International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 06 Issue: 07 | July 2019 www.irjet.net p-ISSN: 2395-0072

Wavelet based Galerkin Method for the Numerical Solution of One Dimensional Partial Differential Equations

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Abstract - *In this paper, we proposed the Wavelet based Galerkin method for numerical solution of one dimensional partial differential equations using Hermite wavelets. Here, Hermite wavelets are used as weight functions and these are assumed bases elements which allow us to obtain the numerical solutions of the partial differential equations. Some of the test problems are given to demonstrate the numerical results obtained by proposed method are compared with already existing numerical method i.e. finite difference method (FDM) and exact solution to check the efficiency and accuracy of the proposed method*

***Key Words*: Wavelet; Numerical solution; Hermite bases; Galerkin method; Finite difference method.**

1. INTRODUCTION

Wavelet analysis is newly developed mathematical tool and have been applied extensively in many engineering fileld. This has been received a much interest because of the comprehensive mathematical power and the good application potential of wavelets in science and engineering problems. Special interest has been devoted to the construction of compactly supported smooth wavelet bases. As we have noted earlier that, spectral bases are infinitely differentiable but have global support. On the other side, basis functions used in finite-element methods have small compact support but poor continuity properties. Already we know that, spectral methods have good spectral localization but poor spatial localization, while finite element methods have good spatial localization, but poor spectral localization. Wavelet bases perform to combine the advantages of both spectral and finite element bases. We can expect numerical methods based on wavelet bases to be able to attain good spatial and spectral resolutions. Daubechies [1] illustrated that these bases are differentiable to a certain finite order. These scaling and corresponding wavelet function bases gain considerable interest in the numerical solutions of differential equations since from many years [2–4].

Wavelets have generated significant interest from both theoretical and applied researchers over the last few decades. The concepts for understanding wavelets were provided by Meyer, Mallat, Daubechies, and many others, [5]. Since then, the number of applications where wavelets have been used has exploded. In areas such as approximation theory and numerical solutions of differential equations, wavelets are recognized as powerful weapons not just tools.

In general it is not always possible to obtain exact solution of an arbitrary differential equation. This necessitates either discretization of differential equations leading to numerical solutions, or their qualitative study which is concerned with deduction of important properties of the solutions without actually solving them. The Galerkin method is one of the best known methods for finding numerical solutions of differential equations and is considered the most widely used in applied mathematics [6]. Its simplicity makes it perfect for many applications. The wavelet-Galerkin method is an improvement over the standard Galerkin methods. The advantage of wavelet-Galerkin method over finite difference or finite element method has lead to tremendous applications in science and engineering. An approach to study differential equations is the use of wavelet function bases in place of other conventional piecewise polynomial trial functions in finite element type methods.

In this paper, we developed Hermite wavelet-Galerkin method (HWGM) for the numerical solution of differential equations. This method is based on expanding the solution by Hermite wavelets with unknown coefficients. The properties of Hermite wavelets together with the Galerkin method are utilized to evaluate the unknown coefficients and then a numerical solution of the one dimensional partial differential equation is obtained.

The organization of the paper is as follows. Preliminaries of Hermite wavelets are given in section 2. Hermite wavelet-Galerkin method of solution is given in section 3. In section 4 Numerical results are presented. Finally, conclusions of the proposed work are discussed in section 5.

2. PRELIMINARIES OF HERMITE WAVELETS

Wavelets form a family of functions which are generated from dilation and translation of a single function which is called as mother wavelet $\psi(x)$. If the dialation parameter a and translation parameter b varies continuously, we have the following family of continuous wavelets [7 , 8]:

$$\psi_{a,b}(x) = |a|^{-1/2}\,\psi\left(\frac{x-b}{a}\right), \quad \forall\ a,b \in R, \quad a \neq 0.$$

If we restrict the parameters a and b to discrete values as $a = a_0^{-k}, \ \ b = n b_0\, a_0^{-k}, \quad a_0 > 1, \quad b_0 > 0.$ We have the following family of discrete wavelets

$$\psi_{k,n}(x) = |a|^{1/2}\,\psi(a_0^k\, x - n b_0), \ \forall\ a,\ b \in R,\ \ a \neq 0,$$

where $\psi_{k,n}$ form a wavelet basis for $L^2(R)$. In particular, when $a_0 = 2$ and $b_0 = 1$,then $\psi_{k,n}(x)$ forms an orthonormal basis. Hermite wavelets are defined as

$$\psi_{n,m}(x) = \begin{cases} \dfrac{2^{\frac{k}{2}}}{\sqrt{\pi}}\tilde{H}_m(2^k x - 2n + 1), & \dfrac{n-1}{2^{k-1}} \le x < \dfrac{n}{2^{k-1}} \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

