International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 06 Issue: 08 | Aug 2019 www.irjet.net p-ISSN: 2395-0072

Analysis of Heat Transfer from Rectangular Finned Surface using Shooting Method

RANJAN SINGH

[1]M.Tech Student, Mechanical Department, CAPGS, BPUT, Rourkela, Odisha, India

---

Abstract - *Numerical and analytical determination of temperature distribution T(x) and fin heat transfer rate of rectangular fin is presented. The shooting method is used which converts the boundary value problem into an initial value problem. Numerical results at different step sizes are compared to the analytical solution. The shooting method starts the solution from a guessed value at the boundary with initial value and produces the final boundary value at the end. The major downside is that the shooting method is computationally time expensive and complex. Calculated the heat transfer for rectangular geometry to get the desired output value. Heat transfer rate, temperature distribution along the length are increased from the tip to the base for rectangular section. All the results are plotted in the form of graph.*

Key Words: Shooting method, Rectangular fin, Temperature distribution and Guessing value.

1. INTRODUCTION

Fins are used in a large number of applications to increase the heat transfer from surfaces. Typically, the fin material has a high thermal conductivity. The design of cooling fins is encountered in many situations and we thus examine heat transfer in a fin as a way of defining some criteria for design. Fins can be of a variety of geometry rectangular, triangular, parabolic, and hyperbolic and can be attached to the inside, outside or to both sides of circular, flat plate. Fins are most commonly used in heat exchanging devices such as radiators in cars, computer CPU heat sinks, and heat exchangers in power plants.

•At wider spacing, shorter fins are more preferred than longer fins.

•The aspect ratio for an optimized fin array is more than that of a single fin for rectangular profile.

1.1 Types of Fin

Fins can be broadly classified as:
1.Longitudinal fin - Rectangular, Trapezoidal and Concave profile.
2. Radial fin - Rectangular and Triangular profile.
3. Pin fin - Cylindrical.

1.2 Rectangular fin using AUTOCAD

The rectangular fin as shown in the above figure, with L, as the length of the fin, as thickness of the fin and B breadth of fin and assuming the heat flow is unidirectional and it is along length and the heat transfer coefficient (h) on the surface of the fin is constant.

Base temperature,T_b = 40 °C, Ambient temperature,T_∞=20 °C, Length of fin=10cm, Breadth of fin =5cm.

2. Methodology

Introduction: The shooting method is used to solve initial value problems. An ordinary differential equation is converted to many first order differential equations. The known boundary values are converted to initial values of the solution. With guess value of unknown boundary value; using trial and error method or some other scientific approach, these one dimensional equations are solved simultaneously.

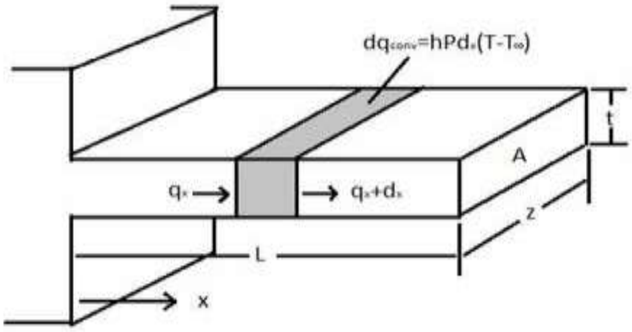

Figure 1: *Geometry of rectangular fin.*

$$Q_x = Q_{x+dx} + Q_c$$

$$Q_c = hP(x)\Delta x(T - T_\infty) = 0 ------ (3.2)$$

Where:

Q_{x+dx}= Energy leaving the control volume,

Q_x =Energy entering the control volume.

Q_c = Heat transfer due to convection.

$$Q_x = -kA(x)\frac{dT}{dx} ------ (3.1)$$

By Using Taylor Series Expansion

$$Q_{x+dx} = Q_x + \frac{d}{dx}(Q_x)\Delta x + Q_c --- (3.3)$$

Using equation (3.1) and (3.3) in (3.2) we get,

$$\frac{d}{dx}(-kA(x)\frac{dT}{dx})\Delta x + hP(x)\Delta x(T - T_\infty) = 0$$

$$\frac{d}{dx}(A(x)\frac{dT}{dx})\Delta x - \frac{hP(x)}{k}(T - T_\infty) = 0 -- (3.4)$$

This is the governing differential equation for finned surface.

