity gives

$$A\frac{\partial \rho}{\partial x} + \psi S = 0 \qquad (2.2)$$

Where S is the inner perimeter of the pipe, and ' is the shear stress on the internal surface of the pipe. The equations of motions of the pipe element are derived as follows.

$$\frac{\partial T}{\partial x} + \psi S - Q\frac{\partial^2 y}{\partial x^2} = 0 \qquad (2.3)$$

Where Q is the transverse shear force in the pipe and T is the longitudinal tension in the pipe. The forces on the element of the pipe normal to the pipe axis accelerate the pipe element in the Y direction. For small deformations,

$$\frac{\partial Q}{\partial x} + T\frac{\partial^2 y}{\partial x^2} - F = m\frac{\partial^2 y}{\partial t^2} \qquad (2.4)$$

Where m is the mass per unit length of the empty pipe. The bending moment M in the pipe, the transverse shear force Q and the pipe deformation are related by

$$Q = \frac{\partial M}{\partial x} = \frac{\partial^3 y}{\partial x^3} \quad (2.5)$$

Combining all the above equations and eliminating Q and F yields:

$$EI\frac{\partial^4 y}{\partial x^4} + (\rho A - T)\frac{\partial^2 y}{\partial x^2} + \rho A\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)^2 Y + m\frac{\partial^2 y}{\partial t^2} = 0 \quad (2.6)$$

The shear stress may be eliminated from equation 3.2 and 3.3 to give:

$$\frac{\partial(\rho A - T)}{\partial x} = 0 \quad (2.7)$$

At the pipe end where x=L, the tension in the pipe is zero and the fluid pressure is equal to ambient pressure. Thus ρ =T=0 at x=L,

$$\rho A - T = 0 \quad (2.8)$$

The equation of motion for a free vibration of a fluid conveying pipe is found out by substituting ρA -T = 0 from equation 2.8 in equation 2.6 and is given by the equation:

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A v^2\frac{\partial^2 y}{\partial x^2} + 2\rho A v\frac{\partial^2 y}{\partial x \partial t} + M\frac{\partial^2 y}{\partial t^2} = 0 \quad (2.9)$$

Where the mass per unit length of the pipe and the fluid in the pipe is given by M = m + ρA. The next section describes the forces acting on the pipe carrying fluid for each of the components of equation (2.9).

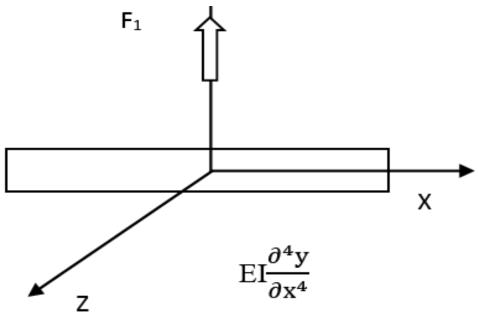

Figure 2.3 Force due to Bending

Representation of the First Term in the Equation of Motion for a Pipe Carrying Fluid

The term $EI\frac{\partial^4 y}{\partial x^4}$ is a force component acting on the pipe as a result of bending of the pipe. Figure (2.3) shows a schematic view of this force F_1.

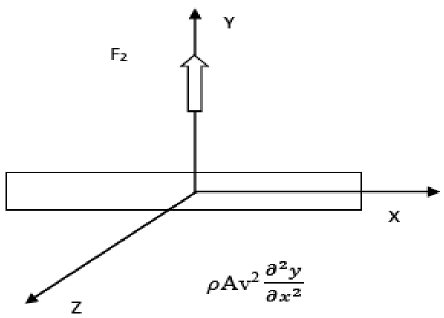

Figure 2.4 Force that Conforms Fluid to the Curvature of Pipe

Representation of the Second Term in the Equation of Motion for a Pipe Carrying Fluid

The term $\rho A v^2 \frac{\partial^2 y}{\partial x^2}$ is a force component acting on the pipe as a result of flow around a curved pipe. In other words the momentum of the fluid is changed leading to a force component F_2 shown schematically in Figure (2.4) as a result of the curvature in the pipe.

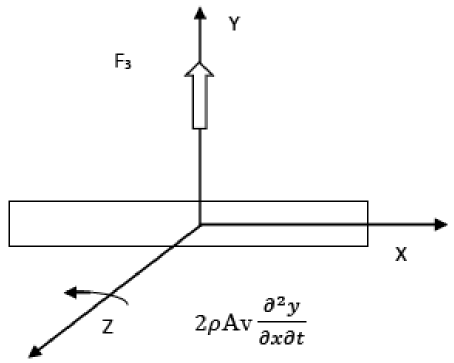

Figure 2.5 Coriolis force

Representation of the Third Term in the Equation of Motion for a Pipe Carrying Fluid

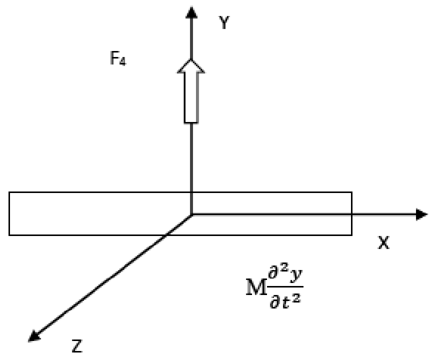

Figure 2.6 Inertia Force

Representation of the Fourth Term in the Equation of Motion for a Pipe Carrying Fluid

The term $M \frac{\partial^2 y}{\partial t^2}$ is a force component acting on the pipe as a result of Inertia of the pipe and the fluid owing through it. Figure (2.6) shows a schematic view of this force F_4.

2.5 Solution For Equation Of Motion Of Cylindrical Pipes Using Galkerin's Method

For a single-span pipe conveying fluid, the equation based on beam theory is given by:

$$EI \frac{\partial^4 y}{\partial x^4} + MU^2 \frac{\partial^2 y}{\partial x^2} + 2MU \frac{\partial^2 y}{\partial x \partial t} + (M+m) \frac{\partial^2 y}{\partial t^2} = 0$$

Where as , EI- Flexural Rigidity Of Cylindrical Beam

U- Steady State Velocity of Fluid

M- Mass of Fluid Flowing in Pipe per unit length (ρA)

A- Cross-sectional rea of pipe

m- Mass of pipe per unit length

The equation of motion can be written in the following non-dimensional form:

$$\ddot{\eta} + 2M_r u_0 \ddot{\eta}' + (u_0^2)\eta'' + \eta'''' = 0 \qquad (2.10)$$

Where $\eta = \frac{y}{L}$

$$\eta'''' = \frac{\partial^4 y}{\partial x^4},\ \eta'' = \frac{\partial^2 y}{\partial x^2},\ \ddot{\eta}' = \frac{\partial^2 y}{\partial x \partial t},\ \ddot{\eta} = \frac{\partial^2 y}{\partial t^2}$$

M_r =(non- dimensional mass ratio) = $\sqrt{\frac{M}{M+m}}$

U_0 = (non- dimensional velocity ratio) = UL $\sqrt{\frac{M}{EI}}$

The motion equation above is in-homogeneous. Then we discretize Eq. (2.10) using the Galerkin's method. Let

$$\eta(\xi,\tau) = \sum_{i=1}^{\infty} \phi i(\xi) qi(\tau) \qquad (2.11)$$

$q_{i(\tau)}$ is an generalized coordinate, $\phi_i(\xi)$ is an comparison function which satisfies all the boundary conditions. Selecting the first three orders conducts researches, which is

$$\eta(\xi,\tau) = \sum_{i=1}^{3} \phi_i(\xi) q_{i(\tau)}$$

For pinned at both ends of pipes, its vibration model function is:

$$\phi_i = \sqrt{2}\sin(\lambda_i, \xi), \qquad i = 1,2,3 \qquad (2.12)$$

Where λ_1, λ_2 and λ_3 are beam eigenvalues $\lambda_1 = \Pi$ $\lambda_2 = 2\Pi$ and $\lambda_3 = 3\Pi$