Where
$$\tilde{H}_m = \sqrt{\frac{2}{\pi}}\, H_m(x) \tag{2.2}$$

where $m = 0,1,\ldots,M-1.$ In eq. (2.2) the coefficients are used for orthonormality. Here $H_m(x)$ are the second Hermite polynomials of degree m with respect to weight function $W(x) = \sqrt{1-x^2}$ on the real line R and satisfies the following reccurence formula $H_0(x) = 1,\ H_1(x) = 2x$,

$$H_{m+2}(x) = 2xH_{m+1}(x) - 2(m+1)H_m(x), \ \text{where}\ m = 0,1,2,\ldots. \tag{2.3}$$

For $k = 1$ & $n = 1$ in (2.1) and (2.2), then the Hermite wavelets are given by

$$\psi_{1,0}(x) = \frac{2}{\sqrt{\pi}},$$

$$\psi_{1,1}(x) = \frac{2}{\sqrt{\pi}}(4x - 2),$$

$$\psi_{1,2}(x) = \frac{2}{\sqrt{\pi}}(16x^2 - 16x + 2),$$

$$\psi_{1,3}(x) = \frac{2}{\sqrt{\pi}}(64x^3 - 96x^2 + 36x - 2),$$

$$\psi_{1,4}(x) = \frac{2}{\sqrt{\pi}}(256x^4 - 512x^3 + 320x^2 - 64x + 2)$$, and so on.

Function approximation:

We would like to bring a solution function $u(x)$ under Hermite space by approximating $u(x)$ by elements of Hermite wavelet bases as follows,

$$u(x) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\,\psi_{n,m}(x) \tag{2.4}$$

where $\psi_{n,m}(x)$ is given in eq. (2.1).

We approximate $u(x)$ by truncating the series represented in Eq. (2.4) as,

$$u(x) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\, \psi_{n,m}(x) \tag{2.5}$$

where c and ψ are $2^{k-1} M \times 1$ matrix.

Convergence of Hermite wavelets

Theorem: If a continuous function $u(x) \in L^2(R)$ defined on $[0,1)$ be bounded, i.e. $u(x) \le K$, then the Hermite wavelets expansion of $u(x)$ converges uniformly to it [9].

Proof: Let $u(x)$ be a bounded real valued function on $[0,1)$. The Hermite coefficients of continuous functions $u(x)$ is defined as,

$$C_{n,m} = \int_0^1 u(x)\psi_{n,m}(x)dx$$

$$= \int_I u(x) \frac{2^{\frac{k+1}{2}}}{\sqrt{\pi}} H_m\left(2^k x - 2n + 1\right)dx, \quad \text{where } I = \left[\frac{n-1}{2^{k+1}}, \frac{n}{2^{k+1}}\right)$$

Put $2^k x - 2n + 1 = z$

$$= \frac{2^{\frac{k+1}{2}}}{\sqrt{\pi}} \int_{-1}^{1} u\left(\frac{z-1+2n}{2^k}\right) H_m(z) 2^{-k} dx$$

$$= \frac{2^{\frac{-k+1}{2}}}{\sqrt{\pi}} \int_{-1}^{1} u\left(\frac{z-1+2n}{2^k}\right) H_m(z) dx$$

Using GMVT integrals,

$$= \frac{2^{\frac{-k+1}{2}}}{\sqrt{\pi}} u\left(\frac{w-1+2n}{2^k}\right) \int_{-1}^{1} H_m(z) dx, \text{ for some } w \in (-1,1)$$

$$= \frac{2^{\frac{-k+1}{2}}}{\sqrt{\pi}} u\left(\frac{w-1+2n}{2^k}\right) h$$

where $h = \int_{-1}^{1} H_m(z) dx$

$$\left| C_{n,m} \right| = \left| \frac{2^{\frac{-k+1}{2}}}{\sqrt{\pi}} \right| \left| u\left(\frac{w-1+2n}{2^k}\right) \right| h$$

Since u is bounded, therefore $\sum_{n,m=0}^{\infty} C_{n,m}$ absolutely convergent. Hence the Hermite series expansion of $u(x)$ converges uniformly.

3. METHOD OF SOLUTION

Consider the differential equation of the form,

$$\frac{\partial^2 u}{\partial x^2} + \alpha \frac{\partial u}{\partial x} + \beta u = f(x) \tag{3.1}$$

With boundary conditions $u(0) = a, \quad u(1) = b$ (3.2)

Where α , β are may be constant or either a functions of x or functions of u and $f(x)$ be a continuous function.

