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A GENERALIZED DELTA SHOCK MAINTENANCE MODEL

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Abstract : In this paper, we give the definitions of some concepts related to a repairable system. A generalized δ-shock model is discussed. The concept of ageing class related to the δ-shock maintenance model are presented.

Keywords: A generalized δ-shock model, Maintenance model, Reliability function, NBU.

1 Introduction

In maintenance modelling of a technical object mathematics find applications in work sampling, inventory control analysis, failure data analysis, establishing optimum preventive maintenance policies, maintenance cost analysis, and project management control. High system reliability can be achieved by maintenance. A classical problem is to determine how reliability can be improved by using mathematical models. Many systems are subject to random shocks from external environment in there working. Shock may bring a certain amount of damage to a system and eventually destroy it with the prolonged, or break the system down immediately. Therefore, the probability of a system to survive the shocks in the time interval $[0,t]$ is a central problem in the field of shock model theory. In the cumulative shock model, system fails because of the cumulative effect of shock, while in the extreme shock model, the system breaks down due to a single shock with a great magnitude, see A-hameed and proschan (1973), Finkelestein and Zarudnij (2001), Gut (1990), Shanthikumar and Sumita (1983; 1984) for references.

The δ-Shock model is a special type of shock model, which was first proposed by Li, Chan, and Yuan (1999). In engineering area, when a electronic or mechanical system is subject to a shock, it usually needs a period of time to recover from the shock. If the arrivals of two successive shocks are too close to each other, the system will break down because it has not recovered from former shock when a new shock comes. This is the motivation to study the δ-shock maintenance model.

2. Preliminaries

In this section, we present some terms and definitions directly used in maintenance models. Definitions of basic notions are given below:

Definition 2.1. A system which, after failing to perform one or more of its functions satisfactorily, can be restored to fully satisfactory performance by any method, other than replacement of the entire system is called a *repairable system*.

Definition 2.2. *Repair* - a restoration where in a failed system (device) is returned to operable condition.

Definition 2.3. *Perfect repair* - a repair under which a failed system is replaced with a new identical one is called perfect repair.

Definition 2.4. *Minimal repair* - a repair of limited effort where in the device is returned to the operable state it was in just before failure.

Definition 2.5. *Mean Time to Repair* (MTTR) - a figure of merit depending on item maintainability equal to the mean item repair time.

Remark. In the case of exponentially distributed times to repair, MTTR is the reciprocal of the repair rate.

Definition 2.6. The process $N(t),\ t \geq 0$ is said to be a *Homogeneous Poisson Process* with rate (or intensity) $\lambda > 0$. If

(i) $N(0) = 0;$ i.e. there are no events at time 0 ;

(ii) the number of events $N(s_2) - N(s_1)$ and $N(t_2) - N(t_1)$ in disjoint time intervals $(s_1, s_2]$ and $(t_1, t_2]$ are independent random variables (independent increment);

(iii) the distribution of the number of events in a certain interval depends only on the length of the interval and not on its position (stationary increments);

(iv) there exists a constant such that

$$\lim_{\Delta t \to 0} \frac{P(N(\Delta t) = 1)}{\Delta t} = \lambda;$$

(v) $$\lim_{\Delta t \to 0} \frac{P(N(\Delta t) \geq 2}{\Delta t} = 0$$

3 Poisson Process as a Model of a Repairable System

If failures occur exponentially, that is the unit fails constantly during a time interval $(t, t+dt]$ irrespective of time t, the system forms a Poisson process. Roughly speaking, failures occur randomly in $(t, t+dt]$ with probability λdt, for constant $\lambda > 0$; and inter arrival times between failures have an exponential distribution $(1-e^{-\lambda t})$. Then, it is said that failures occur in a Poisson process with rate λ.