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_C}(T - T_\infty) = 0 ------------ (3.5)$$

Let$\theta = T - T_\infty => dT = d\theta$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 ------------ (3.6)$$

General solution to the differential equation is:

$$\theta(x) = C_1 e^{-mx} + C_2 e^{mx}$$

$$= C_1 \cos h(mx) + C_2 \sin h(mx) ---- (3.7)$$

Where C_1 and C_2 are two arbitrary unknown constants. The boundary conditions are:

$$\mathrm{x = 0,\ T = T_b(Base\ temperature)} ---- (3.8)$$

Boundary conditions at fin tip:

1. Heat dissipation from an infinitely long fin.

$$(x = \infty, T = T_\infty) ------- (3.8)a$$

2. Heat dissipation from a fin insulated at the tip.

$$\left(x = L, -k\frac{dT}{dx} = h_t(T - T_\infty)\right) ------- (3.8)b$$

3. Heat dissipation from a fin losing heat at the tip.

$$\left(x = L, \frac{dT}{dx} = 0\right) ---------- (3.8)c$$

Heat lost by rectangular fin,

$$Q = k\frac{A_c m\theta_0 h \cos hmL + km \sin hmL}{km \cos hmL + h \sin hmL}$$

Where, $\theta_0 = Temperature\ difference, K$.

k = thermal conductivity, W/MK.

m^2=hL/kA- - - - - - - - - - - - - - - - - - - -(3.9)

A =cross sectional area of fin, m^2.

h = heat transfer coefficient/m^2K.

3. Solution of the differential equation: Solution of the Eq.(3.6) is obtained using analytical method and numerical method.

3.1.1 Analytical method: Using the boundary condition Eq.(3.8) and(3.8)a in Eq.(3.7); the solution obtained is:

$$\theta(x) = \theta_b e^{-mx} -------- (3.10)$$

Using the boundary condition Eq.(3.8) and (3.8)b in Eq.(3.7); the solution obtained is:

$$\theta(x) = \theta_b \frac{e^{m(l-x)} + e^{-m(l-x)}}{e^{ml} + e^{-ml}} ------ (3.11)$$

Using the boundary condition Eq.(3.8) and (3.8)c in the Eq.(3.7); the solution obtained is:

$$\theta(x) = \theta_b \frac{\cosh m(l-x) + \frac{h}{km}\sinh m(l-x)}{\cosh ml + \frac{h}{km}[\sinh(ml)]} -- -- (3.12)$$

3.1.2 Numerical method: For solution of the Eq.(3.6), the shooting technique is used. From Eq.(3.6),

Let z=θ ---------- (3.13)

And y = z^I ---------- (3.14)

The boundary conditions are: At $x = 0, \theta = \theta_b$ and at $x = L, \theta(L) = 0$.

The solution of Eq.(3.13) is:

$$z_{i+1} = z_i + f_1(x_i, \theta_i, z_i)h$$

$$\mathrm{z_{i+1} = z_i + m^2\theta h} -------------- (3.15)$$

And the solution of Eq.(3.14) is: $\theta = \theta_0 + f_2(x_i, \theta_i, z_i)h$

Or $\theta = \theta_0 + zh$

Or $\theta_{i+1} = \theta_i + zh$ ———————————————— (3.16)

The solution starts from x=0, $\theta = \theta_b$ and guess value of z and ends at x=L, $\theta = 0$ using Shooting techniques. Here h is the step size. That is calculated according to the number of points used along the fin surface. So $h = L/n$, where n is the number of points along L.

3.2 Shooting Technique:

Shooting technique is applied for the Eq.(3.15) and (3.16). Using the value of ɵ and guessed value of z as initial value, these values are calculated at different points along the length of the fin till x=L. The solution is said to be converged when for a particular guess value of z; ɵ becomes 0. The initial guess value is obtained using heat and trial method.

3.3 Conclusion:

The differential equation for the fin has been solved using analytical and numerical method. So, the solution procedure has been discussed. Then both the solution must be matched to get the accuracy of the solution.

4. Results and Stimulation:

4.1. Introduction: The formulation to the fin for varying cross sectional area has been derived in the previous chapter. The Eq.(3.6) has been solved using analytical and numerical method. The numerical method of solution consists of Shooting techniques. The results and the required graphs for the said equation are presented below.

4.2. Results: Eq.(3.12) for analytical result and also Eq.(3.16) and (3.17) for numerical results obtained for different values of *x* are presented in table.4.1.

Table:1. Temperature profile along the length of the fin (Rectangular cross section).