Write the equation (3.1) as

$$R(x) = \frac{\partial^2 u}{\partial x^2} + \alpha \frac{\partial u}{\partial x} + \beta u - f(x) \tag{3.3}$$

where $R(x)$ is the residual of the eq. (3.1). When $R(x) = 0$ for the exact solution, $u(x)$ only which will satisfy the boundary conditions.

Consider the trail series solution of the differential equation (3.1), $u(x)$ defined over [0, 1) can be expanded as a modified Hermite wavelet, satisfying the given boundary conditions which is involving unknown parameter as follows,

$$u(x) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \psi_{n,m}(x) \tag{3.4}$$

where $c_{n,m}$'s are unknown coefficients to be determined.

Accuracy in the solution is increased by choosing higher degree Hermite wavelet polynomials.

Differentiating eq. (3.4) twice with respect to x and substitute the values of $\frac{\partial^2 u}{\partial x^2}, \frac{\partial u}{\partial x}, u$ in eq. (3.3). To find $c_{n,m}$'s we choose weight functions as assumed bases elements and integrate on boundary values together with the residual to zero [10].

i.e. $\int_0^1 \psi_{1,m}(x) R(x)\, dx = 0$, $m = 0, 1, 2, \ldots\ldots$

then we obtain a system of linear equations, on solving this system, we get unknown parameters. Then substitute these unknowns in the trail solution, numerical solution of eq. (3.1) is obtained.

4. NUMERICAL EXPERIMENT

Test Problem 4.1 First, consider the differential equation [11],

$$\frac{\partial^2 u}{\partial x^2} + u = -x\,, \quad 0 \le x \le 1 \tag{4.1}$$

With boundary conditions: $u(0) = 0, \quad u(1) = 0$ (4.2)

The implementation of the eq. (4.1) as per the method explained in section 3 is as follows:

The residual of eq. (4.1) can be written as: $R(x) = \frac{\partial^2 u}{\partial x^2} + u + x$ (4.3)

Now choosing the weight function $w(x) = x(1-x)$ for Hermite wavelet bases to satisfy the given boundary conditions (4.2), i.e. $\psi(x) = w(x) \times \psi(x)$

$$\psi_{1,0}(x) = \psi_{1,0}(x) \times x(1-x) = \frac{2}{\sqrt{\pi}} x(1-x) \quad ,$$

$$\psi_{1,1}(x) = \psi_{1,1}(x) \times x(1-x) = \frac{2}{\sqrt{\pi}} (4x-2)\, x(1-x)$$

$$\psi_{1,2}(x) = \psi_{1,2}(x) \times x(1-x) = \frac{2}{\sqrt{\pi}} (16x^2 - 16x + 2)\, x(1-x)$$

Assuming the trail solution of (5.1) for $k=1$ and $m=3$ is given by

$$u(x) = c_{1,0}\,\psi_{1,0}(x) + c_{1,1}\,\psi_{1,1}(x) + c_{1,2}\,\psi_{1,2}(x) \tag{4.4}$$

Then the eq. (4.4) becomes

$$u(x) = c_{1,0}\frac{2}{\sqrt{\pi}}x(1-x) + c_{1,1}\frac{2}{\sqrt{\pi}}(4x-2)x(1-x) + c_{1,2}\frac{2}{\sqrt{\pi}}(16x^2-16x+2)x(1-x) \tag{4.5}$$

Differentiating eq. (4.5) twice w.r.t. x we get,

i.e. $$\frac{\partial u}{\partial x} = c_{1,0}\frac{2}{\sqrt{\pi}}(1-2x) + c_{1,1}\frac{2}{\sqrt{\pi}}(-12x^2+12x-2) + c_{1,2}\frac{2}{\sqrt{\pi}}(-64x^3+96x^2-36x+2) \tag{4.6}$$

$$\frac{\partial^2 u}{\partial x^2} = c_{1,0}\frac{2}{\sqrt{\pi}}(-2) + c_{1,1}\frac{2}{\sqrt{\pi}}(-24x+12) + c_{1,2}\frac{2}{\sqrt{\pi}}(-192x^2+192x-36) \tag{4.7}$$