For fixed at both ends of pipes, its vibration model function is:

$$\phi_i = \cosh(\lambda_i\xi) - \cos(\lambda_i\xi) + \frac{\cosh(\lambda i) - \cos(\lambda i)}{\sinh(\lambda i) - \sin(\lambda i)}[\sin(\lambda_i\xi) - \sinh(\lambda_i\xi)],\ i=1,2,3 \qquad (2.13)$$

Where $\lambda_1 = 4.7300$, $\lambda_2 = 7.8532$, $\lambda_3 = 10.9956$

For fixed at one end and pinned at other end of pipes, its vibration model function is:

$$\phi_i = \cos(\lambda_i\xi) - \cosh(\lambda_i\xi) - \frac{\cos(\lambda i) - \cosh(\lambda i)}{\sin(\lambda i) - \sinh(\lambda i)}[\sin(\lambda_i\xi) - \sinh(\lambda_i\xi)],\ i=1,2,3 \qquad (2.14)$$

Where $\lambda_1 = 3.9267$, $\lambda_2 = 7.0686$, $\lambda_3 = 10.2102$

For cantilever pipe, its vibration model function is:

$$\phi_i = \cosh(\lambda_i\xi) - \cos(\lambda_i\xi) + \frac{\sinh(\lambda i) - \sin(\lambda i)}{\cosh(\lambda i) + \cos(\lambda i)}[\sin(\lambda_i\xi) - \sinh(\lambda_i\xi)],\ i=1,2,3 \qquad (2.15)$$

Where $\lambda_1 = 1.87512$, $\lambda_2 = 4.6941$, $\lambda_3 = 7.85476$

Equation (2.11) is changed into matrix type,

Supposing $\phi = \begin{pmatrix} \phi 1 \\ \phi 2 \\ \phi 3 \end{pmatrix}$, $Q = \begin{pmatrix} q1 \\ q2 \\ q3 \end{pmatrix}$ then

$$\eta(\xi,\tau) = \phi^{T} Q = Q^{T}\phi \qquad (2.16)$$

Plugging equation (3.16) into (3.10), and supposing H = $u02$+Π, then:

$$\phi^{T}Q + 2M_r u_0 \phi'^{T}\dot{Q} + H\phi''^{T}Q + \phi''''^{T}Q = 0 \qquad (2.17)$$

By multiplying $\phi = \begin{pmatrix} \phi 1 \\ \phi 2 \\ \phi 3 \end{pmatrix}$ with two sides of (3.17) and then

$$\phi\phi^{T}\ddot{Q} + 2M_r u_0 \phi\phi'^{T}\dot{Q} + H\phi\phi''^{T}Q + \phi\phi''''TQ = 0 \qquad (2.18)$$

Conducting ξ integral to (3.18) at interval [0, 1], and substitutions based on orthogonally of trigonometric function:

$$\int_0^1 \phi\, \phi^T d\xi = I = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$$\int_0^1 \phi\phi^T d\xi = B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$$\int_0^1 \phi\phi^T d\xi = C = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

ϕ_1, ϕ_2 and ϕ_3 are the first three mode functions for specific boundary conditions.

For pinned at both ends of pipes, the matrix B and C are:

$$B = \begin{pmatrix} 0 & -2.6667 & 0 \\ 2.6667 & 0 & -4.8 \\ 0 & 4.8 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} -\pi^2 & 0 & 0 \\ 0 & -2\pi^2 & 0 \\ 0 & 0 & -3\pi^2 \end{pmatrix}$$

For pinned at both ends of pipes, the matrix B and C are: For fixed at both ends of pipes, the matrix B and C are:

$$B = \begin{pmatrix} 0 & -3.3421 & 0 \\ 3.3421 & 0 & -5.5161 \\ 0 & 5.5161 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} -1.3028 & 0 & 9.7315 \\ 0 & -46.0501 & 0 \\ 9.7315 & 0 & -98.9047 \end{pmatrix}$$

For fixed at one end and pinned at other end of pipes, the matrix B and C are:

$$B = \begin{pmatrix} 0 & -2.9965 & 0.3167 \\ 2.9965 & 0 & -5.1468 \\ -0.3167 & 5.1468 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} -11.5126 & 4.2814 & 3.7993 \\ 4.2814 & -42.8964 & 7.81913 \\ 3.7993 & 7.8191 & -94.0376 \end{pmatrix}$$

For cantilever pipe, the matrix B and C are:

$$B = \begin{pmatrix} 2 & -4.75948 & 3.78433 \\ 0.75948 & 2 & -6.22218 \\ 0.21566 & 2.22218 & 2 \end{pmatrix} \qquad C = \begin{pmatrix} 0.8581 & -11.7433 & 27.4531 \\ 1.8738 & -13.2942 & 9.04205 \\ 1.56453 & 3.22935 & -45.9043 \end{pmatrix}$$

Using equations of (2.18), the discretized equation after reduced order through (2.17) is showed below:

$$I\ddot{Q} + 2M_r u_0 B\dot{Q} + (CH + \Lambda)\, Q = 0 \qquad (2.19)$$

When $\dot{Q} = \Omega_i$, $\ddot{Q} = -\Omega^2$ and Equation (2.19) become;

$$[-I\Omega^2 + 2M_r u_0 B\,\Omega_i + (CH + \Lambda)] = S = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix}$$

Where

$S_{11} = \lambda 1^4 + Hc_{11} + 2M_r u_0 b_{11}\Omega_i - \Omega^2$

$S_{12} = Hc_{12} + 2M_r u_0 b_{12}\Omega_i$

$S_{13} = Hc_{13} + 2M_r u_0 b_{13}\Omega_i$

$S_{21} = Hc_{21} + 2M_r u_0 b_{21}\Omega_i$

$S_{22} = \lambda 2^4 + Hc_{22} + 2M_r u_0 b_{22}\Omega_i - \Omega^2$

$S_{23} = Hc_{23} + 2M_r u_0 b_{23}\Omega_i$

$S_{31} = Hc_{31} + 2M_r u_0 b_{31}\Omega_i$

$S_{32} = Hc_{32} + 2M_r u_0 b_{32}\Omega_i$

$S_{33} = \lambda 3^4 + Hc_{33} + 2M_r u_0 b_{33}\Omega i - \Omega^2$

III. DYNAMIC ANALYSIS OF PIPES CONVEYING FLUID BY ANALYTICAL AS WELL AS COMPUTATIONAL APPROACH

3.1 Problem Description

A straight slender cylindrical pipe made up of aluminium with hollow surface from inside with water flowing inside a pipe with steady velocity. The following parameters of pipe are given below:

Length of the pipe (L) = 1m

Inside diameter of pipe (D0) = 0.011m

Thickness of the pipe (t) = 0.0011m

Mass per unit length of pipe (m) = 0.113 Kg/m

Mass of water per unit length of pipe flowing inside a pipe (ρA) = 0.095 Kg/m

Area moment of inertia of pipe (I) = 7.716× 10−10 m4

Density of aluminium (ρAl) = 2700 Kg/m^3

Modulus of elasticity for Al (E) = 68.9 GPa

Poisson ratio of Al pipe (μ) = 0.33Velocity of fluid (U) = 7.0151 m/s

The various dimensionless parameters can be define as:

$$\text{Mass ratio } (\beta) = \frac{\rho A}{(\rho A + m)}$$

$$\text{Dimensionless frequency } (\Omega) = \omega L^2 \sqrt{\frac{M}{EI}}$$

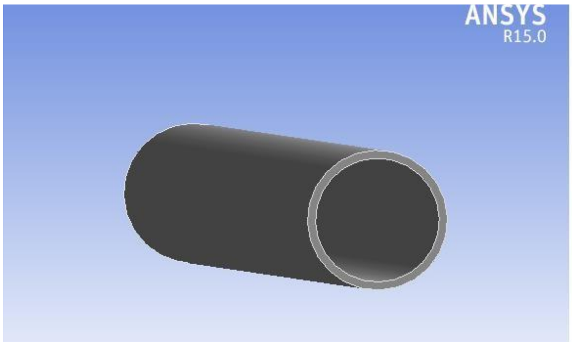

Fig. 3.1: Model of Cylindrical Pipe

$$\text{Velocity ratio } (V_r) = UL\sqrt{\frac{\rho A}{EI}}$$

3.2 Assumptions made

The following assumptions are taken for the cylindrical pipe problem:

1. Neglecting the effect of gravity.
2. The pipe considered to be horizontal.
3. Neglecting the material damping.
4. The pipe is inextensible.
5. Neglecting the shear deformation and rotary inertia.
6. All motion considered small.
7. Neglecting the velocity distribution through the cross-section of the pipe.
8. The flow should be considered as turbulent flow.