X in cm	ɵ(Analytical)	ɵ(Numerical)
0	20	20
1.8	8.85	3.7
3.6	3.92	0.68
5.4	1.74	0.12
7.2	0.77	0.023
9.0	0.34	0.004
10.8	0.15	0.001
12.6	0.07	0.00022
14.4	0.03	0.00020
16.2	0.01	0.00010
18.0	0	0

The geometrical dimension for the rectangular cross section: length = 18cm, width = 0.1cm.

Figure:1. Temperature profile along the length of the fin with uniform rectangular cross section.

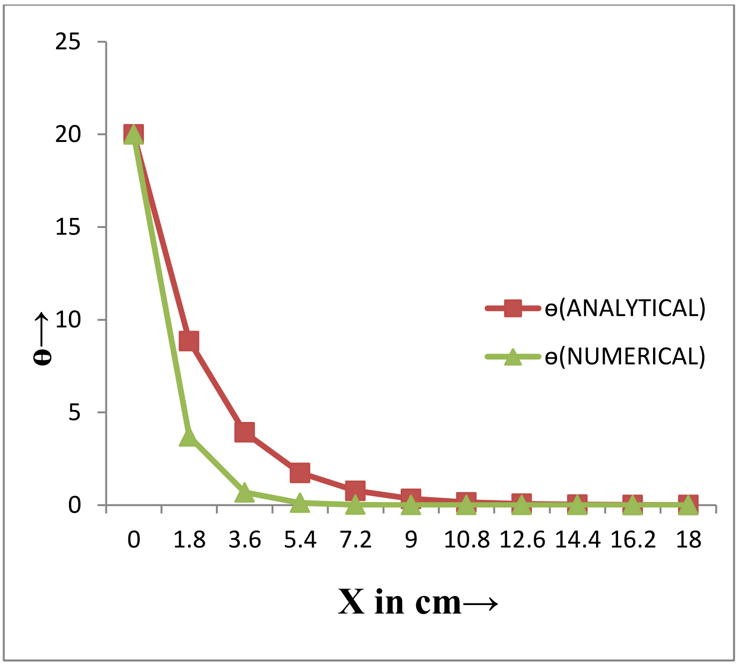

Figure shows that the numerical and analytical solution for the temperature profile gradually decreases along the length of the fin. The numerical result shows greater loss of the temperature than the analytic solution temperature values.

4.3 CONCLUSIONS:

The results show good match at both end of the fin but there is large difference between these two at the central portion of the fin. The numerical result shows greater loss of the temperature than the analytic solution temperature values. In Rectangular fin the guess value taking in the numerical method is -20.57672.

5. CONCLUSIONS

1. Temperature of rectangular fin increases along the length from the tip to the base of the fin.

2. Heat transfer rate increases along the length from the tip to the base for rectangular fin profile.

3. In case of shooting method, maximum heat transfer rate is increased along the length from the tip to the base.

4. In Rectangular fin the guess value taking in the numerical method is -20.57672.

REFERENCES

[1] Saurav Kumar and M.K. Singh,"Comparative analysis of rectangular and triangular fin in free and forced convection", International Journal of Advance Research in Science and Engineering(IJARSE) 6 Issue-7(2017).

[2] M. Vinay Kumar, E. Surendar and T.Vikram, ”Enhancement of heat transfer from the rectangular fins by different perforations”,IJMTARC,5 Issue-19(2017).

[3] Mayank Jain, Mahendra Sankhala, Kanhaiya Patidar, Lokesh Aurangabadkar, “Heat Transfer Analysis and Optimization of Fins by Variation in Geometry”, International Journal of Mechanical And Production Engineering, 5, Issue-7(2017)2320-2092.

[4]Pirompugd W. and Wongwises S. Partially wet fin efficiency for the longitudinal fins of rectangular, triangular, concave parabolic and convex parabolic profiles. Journal of the Franklin Institute, 350(6),(2013)1424-1442.

[5]DogonchiA.S. and Ganji D.D. “Convection-radiation heat transfer study of moving fin with temperature-dependent thermal conductivity, heat transfer coefficient and heat generation”, Applied Thermal Engineering,103(2016)705-712.M. Young, The Technical Writer’s Handbook. Mill Valley, CA: University Science, 1989.

BIOGRAPHIES

Ranjan Singh received B.Tech Mechanical degree from NIET, BPUT in 2016 and studying M.Tech. Degree in Mechanical Engineering from CAPGS, BPUT, Rourkela, Odisha will be completed by September 2019, with specialization in Heat power and thermal engineering. Presently in the final year of Mtech in CAPGS, BPUT, Rourkela, Odisha.