Testing Exponentiality Against New Better than used Average Renewal Failure Rate Ageing Class

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Abstract: - In this paper, a new class of life distribution namely New Better than Used Average Renewal Failure Rate ageing class (NBUARFR) is discussed. The Pitman asymptotic efficiency of the test are calculated. Power estimates of the test is also evaluated.

Keywords: Classes of Life Distribution Ageing class, efficiency, estimates.

1. Introduction

Testing exponentiality against various classes of Life distribution has got a deal life attention with respect to the testing exponential against NBUE, NBUFR and NBUARFR classes. In this paper, real data is given and we desire to test H_O: data is exponential versus the alternative hypothesis H_1: data is not exponential to choose between H_0 and H_1 or to make a decision, we need to the determine how close a specific sample result falls to one of the hypothesis being tested. These comparisons produce new NBU type classes including New Better than Renewal of Used (NBRU), Renewal New is Better than Used (RNBU) and Renewal New is Better than Renewal Used Expectation (RNBRUE) when comparing the residual equilibrium life at a certain age and its equilibrium (stationary) life in expectation.

2. Preliminaries

In this section, we give some definitions that are necessary for further discussion.

Definition 2.1. A life distribution F is New Better than Used in Average Renewal Failure Rate (NBUARFR), if

$$r_F(0)\, t^{-1} \int_0^t r_{W_F}(u)du, \;\; t > 0.$$

Equivalently,

$$r_F(0) \le t^{-1} \ln \overline{W}_F(t),$$

where

$$\overline{W}_F(0) = \mu_F^{-1} \int_0^\infty \overline{F}(u)\, du = 1.$$

Where r_F is the failure rate of a new system is than the average renewal failure rate system.

Definition 2.2. A random variable X is said to be NBRUL, if

$$\int_0^\infty e^{-sx}\, \overline{W}_F(x+t)dt \le \overline{W}_F(t) \int_0^\infty e^{-sx} \overline{F}(x)dx,$$

$$x \ge 0, t \ge 0 \text{ and } s \ge 0.$$

It is obvious that NBRUL contains the NBRU class, that is NBRUL⊂ NBRUL.

Also,

NBRU ⇒ NBRUL ⇒NBRUE.

3. Main Results

In this section, we gives some results related to our study.

Theorem 3.1. The life distribution F or the its survival F having NBUARFR, if and only if

$$\int_t^\infty \overline{F}(u)du \le (\mu)_F e^{-trF^{(0)}}, \;\; t \ge 0.$$

Proof. Let F be an distribution with failure rate r(·), F is NBUARFR means

$$r_F(0) \le t^{-1} \ln \overline{W}_F(t).$$

Then

$$-tr_F(0) \ge \ln \overline{W}_F(t),$$

$$\overline{W}_F(t) \le e^{-trF^{(0)}}.$$

This is equivalent to the form

$$\int_t^\infty \overline{F}(u)du \le (\mu)_F e^{-trF^{(0)}}, \;\; t \ge 0.$$

The above equation is satisfied, then it is easy to prove the NBUARFR property.

■

Theorem 3.2. If X is a random variable with distribution function F and F belongs to NBRUL class, then

$$\frac{1}{s^3}\phi(s)+\frac{1}{s^2}\mu-\frac{1}{s^3}\geq\frac{1}{2s}\,\mu_{(2)}\phi(s), s\geq 0\,. \qquad (1)$$

Where

$$\phi(S)=Ee^{-sx}\int_0^\infty e^{-sx}dF(x).$$

And

$$\mu_2=2\int_0^\infty x\,\bar{F}(x)dx.$$

Proof. Since F is NBRUL then

$$\int_0^\infty e^{-sx}\bar{W}_F(x+t)dx$$

$$\leq\ \bar{W}_F(t)\int_0^\infty e^{-sx}\bar{F}(x)dxdt,$$

$$x,\ t\geq 0.\ .$$

Integrating both sides with respect to t over [0;1), gives

$$\int_0^\infty\int_0^\infty e^{-sx}\,\bar{W}_F(x+t)dxdt$$

$$\leq\int_0^\infty\int_0^\infty e^{-sx}\bar{F}(x)dxdt, \qquad (2)$$

setting

$$I_1=\int_0^\infty\int_0^\infty e^{-sx}\bar{W}(x+t)dxdt.$$

I_1 can be rewritten as

$$I_1=E\int_0^x\left[\frac{1}{s}X+\frac{1}{s^2}e^{-sx}e^{st}-\frac{1}{s^2}-\frac{1}{s}t\right]dt. \qquad (3)$$

So,

$$I_1=\frac{1}{2s}\mu_{(2)}-\frac{1}{s^3}\phi(s)-\frac{1}{s^2}\mu+\frac{1}{s^3}\cdot$$

Similarly, if we write

$$I_2=\int_0^\infty\int_0^\infty e^{-sx}\ \bar{F}(x)\bar{W}_F(t)dxdt.$$

Then

$$I_2=\frac{1}{2s}\mu_{(2)}-\frac{1}{2s}\mu_{(2)}\phi(s). \qquad (4)$$

Substituting the equations (3) and (4) in equation (2), we get

$$\frac{1}{s^3}\phi(s)+\frac{1}{s^2}\mu-\frac{1}{s^3}\geq\frac{1}{2s}\mu_{(2)}\phi(s).$$