Using eq. (4.5) and (4.7), then eq. (4.3) becomes,

$$R(x) = c_{1,0}\frac{2}{\sqrt{\pi}}(-2) + c_{1,1}\frac{2}{\sqrt{\pi}}(-24x+12) + c_{1,2}\frac{2}{\sqrt{\pi}}(-192x^2+192x-36) + \left(c_{1,0}\frac{2}{\sqrt{\pi}}x(1-x) + c_{1,1}\frac{2}{\sqrt{\pi}}(4x-2)x(1-x) + c_{1,2}\frac{2}{\sqrt{\pi}}(16x^2-16x+2)\right) + x$$

$$\Rightarrow R(x) = c_{1,0}\frac{2}{\sqrt{\pi}}(-x^2+x-2) + c_{1,1}\frac{2}{\sqrt{\pi}}(-4x^3+6x^2-26x+12) + c_{1,2}\frac{2}{\sqrt{\pi}}(-16x^4+32x^3-210x^2+194x-36) + x \tag{4.8}$$

This is the residual of eq. (4.1). The “weight functions” are the same as the bases functions. Then by the weighted Galerkin method, we consider the following:

$$\int_0^1 \psi_{1,m}(x)R(x)dx = 0,\ m = 0, 1, 2 \tag{4.9}$$

For $m = 0, 1, 2$ in eq. (4.9),

i.e. $\int_0^1 \psi_{1,0}(x)R(x)dx = 0$, $\int_0^1 \psi_{1,1}(x)R(x)dx = 0$, $\int_0^1 \psi_{1,2}(x)R(x)dx = 0$

$$\Rightarrow (-0.3802)c_{1,1} + (0)c_{1,2} + (0.4487)c_{1,3} + 0.0940 = 0 \tag{4.10}$$

$$(0)c_{1,1} - (0.9943)c_{1,2} + (0)c_{1,3} + 0.0376 = 0 \tag{4.11}$$

$$(0.4487)c_{1,0} + (0)c_{1,1} - (2.3686)c_{1,2} - 0.1128 = 0 \tag{4.12}$$

We have three equations (4.10) – (4.12) with three unknown coefficients i.e. $c_{1,0}$, $c_{1,1}$ and $c_{1,2}$. By solving this system of algebraic equations, we obtain the values of $c_{1,0} = 0.2446$, $c_{1,1} = 0.0378$ and $c_{1,2} = -0.0013$. Substituting these values in eq. (4.5), we get the numerical solution; these results and absolute error = $|u_a(x) - u_e(x)|$ (where $u_a(x)$ and $u_e(x)$ are numerical and exact solutions respectively) are presented in table - 1 and fig - 1 in comparison with exact solution of eq. (4.1) is $u(x) = \dfrac{\sin(x)}{\sin(1)} - x$.

Table – 1: Comparison of numerical solution and exact solution of the test problem 4.1

x	Numerical solution		Exact solution	Absolute error	
	FDM	HWGM		FDM	HWGM
0.1	0.018660	0.018624	0.018642	1.80e-05	1.80e-05
0.2	0.036132	0.036102	0.036098	3.40e-05	4.00e-06
0.3	0.051243	0.051214	0.051195	4.80e-05	1.90e-05
0.4	0.062842	0.062793	0.062783	5.90e-05	1.00e-05
0.5	0.069812	0.069734	0.069747	6.50e-05	1.30e-05

0.6	0.071084	0.070983	0.071018	6.60e-05	3.50e-05
0.7	0.065646	0.065545	0.065585	6.10e-05	4.00e-05
0.8	0.052550	0.052481	0.052502	4.80e-05	2.10e-05
0.9	0.030930	0.030908	0.030902	2.80e-05	6.00e-06