Supper position of Poisson process

Let $N_1(t)$ be the number of failed units from a certain population in $[0,t]$ and $N_2(t)$ be the number of failed units from the other. If the arrival times from two populations are independent and have the respective Poisson processes with rates λ_1, λ_2 the total number $N(t)$ of failed units arriving at a repair shop has the probability

$$P(N(t)=n)=\frac{((\lambda_1+\lambda_2)t)^n}{n!}e^{-(\lambda_1+\lambda_2)t}$$

Thus, the process $\{N(t), t\geq 0\}$ is also a Poisson process with rat $\lambda_1+\lambda_2$.

Decomposition of Poisson process

Let $N(t)$ be the number of failed units occurring in $[0,t]$ and be a Poisson process with rate λ. Classifying into two large groups of failed units, the number $N_1(t)$ of minor ones occurs with probability p and the number $N_2(t)$ of major ones occurs with probability $q=1-p$, independent of N_1(t). Then, because the number N_1(t) hasa binomial distribution with parameter p, given that n failures have occurred, thejoint probability is

$$\mathrm{P}(N_1(t)=k, N_2(t)=n-k) = \frac{(p\lambda t)^k}{k!}e^{-p\lambda t}\,\frac{(pqt)^{n-k}}{(n-k)!}e^{-q\lambda t}$$

Thus, the two Poisson processes $\{N_1(t), t>0\}$ and $\{N_2(t), t>0\}$ are independent and have the Poisson processes with rates $p\lambda$ and $q\lambda$, respectively. In general, when the total number $N(t)$ of failed units with a Poisson process with rate λ is classified into k groups of $N_j(t), j=1,\ldots,k$ with probability p_j where $\sum_{j=1}^{k} N_j(t) = N(t)$ and $\sum_{j=1}^{k} p_j = 1$, the joint probability is

$$\mathrm{P}(N_1(t)=n_1,\ldots,N_k(t)=n_k) = \prod_{j=1}^{k}\frac{(p_j\lambda t)^{n_j}}{n_j\,!}e^{-p_j\lambda t}$$

that is called a multi-Poisson process.

4 A generalized δ-shock model

We consider a generalized δ-shock model for a single component system with two types of shocks and make the following assumptions.

- **[A1]** At time $t=0$, a new system is installed and begins to work.
- **[A2]** The system is subject to external shocks that arrive according to a homogeneous Poisson process $\{N(t), t\geq 0\}$ with intensity λ.
- **[A3]** The shock process includes two types of shocks. Each shock is of type 1 with probability p, type 2 with probability $q=1-p$, independently.
- **[A4]** External shocks are the only cause of system failure. When a shock comes, some damage generated and a certain length of time is needed for the system to recover from it. The system recovery time for a type i shock is $\delta_i, i=1,2$.
- **[A5]** The system fails when a shock occurs and the system still has not recovered from the previous shock.

Remark. From **[A2],** the inter-arrival times $X_n, n=1,2,\ldots$ are independent and exponentially distributed with parameter λ, i.e. $F(t)=1-e^{\lambda t}, t\geq 0$. From **[A3],** the number $N_i(t)$ of type $i\,(i=1,2)$ shocks occurred in the time interval $[0,t]$ yields homogeneous Poisson process with rate λ_i, where $\lambda_1=p\lambda$ and $\lambda_2=q\lambda$, respectively.

Moreover, from the notations we can also see that $\phi_n=2-Z_n$ and $\psi_n=Z_n-1$.

To derive the reliability function of the generalized δ-shock model with two types of shocks, the following lemma is required

Lemma 4.1. Let $\{N(t), t \geq 0\}$ be a homogeneous Poisson process with rate λ, and denote $X_1, X_2, \ldots X_n$ the inter-arrival times of the process. Given $N(t) = n$, then for any fixed constant $a > 0$,

$$P(X_2 \triangleright a, X_3 \triangleright a, \ldots X_n \triangleright a \mid N(t) = n) = \left(-\frac{(n-1)a}{t}\right)_+^n, \quad (1)$$