3.3 Natural Frequency and Mode Shape of Cylindrical Pipes without Conveying Fluid

3.3.1 By using analytical method

The natural frequency of cylindrical pipe as considered beam is given as:

$$\omega_n = (\beta_n L)^2 \sqrt{\frac{EI}{ML^4}}$$

Table (3.1) List for value of (β_nL) for different boundary condition:

Beam Configuration	$(\beta_1 L)^2$ Fundamental mode	$(\beta_2 L)^2$ Second mode	$(\beta_3 L)^2$ Third mode
Pinned-Pinned	9.87	39.5	88.9
Clamped- Pinned	15.4	50.0	104.0
Clamped-Clamped	22.4	61.7	121.0
Clamped-Free	3.52	22.0	61.7

The analytical values for natural frequency of pipe with different boundary conditions for the above problem are shown in the table given below:

Table (3.2) List of Natural Frequency for Different Boundary Condition by Using Analytical Method

Beam Configuration	Fundamental Frequency (in Hz)	Second Frequency (in Hz)	Third Frequency (in Hz)
Pinned-Pinned	34.072	136.36	306.89
Clamped- Pinned	53.16	172.603	359.015
Clamped-Clamped	77.33	212.99	417.7
Clamped-Free	12.15	75.94	212.99

3.3.2 By using computational method

The pipe mode shape and natural frequency is calculated using ANSYS 15.0.The solver used in this is ANSYS Workbench and the value of natural frequency and mode shape found for different boundary condition are listed in the table given below:

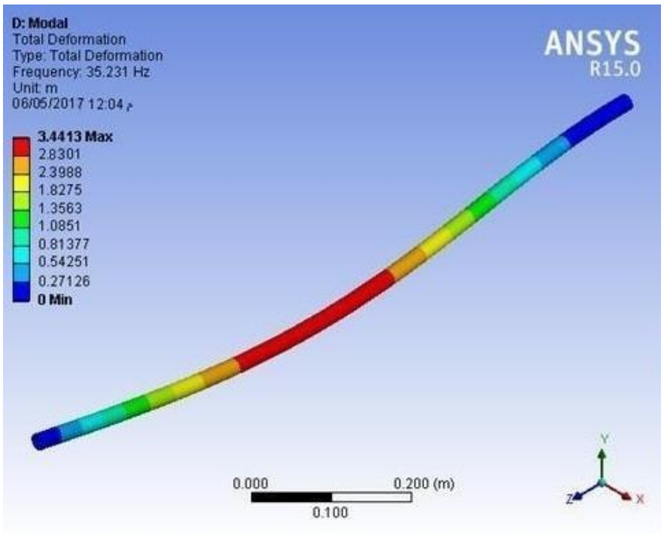

Fig. 3.2: Fundamental natural frequency and mode shape for pinned-pinned pipe

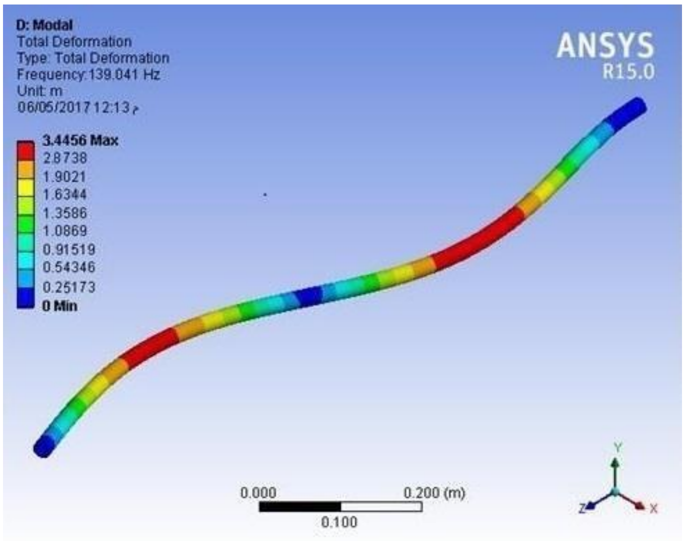

Fig. 3.3: Second natural frequency and mode shape for pinned-pinned pipe

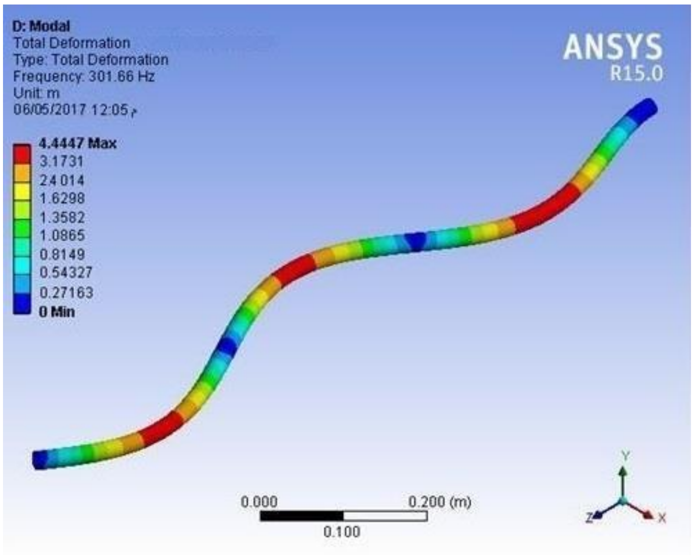

Fig. 3.4: Third natural frequency and mode shape for pinned-pinned pipe

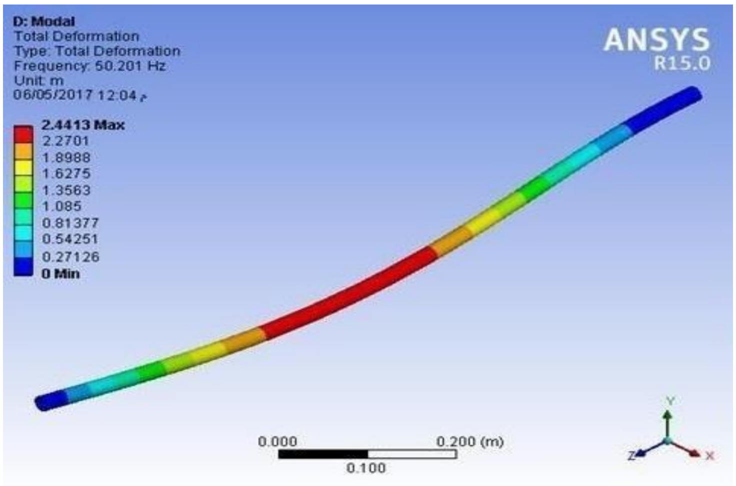

Fig. 3.5: Fundamental natural frequency and mode shape for clamped-pinned pipe

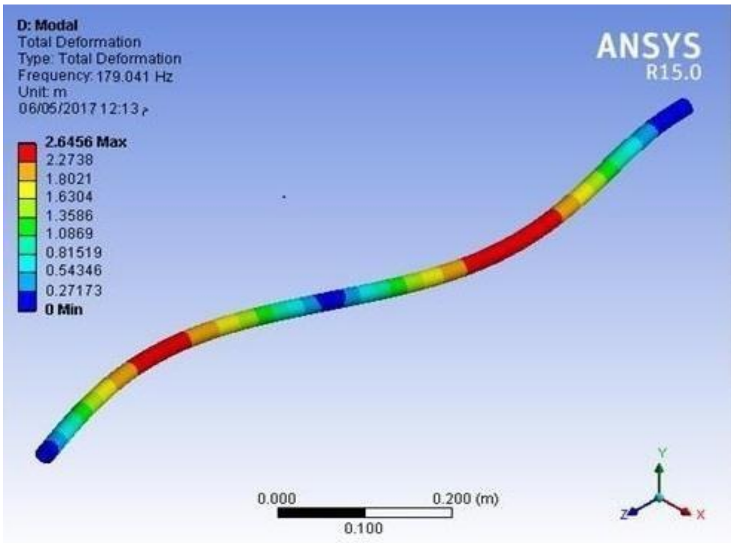

Fig. 3.6: Second natural frequency and mode shape for clamped-pinned pipe

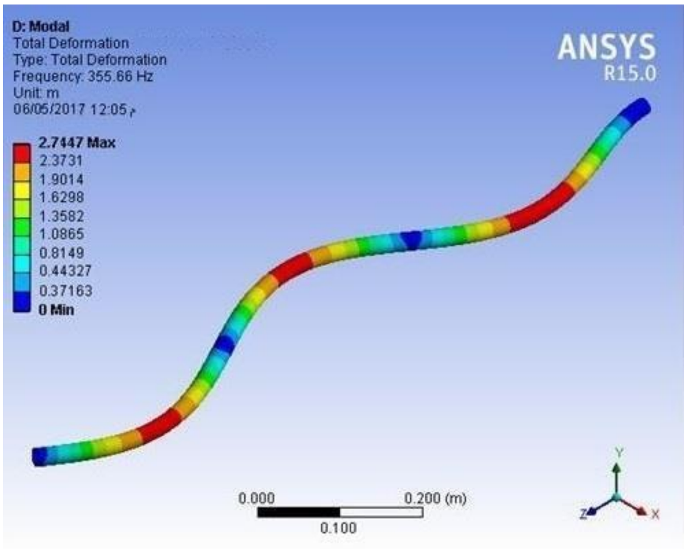

Fig. 3.7: Third natural frequency and mode shape for clamped-pinned pipe

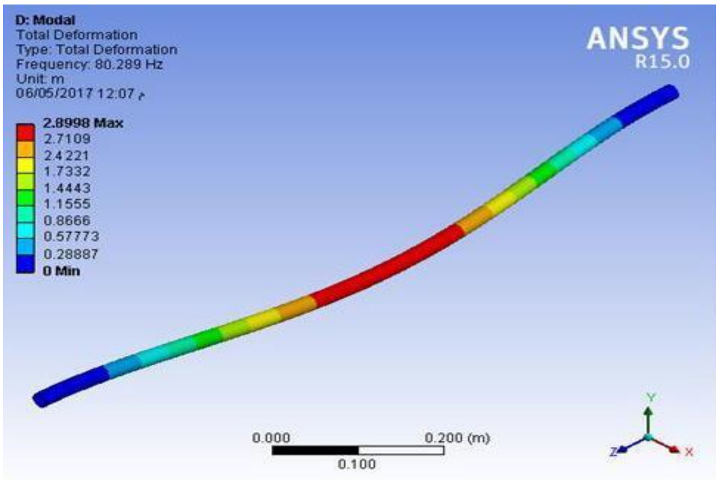

Fig. 3.8: Fundamental natural frequency and mode shape for clamped- clamped pipe

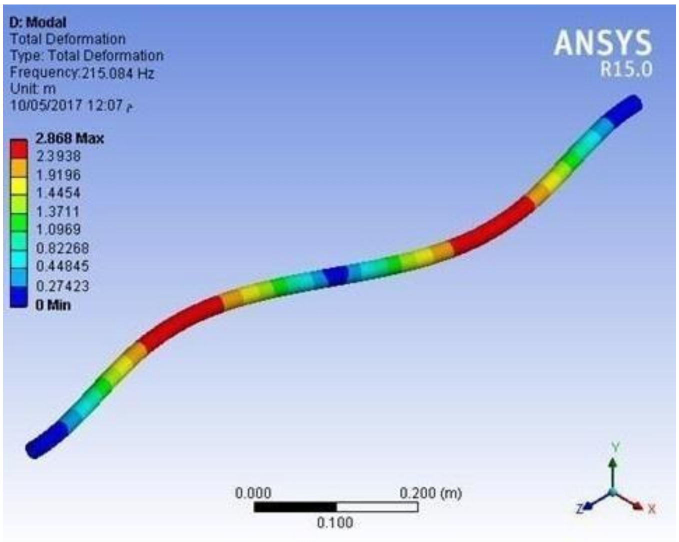

Fig. 3.9: Second natural frequency and mode shape for clamped-clamped pipe

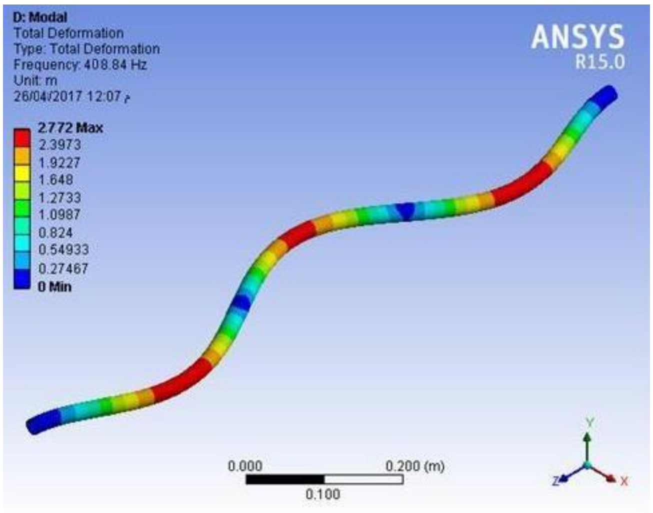

Fig. 3.10: Third natural frequency and mode shape for clamped-clamped pipe

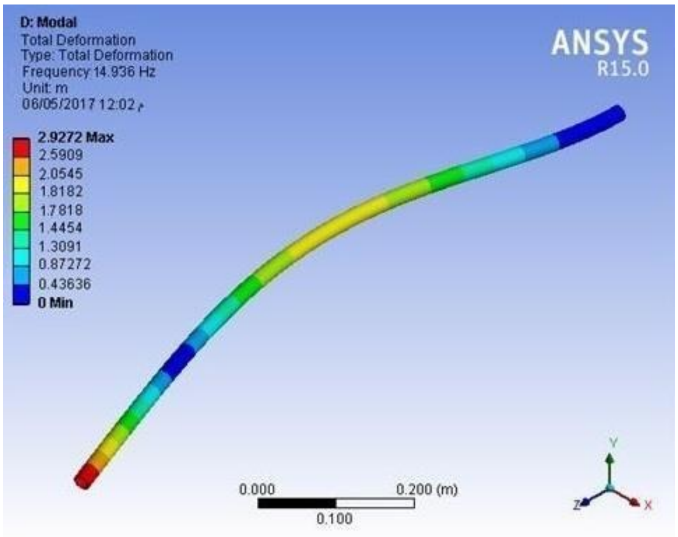

Fig. 3.11: Fundamental natural frequency and mode shape for clamped-free pipe

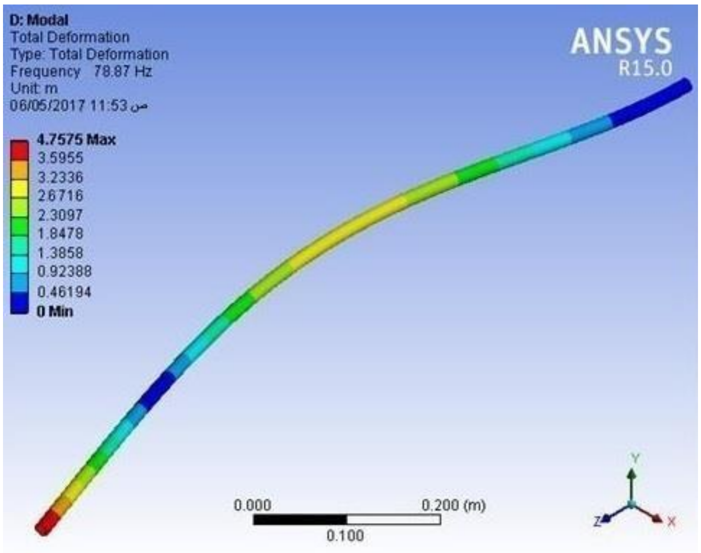

Fig. 3.12: Second natural frequency and mode shape for clamped-free pipe

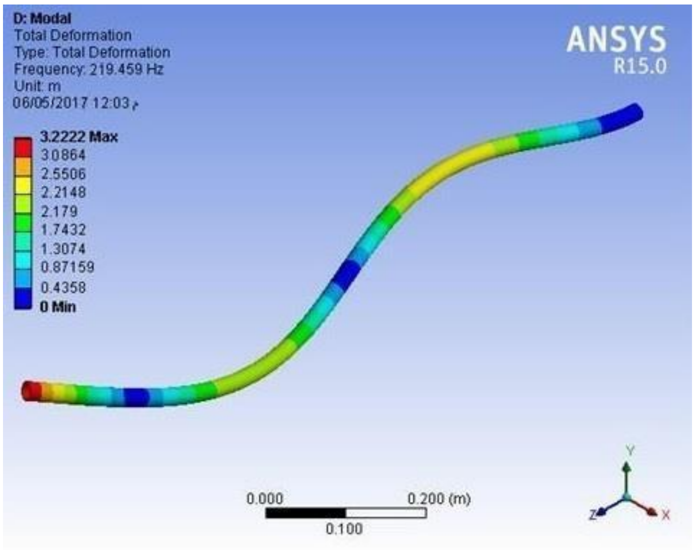

Fig. 3.13: Third natural frequency and mode shape for clamped-free pipe

Table (3.3) List of Natural Frequency for Different Boundary Condition by Using Computational Method

Beam Configuration	Fundamental Frequency (in Hz)	Second Frequency (in Hz)	Third Frequency (in Hz)
Pinned-Pinned	35.231	139.041	301.66
Clamped- Pinned	50.201	179.041	355.66
Clamped-Clamped	80.289	215.084	408.84
Clamped-Free	14.936	78.87	219.459