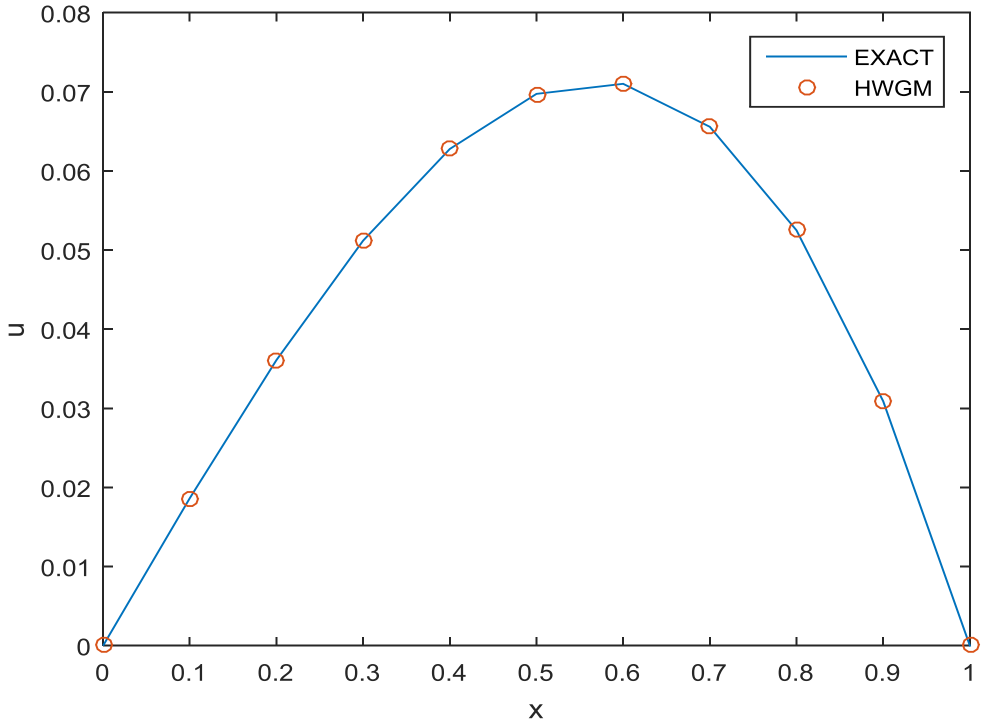

Fig – 1: Comparison of numerical and exact solutions of the test problem 4.1.

Test Problem 4.2 Next, consider another differential equation [12]

$$\frac{\partial^2 u}{\partial x^2} - \pi^2 u = -2\pi^2 \sin(\pi x), \quad 0 \le x \le 1 \tag{4.12}$$

With boundary conditions:

$$u(0) = 0, \quad u(1) = 0 \tag{4.13}$$

Which has the exact solution $u(x) = \sin(\pi x)$.

By applying the method explained in the section 3, we obtain the constants $c_{1,0} = 3.1500$, $c_{1,1} = 0$ and $c_{1,2} = -0.1959$. Substituting these values in eq. (4.5) we get the numerical solution. Obtained numerical solutions are compared with exact and other existing method solutions are presented in table - 2 and fig - 2.

Table – 2: Comparison of numerical solution and exact solution of the test problem 4.2.

x	Numerical solution			Exact solution	Absolute error		
	FDM	Ref [11]	HWGM		FDM	Ref [11]	HWGM
0.1	0.310289	0.308865	0.308754	0.309016	1.27e-03	1.51e-04	2.60e-04
0.2	0.590204	0.587527	0.588509	0.588772	1.43e-03	1.25e-03	2.60e-04
0.3	0.812347	0.808736	0.809554	0.809016	3.33e-03	2.80e-04	5.40e-04
0.4	0.954971	0.950859	0.950670	0.951056	3.92e-03	1.97e-04	3.90e-04
0.5	1.004126	0.999996	0.999123	1.000000	4.13e-03	4.00e-06	8.80e-04
0.6	0.954971	0.951351	0.950670	0.951056	3.92e-03	2.95e-04	3.90e-04
0.7	0.812347	0.809671	0.809554	0.809016	3.33e-03	6.55e-04	5.40e-04

0.8	0.590204	0.588815	0.588509	0.587785	2.42e-03	1.03e-03	7.20e-04
0.9	0.310289	0.310379	0.308754	0.309016	1.27e-03	1.36e-03	2.60e-04