Where $y_+ = \max(y, 0)$,

Theorem 4.1. The reliability function of the generalized δ-shock model is

$$\begin{aligned} P(T > t) = e^{-(\lambda_1+\lambda_2)t}\{1 + \sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty} \frac{n_1}{n_1+n_2}\frac{(\lambda_1)_1^n}{n_1!}\frac{(\lambda_2)^{n_2}}{n_2!} \\ \times [t-(n_1-1)\delta_1 - n_1\delta_2]_+^{n_1+n_2} \\ + \sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty} \frac{n_1}{n_1+n_2}\frac{(\lambda_1)^{n_1}}{n_1!}\frac{(\lambda_2)_2^n}{n_2!} \\ \times [t - n_1\delta_1 - (n_2-1)\delta_2]_+^{n_1+n_2}\}, t > 0, \quad (2) \end{aligned}$$

where $\lambda_1 = p\lambda, \lambda_2 = q\lambda$.

Corollary 4.1. If $\delta_1 = \delta_2 = \delta$, then the reliability function of the generalized δ-shock model becomes

$$P(T > t) = e^{-\lambda t}\sum_{n=0}^{\infty}\frac{\lambda^n[t-(n-1)\delta]_+^n}{n!}, n = 0,1,2,\ldots \quad (3)$$

Remark. If the two types of shocks have the same recovery time, i.e. $\delta_1 = \delta_2 = \delta$, then the generalized δ-shock with two types of shocks reduces to the $\delta -$shock model with single type of shocks.

The Mean Lifetime of the Generalized δ-shock Model

The reliability function (2) of the lifetime T is very complex, so the computing of $E(T)$ based on (2) is difficult. To calculate the mean and the variance of the lifetime T, we need the concept of stopping time. The lifetime T of the generalized δ-shock model can be written as

$$T = X_1 + X_2 + \ldots + X_{\tau}, \quad (4)$$

where $\tau = \min\{n \geq 2 \mid X_2 \leq \delta_{Zn-1}\}$ is the number of shocks the system experienced before failure. it is easy to see that τ is a stopping time for $\{X_n, n \geq 1\}$. By employing the Wald's Equation, we have

$$E(T) = E(X_1)\cdot E(\tau) \quad (5)$$

Furthermore, the variance of the lifetime T can also be derived accordingly. The formula for calculating $Var(T)$ is

$$Var(T) = E[Var(T \mid \tau)] + Var[E(T \mid \tau)]. \quad (6)$$

The conditional expectation and conditional variance can be computed as

$$E(T \mid \tau) = E(X_1 + X_2 + \ldots + X_{\tau} \mid \tau) = \frac{\tau}{\lambda}, \quad (7)$$

and

$$Var(T \mid \tau) = Var(X_1 + \ldots + X_{\tau} \mid \tau) = \frac{\tau}{\lambda^2}. \quad (8)$$

substituting the equations (7) and (8) into equation (6), we have

$$Var(T) = E\left[\frac{\tau}{\lambda^2}\right] + Var\left(\frac{\tau}{\lambda}\right) = \frac{1}{\lambda^2}\left(1 + \frac{1}{\alpha^2}\right), \quad (9)$$

where

$$Var(\tau) = Var(\tau - 1) = (1-\alpha)/\alpha^2.$$

Based on the above analysis, we present the following theorem.

Theorem 4.2. The mean and the variance of the lifetime t of the generalized δ-shock model are given as

$$E(T) = \frac{1}{\lambda}\left(1 + \frac{1}{\alpha}\right), \quad (10)$$

$$Var(T) = \frac{1}{\lambda^2}\left(1 + \frac{1}{\alpha^2}\right). \quad (11)$$

Remark. Based on the above analysis, we can also obtain the probability of a type i failure occurrence when the system fails. Without loss of generality, we assume that the system fails at the $k-th$ shock, i.e. $\tau = k, k \geq 2.$ The conditional probability that the failure is of type 1 can be computed as follows.