3.4 Natural Frequency and Mode Shape of Cylindrical Pipes With Conveying Fluid

3.4.1 By Using Analytical Method

For determining the natural frequency of the cylindrical pipe conveying water for different boundary condition are by using Galkerin's Method mentioned in section 3.5. The characteristic equations for non- dimensional natural frequency for different boundary condition are given below:

For clamped-pinned condition, the characteristic equation is:

$$-\Omega^6 + 13535.92\Omega^4 - 29949559.81\Omega^2 + 6216757740 = 0$$

For clamped-clamped condition, the characteristic equation is:

$$\Omega^6 - 22463.9336\Omega^4 + 91059863.11\Omega^2 - 45620927850 = 0$$

For clamped-free condition, the characteristic equation is:

$$\Omega^6 - \Omega^4 (5825.29 + 425.33i) + \Omega^2 (3696994.086 + 897956i) - (50984595.61 + 85574547i) = 0$$

Whereas Ω is non-dimensional natural frequency and actual natural frequency is calculated from the formulae is given below:

$$\text{Dimensionless frequency } (\Omega) = \omega L^2 \sqrt{\frac{M}{EI}}$$

By solving these six degree equations the natural frequency of cylindrical pipe for different boundary conditions is listed in the table given below:

Table (3.4) List of Natural Frequency for Different Boundary Condition by Using Analytical Method

Beam Configuration	Fundamental Frequency (in Hz)	Second Frequency (in Hz)	Third Frequency (in Hz)
Clamped- Pinned	14.21	48.03	101.24
Clamped-Clamped	19.74	54.91	107.78
Clamped-Free	4.254	21.96	58.42

3.4.2 By Using Computational Method

The pipe mode shape and natural frequency is calculated using ANSYS 15.0.The solver used in this is ANSYS Workbench and the value of natural frequency and mode shape found for different boundary condition are listed in the table given below:

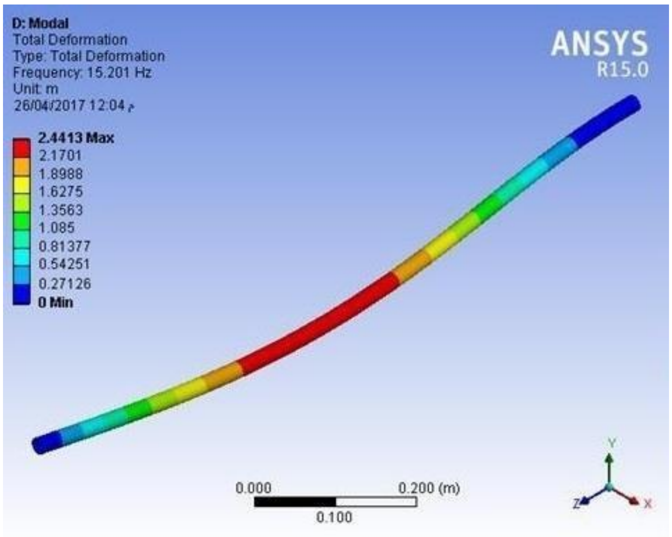

Fig. 3.14: Fundamental natural frequency and mode shape for clamped-pinned pipe

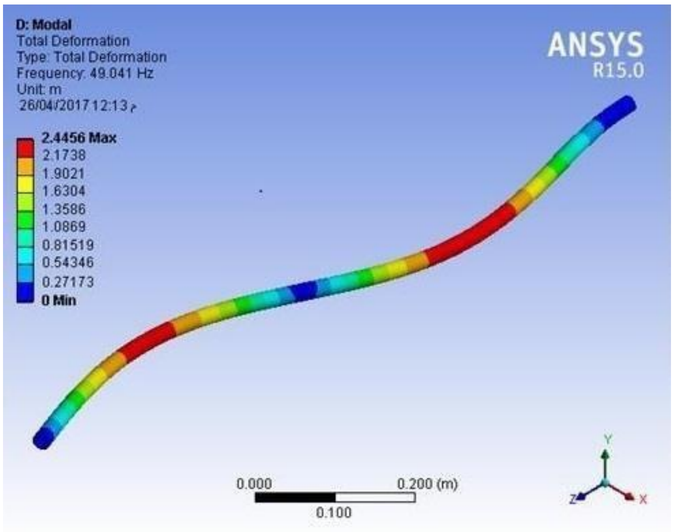

Fig. 3.15: Second natural frequency and mode shape for clamped-pinned pipe

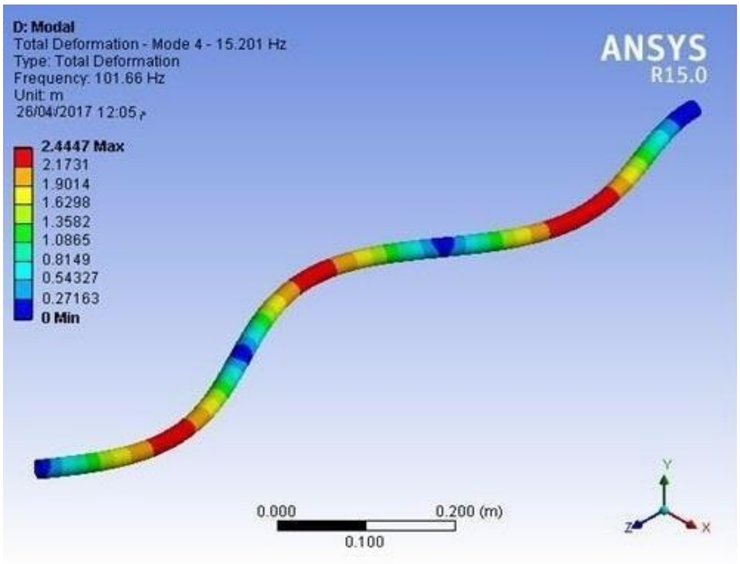

Fig. 3.16: Third natural frequency and mode shape for clamped-pinned pipe

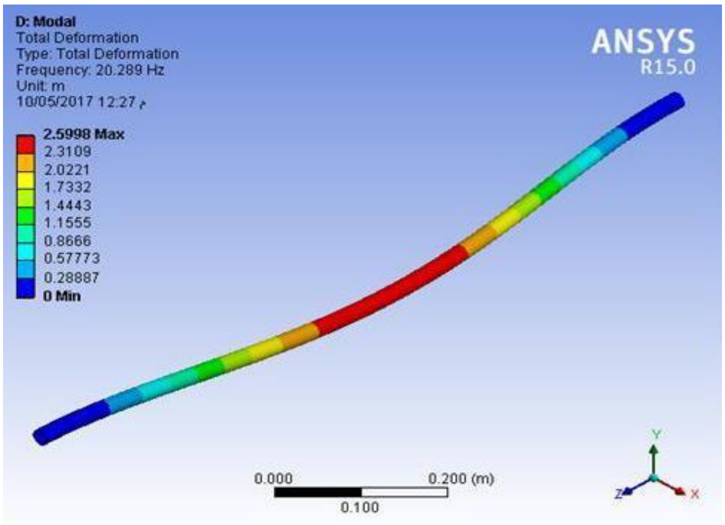

Fig. 3.17: Fundamental natural frequency and mode shape for clamped- clamped pipe

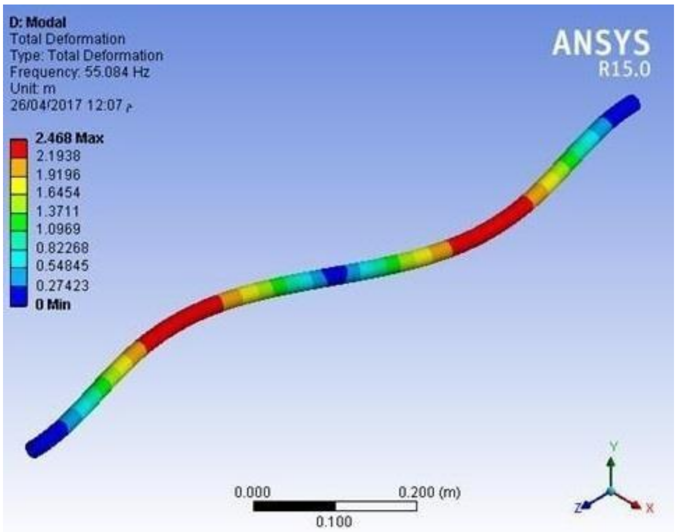

Fig. 3.18: Second natural frequency and mode shape for clamped-clamped pipe

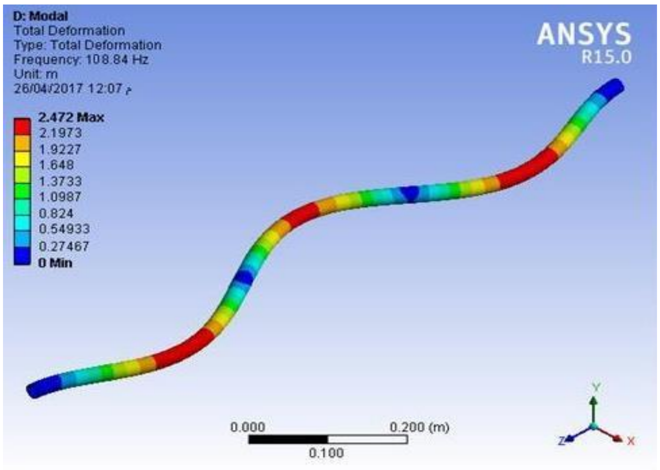

Fig. 3.19: Third natural frequency and mode shape for clamped-clamped pipe

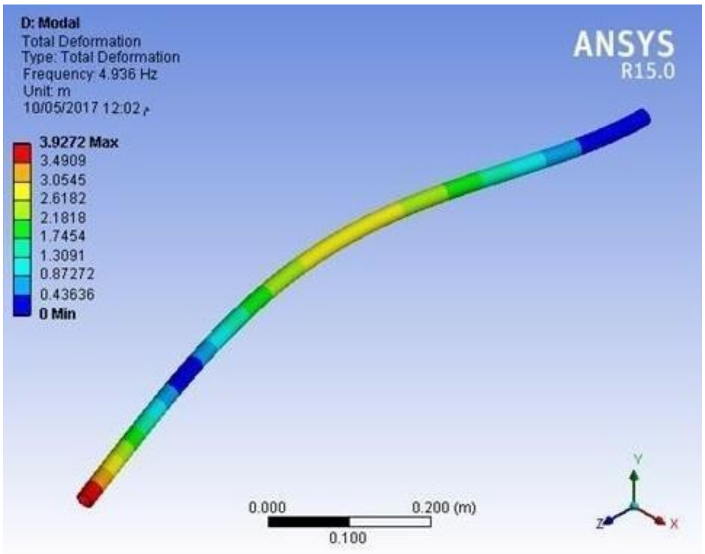

Fig. 3.20: Fundamental natural frequency and mode shape for clamped-free pipe

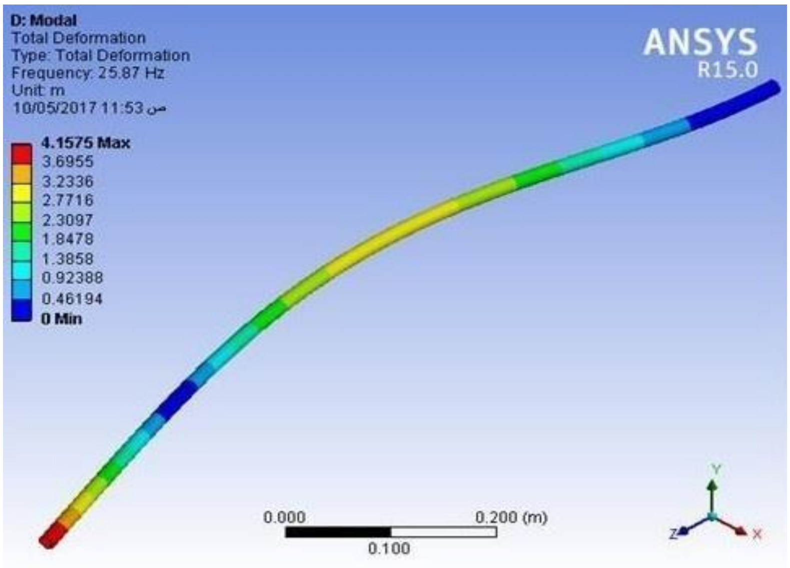

Fig. 3.21: Second natural frequency and mode shape for clamped-free pipe

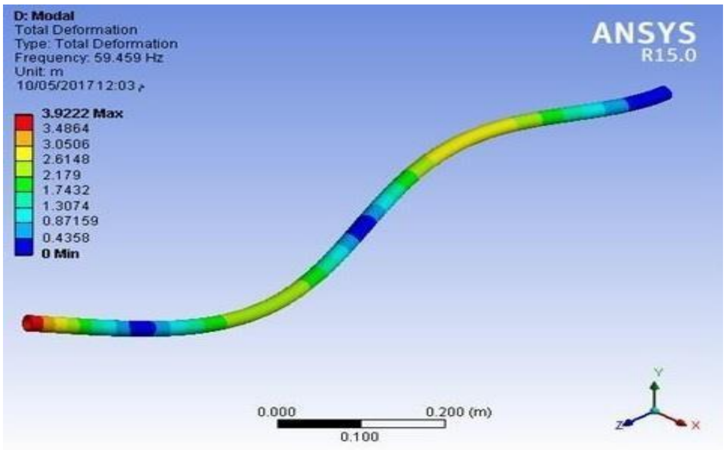

Fig. 3.22: Third natural frequency and mode shape for clamped-free pipe

Table (4.5) List of Natural Frequency for Different Boundary Condition by Using Computational Method