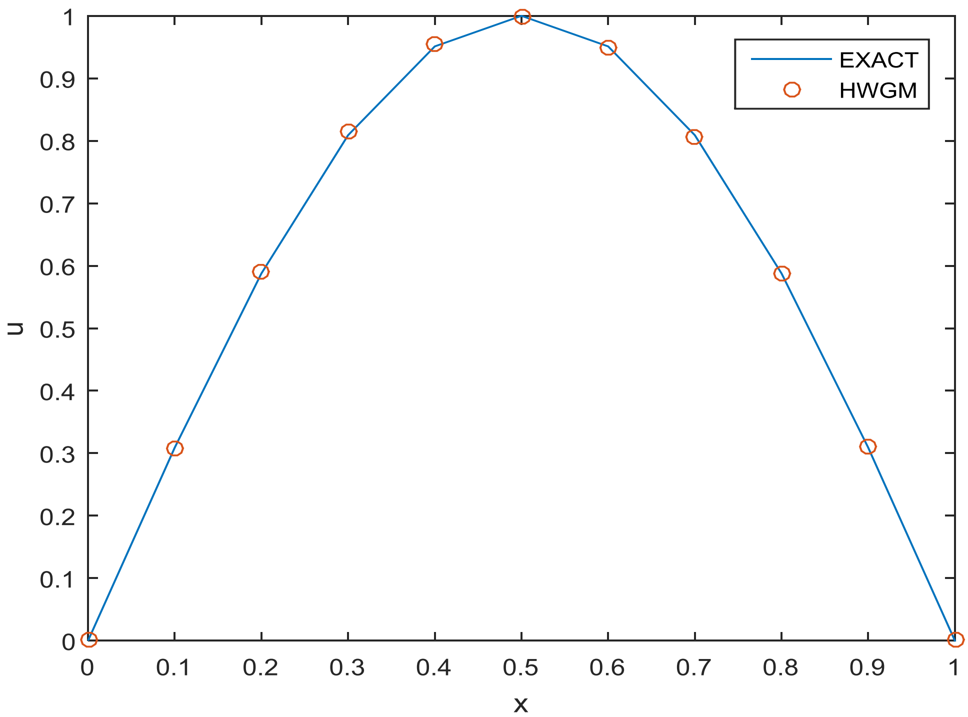

Fig – 2: Comparison of numerical and exact solutions of the test problem 4.2.

Test Problem 4.3 Consider another differential equation [13]

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial x} = -\left(e^{x-1}+1\right), \quad 0 \le x \le 1 \tag{4.14}$$

With boundary conditions: $u(0)=0, \quad u(1)=0$ (4.15)

Which has the exact solution $u(x)=x\left(1-e^{x-1}\right)$.

By applying the method explained in the section 3, we obtain the constants $c_{1,0}=0.7103$, $c_{1,1}=0.0806$ and $c_{1,2}=0.0064$. Substituting these values in eq. (4.5) we get the numerical solution. Obtained numerical solutions are compared with exact and other existing method solutions are presented in table - 3 and fig - 3.

Table - 3: Comparison of numerical solution and exact solution of the test problem 4.3.

x	Numerical solution			Exact solution	Absolute error		
	FDM	Ref [12]	HWGM		FDM	Ref [12]	HWGM
0.1	0.061948	0.059383	0.059339	0.059343	2.61e-03	4.00e-05	4.00e-06
0.2	0.115151	0.110234	0.110138	0.110134	5.02e-03	1.00e-04	4.00e-06
0.3	0.158162	0.151200	0.151031	0.151024	7.14e-03	1.76e-04	7.00e-06
0.4	0.189323	0.180617	0.180479	0.180475	8.85e-03	1.42e-04	4.00e-06
0.5	0.206737	0.196983	0.196733	0.196735	1.00e-02	2.48e-04	2.00e-06
0.6	0.208235	0.198083	0.197803	0.197808	1.04e-02	2.75e-04	5.00e-06
0.7	0.191342	0.181655	0.181421	0.181427	9.92e-03	2.28e-04	6.00e-06
0.8	0.153228	0.145200	0.145008	0.145015	8.21e-03	1.85e-04	7.00e-06
0.9	0.090672	0.085710	0.085637	0.085646	5.03e-03	6.40e-05	9.00e-06

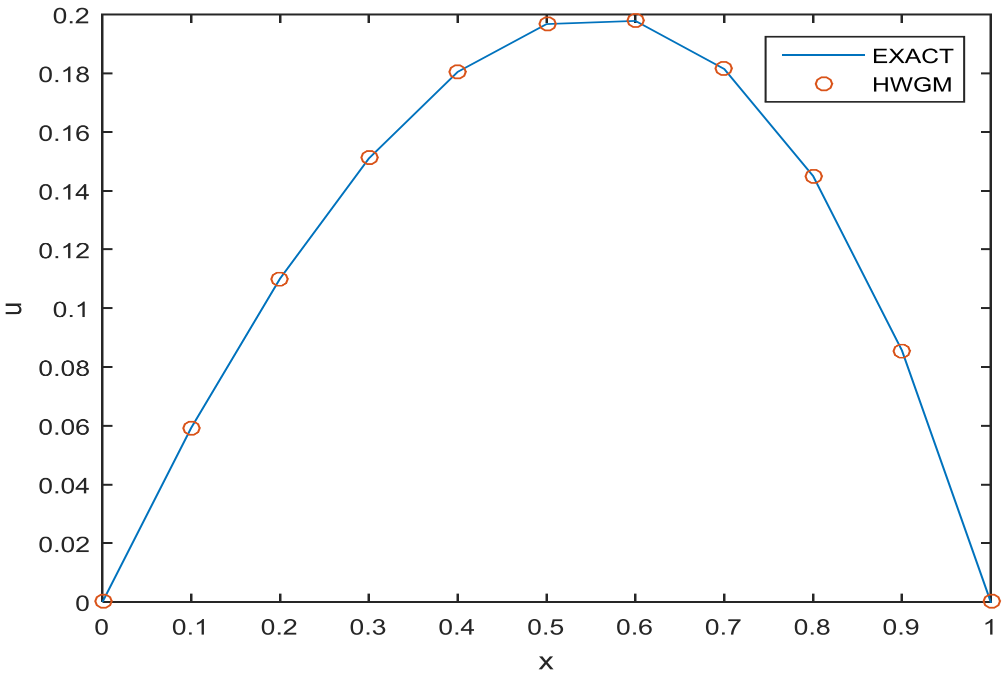

Fig – 3: Comparison of numerical and exact solutions of the test problem 4.3.