$$\begin{aligned} & P(Z_{k-1}=1 \mid \tau=k) \\ & =P(Z_{k-1}=1 \mid X_2 \rhd \delta_{Z_1}, \ldots X_{k-1} \rhd \delta_{z_{k-2}}, X_k \leq \delta_{z_{k-1}}) \\ & =\frac{P(Z_{k-1}=1, X_2 \rhd \delta_{z_1}, \ldots, X_{k-1} \rhd \delta_{z_{k-2}}, X_k \leq \delta_{z_{k-1}})}{P(X_2 \rhd \delta_{z_1}, \ldots, X_{k-1} \rhd \delta_{z_{k-2}}, X_k \leq \delta_{z_{k-1}})} \\ & =\frac{p(1-e^{-\lambda\delta_1})}{1-pe^{\lambda\delta_1}-qe^{-\lambda\delta_2}}. \qquad (12) \end{aligned}$$

which is unrelated to the number k. So the probability that a system failure is of type 1 is given, and the probability of type 2 failure is $q(1-e^{-\lambda\delta_2})/(1-pe^{\lambda\delta_1}-qe^{-\lambda\delta_2})$

Life Time Properties

NBU property of life-time T

Let T_0 be the lifetime of a special $\delta-$shock model that the system is subject to a shock at time $t=0$ and the shock is of type 1 with probability p and is of type 2 with probability $1-p$. Other assumptions are the same as the generalized δ-shock model given previously. Denote by H_0 the (Common Cumulative Distribution Function) CDF of T_0, the number of shocks the system experienced in $[0,t]$. To prove the NBU property of the lifetime T, we give the following lemma without proof.

Lemma 4.2. For any $t \geq 0, \bar{H}(t) \geq \bar{H}_0(t)$, That is, T, is stochastically lager than T_0.

Remark. If X_0 yields exponential distribution with parameter λ and is independent of T_0 we can see that X_0+T_0 has the same distribution with T. Therefore, Lemma holds obviously. The lifetime T is New Better than Used (NBU) if and only if $P(T>t) \geq P(T>s+t \mid T>s),$ or $\bar{H}(s+t)\bar{H}(s)\bar{H}(t)$ for any $s,t>0.$ An equivalent expression of the condition can also be written as $\bar{H}_s(t) \leq \bar{H}(t),$ for any $s,t>0,$ where $\bar{H}_s(t)$ is the reliability function of the residual lifetime T_s of T at time $s>0.$

By applying Lemma, we have the following theorem.

Theorem 4.3. The lifetime T of generalised δ-shock model is NBU.

Proof. Denote $S_{N(s)}$ the arrival time of the last shock before time s For any $t \geq 0,$ we have

$$\begin{aligned} \bar{H}_s(t) &= P(T \rhd s+t \mid T \rhd s) \\ &= P(T \rhd s+t \mid T \rhd s, s-S_{N(s)} \rhd \delta Z_{N(s)} \cdot \\ &\qquad P(s-S_{N(s)} \geq \delta_{Z_{N(s)}})) \\ &\leq \bar{H}(t)\, P(s-S_{N(s)} \geq \delta_{Z_{N(s)}}) \\ &\quad + E[\bar{H}_0(t+s-S_{N(s)})] P(s-S_{N(s)} \geq \delta_{Z_{N(s)}}) \\ &\leq \bar{H}(t)\, P(s-S_{N(s)} \geq \delta_{Z_{N(s)}}) \\ &\qquad + \bar{H}_0(t) P(s-S_{N(s)} \geq \delta_{Z_{N(s)}}) \\ &\leq \bar{H}(t) \end{aligned}$$

The last inequality follows from Lemma 4.1, and the proof is complete.

Remark. can be interpreted in a rather straight forward way. By **[A1]**, the system is new at $t=0.$ At any time $t>0,$ if the system is still alive, it is either in a recovery state of a shock, or in as good as new state due to the memoryless property of exponential distribution. Therefore, the NBU property holds.

5 Conclusion

In this paper, we have given the definitions of some concepts related to a repairable system. A generalized $\delta-$shock model is discussed. The concept of ageing is class is related to this $\delta-$shock maintenance model are presented.

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