Vr = 0.7015 and β = 0. 40145 for aluminium pipe

Beam Configuration	Fundamental Frequency (in Hz)	Second Frequency (in Hz)	Third Frequency (in Hz)
Clamped- Pinned	15.201	49.041	101.66
Clamped-Clamped	20.289	55.084	108.84
Clamped-Free	4.936	25.87	59.459

3.5 ANSYS CFX Fluid Model for Cylindrical Pipe with Conveying Fluid

The ANSYS 15.0 CFX solver uses finite elements to discretize the domain. Large eddy simulation (LES) is used as a turbulence model. The largest eddies, having dynamic and geometric properties related to the mean fluid flow; contain more energy than the smallest eddies. The LES approach makes use of this fact by applying spatial filters to the governing equations to remove the smallest eddies while the large eddies are numerically simulated. LES does take more computation than the commonly used Reynolds averaged Navier-Stokes (RANS) turbulence models, but RANS models do not resolve the turbulent pressure fluctuations responsible for the pipe vibration of interest.

The next step of the analysis involves applying the appropriate boundary and Initial conditions. The cylindrical surface of the domain is defined as a solid wall with the no-slip, smooth wall pipes condition being enforced. The periodic boundary will let the flow develop as it effectively re-enters the inlet each time it has passed through the entire domain.

Pressure distributions in a pipe conveying fluid (water) along the length of wall pipe is shown in figure 3.22 as shown below:

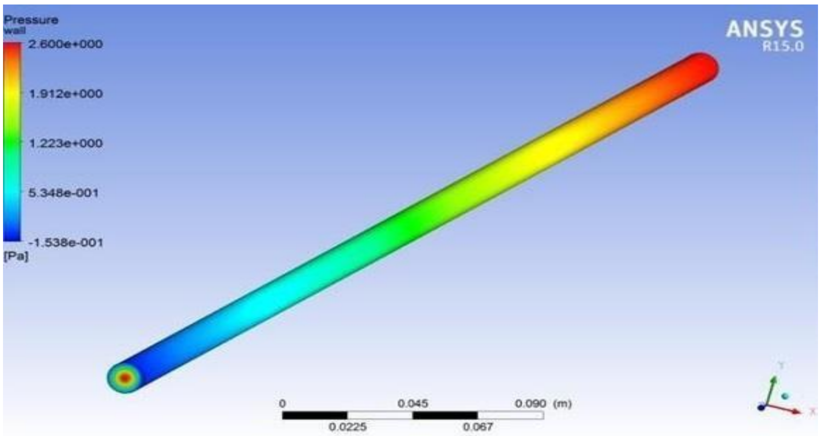

Fig. 3.23: Pressure Distribution along the wall of Pipe

3.6 Structural analysis of cylindrical pipe

The structural model of the pipe was created using ANSYS 15.0. This model experiences transient deformation in response to pressure fluctuations calculated by the fluid model. It uses the finite element method (FEM). The physical properties like modulus of elasticity (E) and Poisson's ratio is defined in engineering material sub-section and the fluid pressure results obtained from the ANSYS CFX V15.0 can be mapped into ANSYS Structural workbench V15.0 as imported pressure loading for the static stress analysis. After that, boundary condition is applied. The result obtained for von-mises stress, von-mises strain and total deformationfor pinned-clamped condition is shown below:

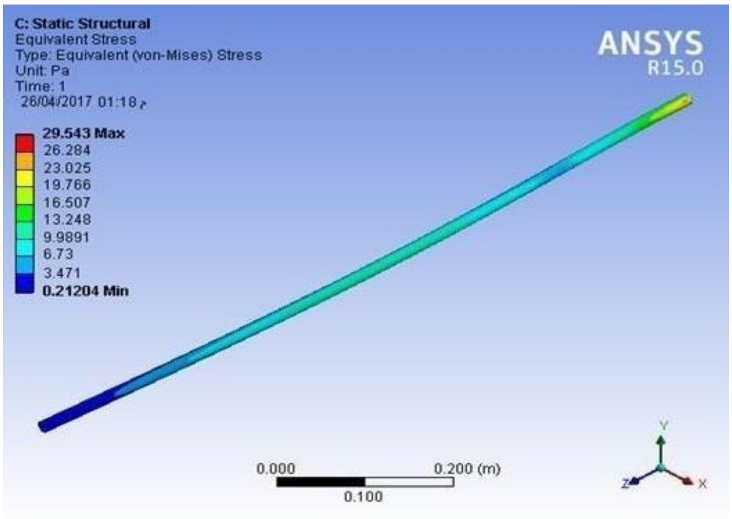

Fig. 3.24: Equivalent von-mises stress for clamped –pinned pipe

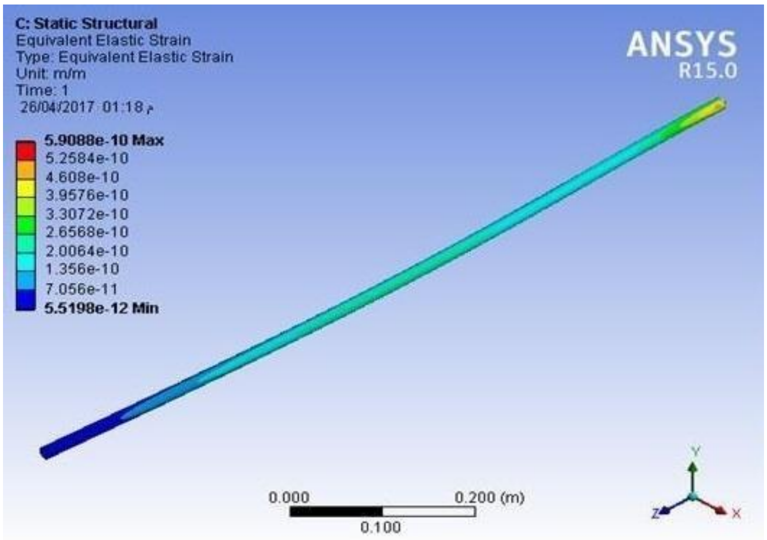

Fig. 3.25: Equivalent von-mises strain for clamped-pinned pipe

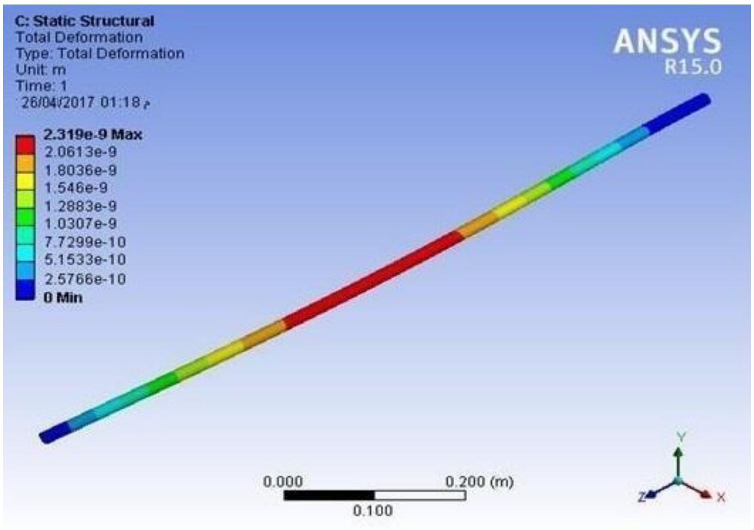

Fig. 3.26: Total deformation for clamped-pinned pipe

V. RESULTS AND DISCUSSIONS

4.1 Natural Frequency and Mode Shape

The calculated values for first three lowest natural frequencies by using analytical method of the cylindrical pipe conveying fluid (water) are presented in table (). By, comparing with the computational value there is a slight error but shows good agreement between these two. Comparison of natural frequencies by these two methods and percentage error is shown in the table given below:

Table (5.1) Comparison and percentage error of natural frequency for a pipe conveying fluid by using analytical and computational method

Boundary Condition	Mode No.	Natural Frequency		% Error
		Analytical	ANSYS	
Clamped - Pinned	1	14.21	15.201	6.974
	2	48.03	49.041	3.105
	3	101.24	101.66	2.415
Clamped - Clamped	1	19.74	20.289	2.78
	2	54.91	55.084	3.317
	3	107.78	108.84	4.984
Clamped - Free	1	4.254	4.936	16.032
	2	21.96	25.87	17.81
	3	58.42	59.459	3.78

4.2 Effect of Fluid Velocities on Natural Frequency for Different Boundary Conditions

The variations of fundamental natural frequency versus velocity of fluid flow for different boundary conditions are shown in the figure given below. By increasing flow velocity the natural frequency of pipe decreases for clamped-pinned and clamped-clamped boundary conditions. But, for clamped-free pipe this is not true. So, at a certain velocity the natural frequency is zero and divergence or buckling occur in the pipe for clamped-pinned and clamped-clamped pipe. For cantilever pipe the instability depend on imaginary part of natural frequency. If it is less than zero then, vibration occurs in expanding envelope.

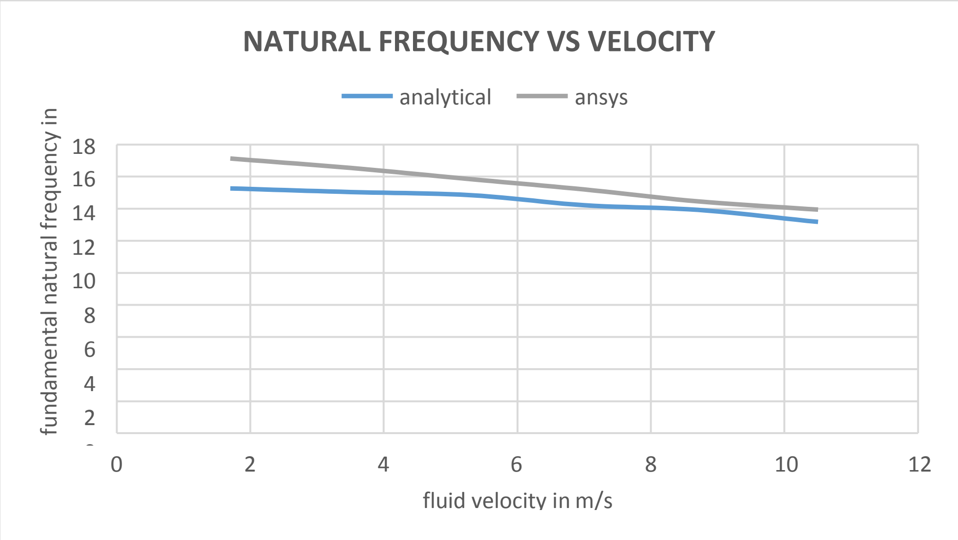

Fig. 4.1: For Clamped-Pinned Boundary Condition

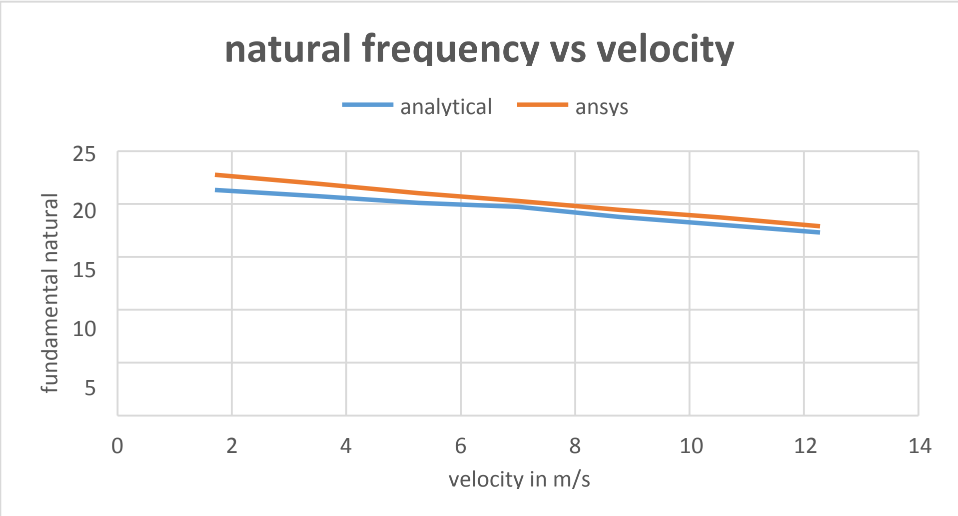

Fig. 4.2: For Clamped-Clamped Boundary Condition

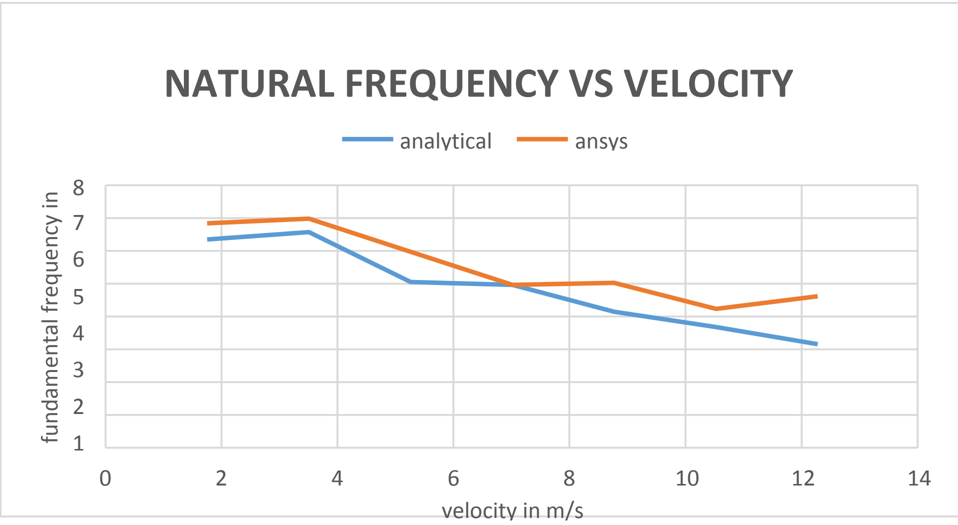

Fig. 4.3: For Clamped-Pinned Boundary Condition

V. CONCLUSIONS

- The analytical solution for the pipe conveying fluid is derived in this thesis. And, for different boundary condition the natural frequency are obtained analytically in terms of pipe parameters.

- The natural frequency of pipe conveying fluid depends on the velocity of fluid flow is observed. Increase in fluid velocity reduces the natural frequency of cylindrical pipe.

- At a certain velocity, the instability of pipe is observed. It affects the natural frequency of vibration differently for different boundary condition. For, pinned-clamped, pinned- pinned and clamped-clamped the natural frequency is zero at the onset of divergence. But, for cantilever type it depends upon imaginary part of natural frequency.

- The pressure variation along the length of pipe at the wall boundary for turbulent flow cause vibration in pipes was observed.

- This pressure variation along the length of pipe also causes stress, strain and deformation in the pipe.

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