Test Problem 4.4 Now, consider singular boundary value problem [12]

$$\frac{\partial^2 u}{\partial x^2} + \frac{2}{x}\frac{\partial u}{\partial x} - \frac{2}{x^2}u = 4x^2, \quad 0 \le x \le 1 \tag{4.16}$$

With boundary conditions: $u(0) = 0, \quad u(1) = 0$ (4.17)

Which has the exact solution $u(x) = x^2 - x$.

By applying the method explained in the section 3, we obtain the constants $c_{1,0} = -0.8945$, $c_{1,1} = 0.0047$ and $c_{1,2} = -0.0046$. Substituting these values in eq. (4.5) we get the numerical solution. Obtained numerical solutions are compared with exact and other existing method solutions are presented in table - 4 and fig - 4.

Table - 4: Comparison of numerical solution and exact solution of the test problem 4.4.

x	Numerical solution		Exact solution	Absolute error	
	FDM	HWGM		FDM	HWGM
0.1	-0.011212	-0.091865	-0.090000	7.88e-02	1.90e-03
0.2	-0.027274	-0.162047	-0.160000	1.33e-02	2.00e-03
0.3	-0.044247	-0.211369	-0.210000	1.66e-02	1.40e-03
0.4	-0.060551	-0.240457	-0.240000	1.79e-02	4.60e-04
0.5	-0.074699	-0.249739	-0.250000	1.75e-02	2.60e-04
0.6	-0.084704	-0.239439	-0.240000	1.55e-02	5.60e-04
0.7	-0.087649	-0.209587	-0.210000	1.22e-02	4.10e-04
0.8	-0.079213	-0.160010	-0.160000	8.08e-02	1.00e-05
0.9	-0.053056	-0.090338	-0.090000	3.69e-02	3.40e-04

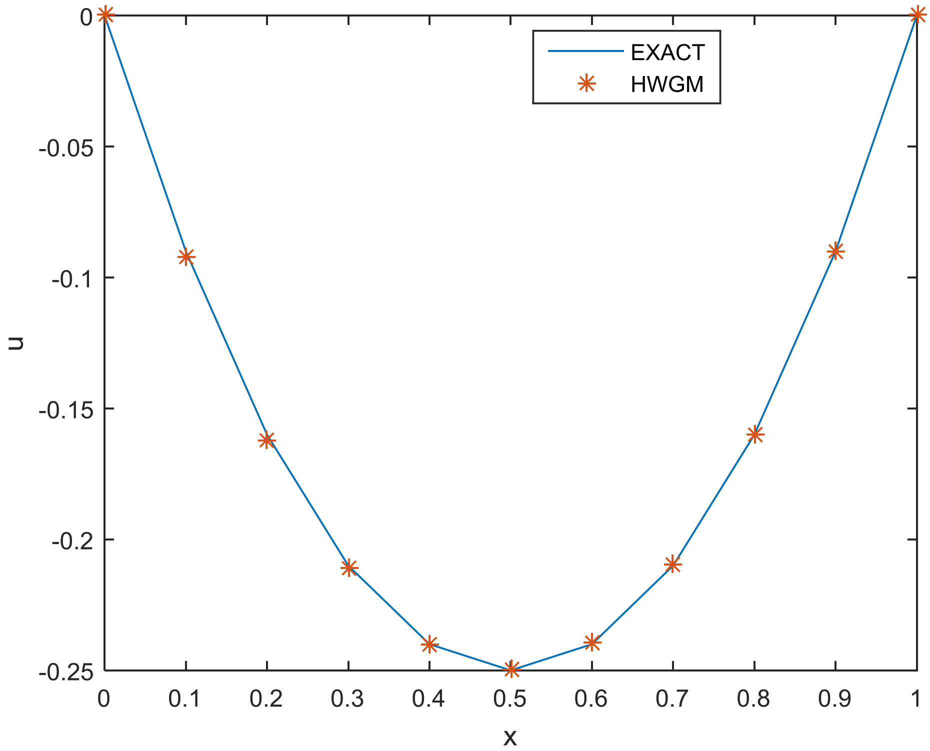

Fig – 4: Comparison of numerical solution and exact solution of the teat problem 4.4.