Which is completes the proof. ∎

Theorem 3.3. As $n\to\infty$, $[\delta_n(s)-\omega(s)]$ is asymptotically normal with mean 0 and variance

$$\sigma^s=Var\left[\left(\frac{1}{s^3}-\frac{1}{2s}X^2\right)E(e^{-sx})+\frac{1}{s^2}X+\frac{1}{s^3}e^{-sx}-\frac{1}{2s}e^{-sx}\mu_{(2)}+\frac{1}{s^2}\mu-\frac{2}{s^3}\right].$$

U and H_0 variance to

$$\sigma_0^2(s)=\frac{s+5}{(s+1^3)(2s+1)}\cdot$$

Proof. Using standard U-statistic theory

$$\sigma^2=\mathrm{V}\{E[\phi(X_1,X_2)|X_1]+E[\phi(X_1,X_2)|X_2]\}$$

Then

$$\mathrm{E}\big(\phi(X_1,X_2)|(X_1)\big)$$

$$=\left(\frac{1}{s^3}-\frac{1}{2s}X_2^1\right)\int_0^\infty e^{-sx}dF(x)$$

$$+\frac{1}{s^2}X_1-\frac{1}{s^3},$$

and

$$\mathrm{E}(\phi(X_1,X_2)|(X_2))=\frac{1}{s^3}e^{-SX_2}-\frac{1}{2s}e^{-SX_2}$$

$$\int_0^\infty x^2dF(x)\frac{1}{s^2}xdF(x)$$

$$-\frac{1}{s^3},$$

therefore

$$\sigma^s=Var\left[\left(\frac{1}{s^3}-\frac{1}{2s}X^2\right)E(e^{-sx})+\frac{1}{s^2}X+\frac{1}{s^3}e^{-sx}-\frac{1}{2s}e^{-sx}\mu_{(2)}\quad+\frac{1}{s^2}\mu-\frac{2}{s^3}\right].$$

Under H_0

$$\sigma_0^2(s)=\frac{s+5}{(s+1^3)(2s+1)}\cdot$$ ∎

4 The Pitman Asymptotic Efficiency (PAE)

To judge on the quality of this procedure Pitman Asymptotic Efficiencies (PAE) are computed and compared with some other tests for the following alternative distribution

(i) The Weibull distribution

$$F_1(X)=e^{-x^\theta}, x\geq 0, \theta\geq 1.$$

(ii) The Linear Failure Rate distribution

$$F_2(X) = e^{-x-\frac{\theta}{2}x^2}, \quad x \geq 0, \theta \geq 0.$$

(iii) The Makeham distribution

$$F_3(x) = e^{-x-\theta(x+e^{-x}+1)}, x \geq 0, \theta \geq 0.$$

Note that if $\theta = 1$ and $\bar{F}_1$ goes to the exponential distribution then if $\theta = 0$ and $\bar{F}_2$ $\bar{F}_3$ reduce to the exponential distribution. The PAE is defined by

$$PAE\left(\Delta_n(s)\right) = \frac{1}{\sigma_0(s)}\left|\frac{d}{dt}\delta(s)\right|_{\theta\to\theta_0}.$$

When s =2.5, this leads to

$PAE[\Delta_n(2.5),\ Weibull] = 1.17243.$

$PAE\ [\Delta_n(2.5), LFR] = 0.956183.$

And

$PAE[\Delta_n(2.5),\ Makeham]$= 0.278887.

Where $\sigma_0(2.5) = 0.170747$.

Test	Weibull	LFR	Makeham
Ahmed [2]	0.132	0.433	0.144
Bryson and Siddique [3]	0.170	0.408	0.039
Abdul-Aziz [1]	0.223	0.535	0.184
Kango [7]	0.050	0.217	0.144
Our test $\Delta_n(2.5)$	1.172	0.956	0.279

Table 1. Compare between the PAE of our test and some others tests.

From table 1, it is obvious that $\Delta_n(5)$ is better than the others based on PAE.

5. Power Estimate of the Test $\Delta_n(2.5)$

In the paper, the power of our $\Delta_n(2.5)$ is estimated at $(1-\alpha)\%$ confidence level, α=0.05 with suitable parameters values of θ at n=10, 20 and 30 with respect to three alternatives Weibull, Linear Failure Rate (LFR) and Gamma distribution based on 10000 samples.

N	θ	Weibull	LFR	Gamma
10	2	0.8043	0.2511	0.2737
	3	0.9972	0.2850	0.7407
	4	0.9999	0.2821	0.9149
20	2	0.9851	0.4897	0.5689
	3	1.0000	0.5571	0.9502
	4	1.0000	0.5892	0.9976
30	2	0.9996	0.6864	0.7393
	3	1.0000	0.7682	0.9929
	4	1.0000	0.8033	0.9997

Table 2. The power estimates of $\Delta_n(2.5)$.

Table 2, when that the power estimates of test $\Delta_n(2.5)$ are good for all alternative and increase when the value of the parameters θ and the samples size increase.

6. Conclusion

Testing exponentiality against NBUARFR distribution is considered. A new class of the distribution is added to the family of renewal classes of life distribution. The Pitman asymptotic efficiency of the test are calculated compared with other test PAE and power estimates of the test. Quality criteria of the test is shown by the famous criteria which is Pitman efficiency.

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