Test Problem 4.5 Finally, consider another singular boundary value problem [14]

$$\frac{\partial^2 u}{\partial x^2}+\frac{8}{x}\frac{\partial u}{\partial x}+xu = x^5-x^4+44x^2-30x, \quad 0 \le x \le 1 \tag{4.16}$$

With boundary conditions: $u(0)=0, \quad u(1) = 0$ (4.17)

Which has the exact solution $u\,(x)=-x^3+x^4$.

By applying the method explained in the section 3, we obtain the constants and substituting these values in eq. (4.5) we get the numerical solution. Obtained numerical solutions are compared with exact and other existing method solutions are presented in table - 5 and fig - 5.

Table – 5: Comparison of numerical solution and exact solution of the test problem 4.5.

x	Numerical solution		Exact solution	Absolute error	
	FDM	HWGM		FDM	HWGM
0.1	0.024647	-0.000900	-0.000900	2.55e-02	0
0.2	0.024538	-0.006401	-0.006400	3.09e-02	1.00e-06
0.3	0.016024	-0.018904	-0.018900	3.40e-02	4.00e-06
0.4	-0.000072	-0.038407	-0.038400	3.83e-02	7.00e-06
0.5	-0.022021	-0.062512	-0.062500	4.05e-02	1.20e-05
0.6	-0.045926	-0.086417	-0.086400	4.05e-02	1.70e-05
0.7	-0.065532	-0.102920	-0.102900	3.74e-02	2.00e-05
0.8	-0.072190	-0.102420	-0.102400	3.02e-02	2.00e-05
0.9	-0.054840	-0.072914	-0.072900	1.81e-02	1.40e-05

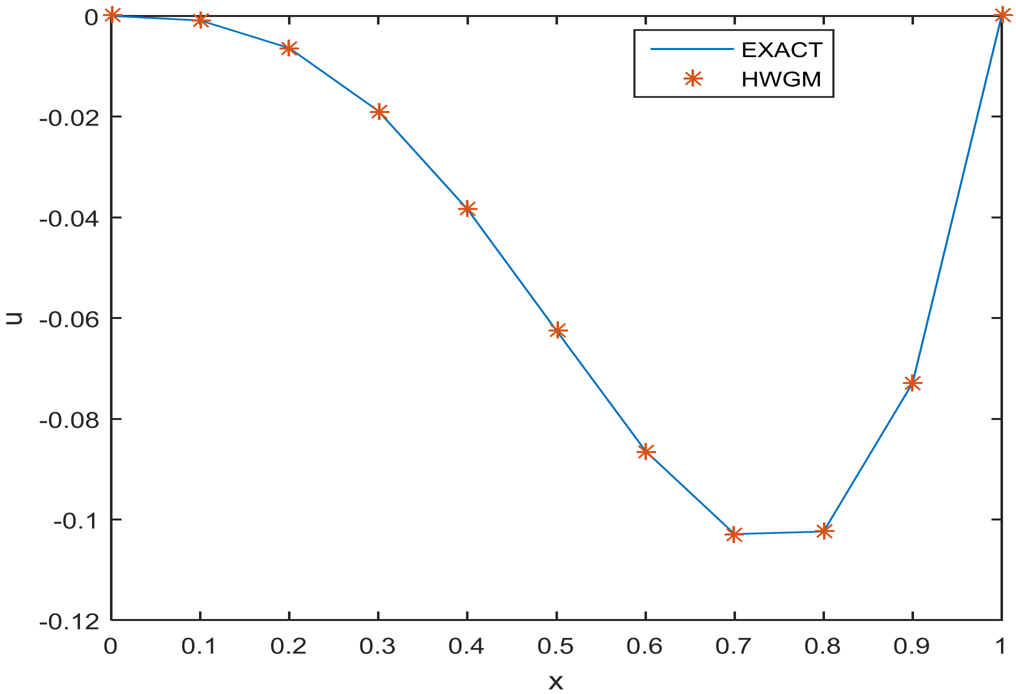

Fig - 5: Comparison of numerical solution and exact solution of the teat problem 4.5.

5. CONCLUSION

In this paper, we proposed the wavelet based Galerkin method for the numerical solution of one dimensional partial differential equations using Hermite wavelets. The efficiency of the method is observed through the test problems and the numerical solutions are presented in Tables and figures, which show that HWGM gives comparable results with the exact solution and better than existing numerical methods. Also increasing the values of M, we get more accuracy in the numerical solution. Hence the proposed method is effective for solving differential equations.

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